Puzzle Corner

INTRODUCTION

I write this column during a tragic time for the world in general and for my fellow Baker House alumni in specific. There is nothing I can add to the horrible story of the terrorist attacks against NYC and Washington, DC, in September or the heroic acts that immediately followed.

This past summer a Baker colleague of mine, Dave Lapin, was killed in an automobile accident, and his wife, Sue, remains in intensive care. This is especially hard on me as Sue was a year behind me in high school and I introduced Dave and her 35+ years ago. I dedicate this column to Dave's memory, to Sue's forthcoming complete recovery, and to her strength in adjusting to a new life.

Let me end on a happy note by remarking that our longtime contributor, Mary Lindenberg, was named the March 2001 "Artist of the Month" by the Fall River, MA, Gueguechan Club.

PROBLEMS

Dec 1. Robert Bart notes that in the hand below South's opening 4-spade bid was passed out. He wonders whether, assuming best play on both sides, the contract will succeed or fail.

North
♠ 9 3
♥ 10 7
♦ Q 10 7 4 2
♣ A K 6 5

West
♠
♥ K Q 9 8
♦ A J 8 6
♣ J 10 7 4 2

East
♠ 10 8 5 4 2
♥ A J 5
♦ K 9 5 3
♣ Q 9

South
♠ A K Q J 10 5 2
♥ 6 4 3 2
♦
♣ 8 3

Dec. 2. Victor Barocas and Eric Lehman have a circle of radius R>>1. Can they fit in more squares of area 1 or equilateral triangles of area 1?

Dec. 3. Norman Spencer has a one-inch cube that he wants to give as a present. What is the area of the smallest rectangular piece of wrapping paper that can be folded to cover the cube? No cutting is permitted.

SPEED DEPARTMENT

Walter Cluett has a sequence of seven consecutive two-digit positive integers, the first, third, and seventh of which have a numerical property that the other four do not have. What are the numbers and what is the property?

SOLUTIONS

J/A 1. Larry Kells wants you to construct a hand in which South can make 5 diamonds despite an opponent holding KJ97543 of diamonds and 14 high-card points overall.

The following solution is from MIT economics professor Robert Bishop.

North
♠ 6 5 4 3 2
♥ A K 6 5 4
♦ 2
♣ A K

West
♠ 10 9 8 7
♥ 10 9 8 7
♦
♣ 10 9 8 7 6

East
♠ K J
♥ Q J
♦ K J 9 7 5 4 3
♣ Q J

South
♠ A Q
♥ K 3 2
♦ A Q 10 8 6
♣ 5 4 3 2

In winning the ace and king of both hearts and clubs, declarer uses three of these four entries to dummy for (1) a spade lead to win both the ace and queen, (2) a trump lead to just top whatever East plays, and (3) a return to dummy to keep the lead there after winning the first seven tricks. With East reduced to just six trumps, a spade or heart lead allows South to win another cheap trump trick. South now endplays East twice, throwing him in each time with a club lead and winning cheaply East's forced trump return. South also wins the last trick with a high trump.

J/A 2. Ramon Mireles enjoyed this problem from Long's elementary Number Theory. Prove that if p is a prime not 2 or 5, then p must divide infinitely many of the numbers 9, 99, 999, ...

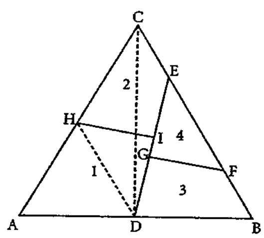
I was torn between using short solutions employing Fermat's little theorem and longer solutions that did not, eventually deciding on the latter, exemplified by Michael Fisher's response. Rino Sánchez notes that similar arguments show that any sequence of repeating numbers such as {23, 2323, 232323, ...} also has, for any prime p unequal to 2 or 5, infinitely many members divisible by p. Fisher writes,

"If p divides a number of the form $m = k \times \sum_{i=0}^{j} 10^{(n+1)i}$, then p also divides all numbers of the form $k = 9 \times \sum_{i=0}^{n} 10^i$, for any positive integer j. Thus if p divides k, then it also divides an infinite subsequence of {9, 99, 999, ...}. So it suffices to show that p divides a single number of the form k. Since k has a factor of 9, the prime $p = 3$ is shown to work for every element in the sequence, and we can consider only numbers of the form $q = k/9$. Thus, we must show that p divides some number of the form 111...1. Consider the set of elements {1, 11, 111, ..., 111...1}, where $e_1 = 1$, $e_2 = 11$, etc., and the largest element e_p has p digits, all 1's. One of these elements must be divisible by p, i.e., equal to 0 mod p. If not, since there are p elements in the set, then at least two of them, e_i and e_j, must be equal mod p. Then, assuming $j > i$, $e_j - e_i = 0 \bmod p$.

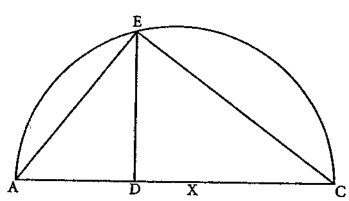
But $0 \bmod p = e_j - e_i = 10^i \times \sum_{h=0}^{j-i} 10^h = 10^i \times f$. Thus p divides $10^i \times f$. Since p is a prime not 2 or 5, p does not divide 10^i, and so must divide f. But f is a member of the set {1, 11, 111, ...,111...1}. QED."

J/A 3. Roy Sinclair wants you to cut an equilateral triangle with straight line segments into 4 pieces and reassemble the pieces to form a square. He believes that this was first solved by H. E. Dudeney in 1902.

I received solutions that constructed a family of cuts all of which gave rectangles and one of which necessarily gave a square. However, determining this particular cut seems to require more than ruler and compass. Robert Miller sent us the following complete solution:



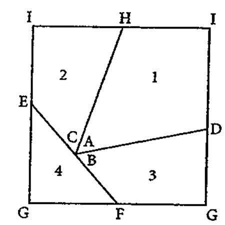
Construction of Triangle (Fig. 1):
1. Construct equilateral triangle ABC.
2. Determine midpoint D of side AB.
3. Erect altitude DC.
4. Area of equilateral triangle ABC = AD × DC.



Construction of Side of Square (Fig. 2):
5. Construct line ADC of segments AD (1/2 side of triangle ABC) + DC (altitude of triangle ABC).
6. Determine midpoint X of line ADC.
7. Construct semicircle about X passing through points A and C.
8. Erect perpendicular DE to line ADC at point D intersecting semicircle at point E.
9. Connect AE and CE forming right triangle AEC.
10. Triangles ADE and CDE are similar to triangle AEC and to each other.
11. AD:DE = DE:DC
12. DE × DE = AD × DC
13. DE = side of square of area = area of equilateral triangle ABC.

Division of Equilateral Triangle (Fig. 1):
14. Construct line DE (of length = side of square, Fig. 2) from midpoint of side AB of triangle ABC, intersecting side BC of triangle ABC at point E.
15. Determine point F on side BC of triangle ABC such that segment EF = 1/2 side BC.
16. Erect line FG at point F perpendicular to DE and intersecting DE at point G.
17. Determine midpoint H of side AC of triangle ABC.
18. Erect line HI at point H perpendicular to DE and intersecting DE at point I.
19. Identify four pieces of equilateral triangle ABC as follows:

Quadrilateral	ADIH – piece #1
Quadrilateral	CEIH – piece #2
Quadrilateral	BDGF – piece #3
Triangle	EGF – piece #4



Construction of Square (Fig. 3):
20. Rotate piece #2 about point H so that side CH of piece #2 coincides with side AH of piece #1, and side IH of piece #2 and side HI of piece #1 form equal segments of line IHI.
21. Rotate pieces #3 and #4 about point D so that side BD of piece #3 coincides with side AD of piece #1 and side GD of piece #3 and side DI of piece #1 form segments of line GDI.
22. Rotate piece #4 about point F so that side FE of piece #4 coincides with side FB of piece #3 and side CE of piece #2; side GE of piece #4 and side EI of piece #2 form segments of line GEI; and side GF of piece #4 and side FG of piece #3 form equal segments of line GFG.

BETTER LATE THAN NEVER

Mar 3. Matthew Fountain notes that more generally any triangle may be cut and the pieces arranged to form two triangles similar to it with no restriction on the ratio of the sizes (of course the areas must sum to the original). The proof can be found in *Recreational Problems in Geometrical Dissections and How to Solve Them.*

OTHER RESPONDENTS

Responses have also been received from J. Andresen, J. Bross, M. Fountain, J. Harmse, E. Kutin, M. Lindenberg, T. Maloney, J. Manson, R. Millar, M. Moss, A. Peralta, H. Sard, and G. Webb.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The numbers are 41, 42,..., 47, and the property is primality. [There are other sequences (e.g., 11, 12,..., 17) as well —ed.]

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York NY 10003, or to gottlieb@nyu.edu.