Puzzle Corner

INTRODUCTION

This being the first issue of a calendar year, we offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 2) and the arithmetic operators. The problem is stated below. Due to space considerations the solution to the 2001 yearly problem does not appear here, but it can be obtained from the editors or at http://allan.ultra.nyu.edu/~gottlieb/tr/Y2001.pdf.

PROBLEMS

Y2002. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 2 exactly once each and the operators +, -, * (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 2 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

March 1. A detective story from Larry Kells: An unusual occurrence at my duplicate Bridge club the other night left us with a mystery concerning who was the winner of that session. By the later rounds of play, it was a neck-and-neck battle between two long-time rival pairs. The score was so close that the outcome depended on the result of the final board at one last table where the play was slow to finish. The exact matchpoint scores of the two contending pairs were riding on the play at that one table, and the result would decide the winner. As the other players finished, some of them started gathering around that table while the tension in the air continued to mount. But shockingly, just as soon as the hand was finished, the four who were playing that table jumped up and bolted without even writing down their score. Nobody had ever seen them before, and nobody ever saw them again. They vanished into the night.

The director questioned the witnesses trying to find out what the result had been at that table so he could score it. But nobody seemed to know in full what had happened. The two contending pairs gave conflicting reports. Finally, after sorting out who was impartial and reliable, the director was able to ascertain the following facts concerning what had occurred:

1. North and South were vulnerable.
2. North was declarer.
3. North and South bid and made game (but it was unknown whether there were any overtricks).
4. The contract was not doubled.
5. All four deuces won tricks.
6. There were no irregularities in the play.

The director peered over the scoresheet and said this was not enough information to determine the score accurately enough to declare a winner. Or was it? Is this enough information to determine with certainty the exact score on the hand? Also, do you have any clue why the four players might have bolted as soon as they were done?

March 2. Roy Sinclair's favorite possession is a frictionless hemisphere of radius R that he received as a gift from his old physics professor K. Kringle. He keeps this gift on a perfectly horizontal table (with the circular edge down). Yesterday he placed a small mass m very near the top of the hemisphere and let go. How far did m travel before leaving the hemisphere and where did it land on the table?

SPEED DEPARTMENT

George Peckar has a two-room house. In one room there are three wall switches in the off position. In the other room there are three lamps, each one controlled by one of the three switches. You are in the hallway between the rooms and may visit each room only once. Determine which switch controls which lamp.

SOLUTIONS

Oct 1. We begin with a Bridge problem from Larry Kells, who writes that this problem was solved at a tournament table by exactly one declarer, to the amazement of onlookers. Kells believes that he would never have solved it at the table and wonders if you will be able to solve it away from the table assuming that after the opening lead the defenders play perfectly.

North	South
♠ AQ9	♠ J108542
♥ AQ75	♥ K
♦ A87	♦ 3
♣ 743	♣ AK852

After a 1NT opening by North, the dealer, and a weak 3-diamond jump overcall by East you have arrived at 6 spades. Your side is vulnerable and the opening lead is the diamond king. Plan the play to give yourself the best possible chance of success assuming that East is neither overly cautious nor overly aggressive with his preemptive bids.

The proposer sent us a detailed analysis that we unfortunately lack the space to print. A copy can be obtained from the editors or at http://allan.ultra.nyu.edu/~gottlieb/tr/2001-oct-1.pdf.

Oct 2. Nob Yoshigahara reports that no less than Don Knuth likes the following problem in which you are to replace each * with a different digit from 1-9 to yield a true equation (each ** is a two-digit number).

$$\frac{*}{**} + \frac{*}{**} + \frac{*}{**} = 1$$

Many readers solved this by writing a program that tried all possibilities. Philip Latham reports that the problem was popular with his students and with other math teachers at the high school. The following "hand solution" is from Donald Aucamp.

The final solution is 9/12 + 5/34 + 7/68 = 1, which I got in my second trial using the method briefly outlined below:

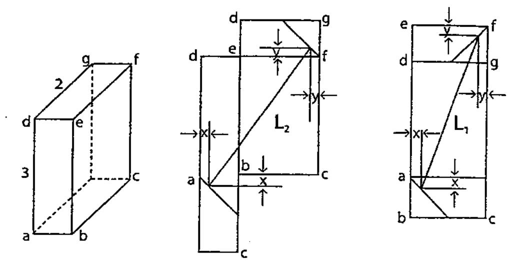
This problem is of the form $a/b + c/d + e/f = 1$. Suppose we arbitrarily assume $b < d < f$. Note that most trial solutions turn out to be less than unity. Thus, to reach unity or higher, we must match up large numerators with small denominators. It is easy to see, for example that $12 \le b \le 17$ is required, since 9/18 + 7/24 + 6/35 = .963 is the largest possible value when $b \ge 18$.

Now reduce all 3 fractions to their lowest terms. For example, $a/b = 9/12$ gets reduced to $A/B = 3/4$, and similarly c/d becomes C/D and e/f becomes E/F. Since $C/D + E/F = (CF+ED)/(DF)$ it is clear that any single reduced denominator, such as B, must be a factor in the product of the other two reduced denominators. Also, a fortiori, any given reduced denominator must be a factor of the product of the other two non-reduced denominators. Thus, if $B = 4$, then df must be divisible by 4. This is a necessary but not a sufficient condition. We now use this restriction, along with the high numerator/low denominator restriction, to find the solution to this problem on the second try.

We start with the highest possible fraction, namely a/b = 9/12, which yields $A/B = 3/4$. This fraction is not only the highest possible (thereby giving us the greatest flexibility in the choice of the other fractions), it also leaves us with very easily met denominator requirements for d and f (i.e., it is easy to choose df to be divisible by 4). The easiest way to meet all the constraints is to try $d = 2K$ and $f = 4K$, where K is any integer. Then, if the reduced values are unchanged (i.e:, $D = 2K$ and $F = 4K$), we meet the necessary conditions since each reduced denominator ($B = 4$, $D = 2K$, and $F = 4K$) is a factor of the product of the other two denominators. Next, in an attempt to match the highest possible remaining numerators with the lowest possible remaining denominators, we set $c/d = 8/34$. As $f = 2d$, this forces us to set $f = 68$, which in turns requires us to set $e = 5$ (the only remaining number). This yields the following trial solution: 9/12 + 7/34 + 5/68 = 1.029, which exceeds unity. The only other possiblity with $a/b = 9/12$ and these denominators is to set c = 5 and $e = 7$, which yields the correct answer, 9/12 + 5/34 + 7/68 = 1. Incidentally, had we not started with $b = 12$, we would have quickly been able to rule out 13, 14, 15, 16, and 17 as candidate values for b, so we would have quickly arrived at $b = 12$ as the only possibility.

Oct 3. Richard Hess has sent us a two-part problem entitled "The Spider and the Fly." (a) A spider sits in the corner of a 1 x 2 x 3 box and can only crawl over the walls of the box. Where should a fly position itself to cause the spider the longest crawl, and how long is this crawl? (b) Now assume the fly can position both the spider and itself. What should he do to maximize the crawl and how long is it?

Eugene Sard, using the solution to 1999 S/O 3 as a guide, was able to combine both parts of Hess's problem. The original box and two developments of the box faces are shown below. The ends of the developed straight crawl lines L_1 and L_2 lie on 45° lines drawn from diagonally opposite corners a and f, with the distances x and $y < 1/2$ to ensure no other shorter paths. Then, $L_1^2 = (2-x-y)^2 + (4+x-y)^2$ and $L_2^2 = (3-x-y)^2 + (3+x-y)^2$. Setting $L_1^2 = L_2^2$ as in 1999 S/O 3 gives $y = (2x+1)/(2x+6)$. The limiting cases are $x = 0$, $y = 1/6$, which answers part (a), and $x = 1/2$, y = 2/7, which answers part (b). The corresponding crawl lengths are $\sqrt{18^1/_{18}} = 4.249$ and $\sqrt{19^{23}/_{98}} = 4.386$.



BETTER LATE THAN NEVER

2000 N/D 3. Art Delagrange writes that Philip Cassady (*TR* October 2001) has solved a puzzle different from the one he intended, i.e., the problem was not specified carefully enough. What Cassady says is true for a propeller plane. There the prop has to turn faster at higher speeds to provide the same thrust, requiring higher power. Delagrange was thinking of a rocket-like engine which provides a given thrust, independent of the speed of the plane. There the power delivered is proportional to the plane's speed, cancelling out Cassady's extra term. Delagrange never got closer to course XVI than Air Force ROTC, but he believes a jet engine is close to a rocket. It is rated in terms of thrust; power can be specified only in a given airframe, as it depends on the maximum speed.

OTHER RESPONDENTS

Responses have also been received from D. Aucamp, R. L. Bishop, G. Blum, N. Borland, E. Collins, C. Costea, C. Dale, D. Dechman, R. C. Ellis, J. Feil, C. Garrett, J. Grossman, R. I. Hess, R. L. Bishop, P. Latham, M. Lindenberg, J. Mahoney, R. Marks, L. McGee, B. McLaughlin, J. K. Menendez, R. A. Moeser, C. E. Muehe, B. Norris, M. Ober, A. Ornstein, A. Taylor, A. P. Taylor, B. Peak, J. Peltier, K. L. Rosato, C. Sameck, E. W. Sard, P. Sheu, H. Snyder, J. D. Teare, H. Thiriez, M. Turner, D. Wellington, and P. A. Wyrick.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Enter the room with the three switches. Turn switch one on and wait a minute. Turn switch one off and turn switch two on. Enter the other room. The lamp that is on is controlled by switch two, the lamp with a warm bulb is controlled by switch one, and the remaining lamp is controlled by switch three.

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 7th floor, New York NY 10003, or to gottlieb@nyu.edu.