Puzzle Corner

INTRODUCTION

This being the first "Puzzle Corner" issue of the calendar year, we again offer a "yearly problem," in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 1) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2000 yearly problem is in the "Solutions" section.

PROBLEMS

YEAR 2001. How many integers from 1 to 100 can you form using the digits 2, 0, 0 and 1 exactly once each and the operators +, –, x (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 2, 0, 0 and 1 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

MARCH 1. Ken Fan notes that one can cut a square along both diagonals to obtain four congruent triangles and then pair up the triangles along their long sides to obtain two congruent squares. He is interested in the same problem for triangles. Specifically, can one start with an equilateral triangle, cut it into pieces, and rearrange them to form two congruent equilateral triangles? If so, how? If not, why not?

MARCH 2. John Sampson enjoys Pythagorean triples (positive integers a, b and c satisfying $a^2 + b^2 = c^2$). He wants you to show that every such triple has one number divisible by 3, one number divisible by 4 and one number divisible by 5. For example, in the triple (5, 12, 13), 12 is divisible by 3, 12 is divisible by 4, and 5 is divisible by 5.

SPEED DEPARTMENT

You can think of this as the yearly problem on steroids. Larry Kells wants you to express the quantity e^{π} using only a finite number of integers and the same five arithmetic operators as in the yearly problem.

SOLUTIONS

YEAR 2001. There are clearly very few solutions for the year 2000. It starts getting better with the new year, 2001. I am not a fan of declaring $0^0 = 1$ and decided not to permit leading zeros (which helps very little). So the best we have is:

$$1 = 200^0$$
$$2 = 2+0+0+0$$
$$20 = 20+0+0$$

S/O 1. Here is a rather weird bridge problem from Larry Kells:

North-South have arrived in a contract that is makeable with best play and defense for almost all distributions of cards between their opponents. Indeed, the probability of defeat (again with best play and defense) is greater than 0 but less than 1 in 10 million. What sort of hands can they hold?

Since 26C13 is 10,400,600, there must be only one possible distribution for the odds to exceed 1 in 10,000,000. The following solution is from Michael Andresen:

North
♠ void
♥ KQ.....32
♦ void
♣ A

West
♠ K
♥ void
♦ KQ.....32
♣ void

East
♠ 2
♥ void
♦ void
♣ KQ.....32

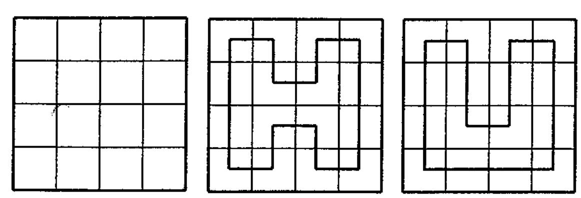
South
♠ AQJ.....43
♥ A
♦ A
♣ void

E: 6C S: 6S W, N: all pass

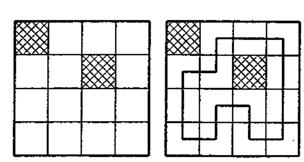
West leads any diamond. East ruffs with the spade 2 and returns any club, giving West an uppercut and allowing him to score his spade king. Any other combination of cards distributed between East and West will allow South to make his contract.

S/O 2. Nob Yoshigahara attributes the following problem to Professor Kotani.

In the 4 x 4 room shown below, there are two types of grand tour routes that visit each room exactly once.

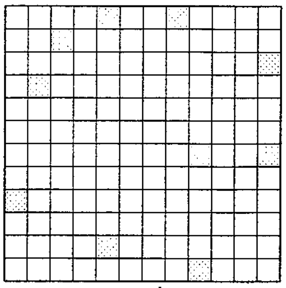


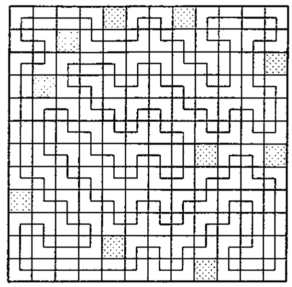
Including their rotations, six routes are possible. But if two rooms are closed, as in the figure shown below, only one route remains.



In a 12 x 12 room (figure next page, on left), 10 rooms are closed. Find its grand tour route. The solution is unique!

John Goodman submitted a lovely solution to this problem (figure next page, on right) and kindly placed a ZIP archive of the graphics at www.qlam.com/puzzle.zip.





Here, we provide only the final answer he found; in his ZIP archive, he goes through the step-by-step process of solution.

S/O 3. Ernest Steel wants to know the resistance between diagonal nodes of a four-resistor loop in a two-dimensional infinite lattice of identical resistors.

This is not an easy problem. The solutions from Tim Barrows and my NEC research colleague Warren Smith (with help from Richard Linke) look reasonable to me, but my only course VI experiences were one computer software course and my junior year in Baker House, when I was near an EE wizard who built transistor-driven audio amplifiers before they were commercially available. Neither of these activities helped for the present problem.

Barrows, through some clever circuit arguments, produced a matrix equation encoding Kirchoff and Ohm's laws for a grid of finite size. When solved for reasonably large grids, the resistance appears to converge to $2/\pi$ ohms.

Smith also solved the Kirchoff-Ohm equations on finite sized grids, which again gave answers very close to $2/\pi$. He then pulled out the big guns and presented an analytic method to obtain the resistance between the nodes (0,0) and an arbitrary node (A, B): the voltage at each node is a simple function of the voltage at its four neighbors. When a discrete Fourier transform is applied, the resulting function can be solved in closed form. The original voltage requires an inverse transform, which yields:

The resistance from (0,0) to (A,B) equals the double integral, as x goes from 0 to 1 and as y goes from 0 to 1, of:

$$\frac{1 - \cos(2\pi(xA + yB))}{2 - \cos(2\pi x) - \cos(2\pi y)}$$

For A = B = 1, the computer algebra program MAPLE evaluates this integral to $2/\pi$. For general A, B Smith solves the inner integral via the Cauchy residue theorem, but the outer integral remains difficult.

Space constraints do not permit printing either solution in its entirety. Barrows's can be obtained by writing to the editor. Smith is writing his up and it will be available both in hard copy from the editor or in electronic form from Smith himself at wds@research.nj.nec.com. Interested readers are also encouraged to see Flander's paper in *Math. Anal. and Appl.*, 1972, pp. 30-35.

BETTER LATE THAN NEVER

2000 M/J 2. I erred in saying that Stern *corresponded* with "a freshly minted MBA." That communication was, in fact, by telephone.

M/J 3. Tom Harriman and others believe we did not do justice to this problem. Harriman writes: "M/J 3 was a fascinating and heuristic exercise. Too bad all of our reported submittals treated it as a 'Speed Problem!' Its two elegant concepts were (A) that an arc most efficiently partitions off a required area bounded by the sides at a vertex, and (B) that a node should have 120 degrees between adjacent legs (bees and honeycomb). The solution quoted for triangle into four parts was inefficient by inspection! (Its aggregate dissector of 1.429 isn't even close to the optimum 1.305.) Square into five parts as quoted didn't even begin to take advantage of (B), and therefore missed by 2.524 to 2.502. The last, triangle into five parts, was just plain naive in using straight dissectors—missed by 1.701 to 1.602." Copies of T. Harriman's, F. Morgan's and Bleicher's contributions can be obtained by writing to *Technology Review.*

M/J SD. Warren Himmelberger and Alan Taylor found other words where no re-arranging of letters is needed: dragoons, dragons, dragon, drago, drag, rag, ra, a; startling, starting, staring, string, sting, sing, sin, in, i.

OTHER RESPONDERS

Responses have also been received from M. Ahlgren, J. Andresen, M. Attlil, C. Blair, S. Brown, N. Cohen, R. Currier, C. Dale, D. Dechman, A. Delagrange, J. Feil, E. Feltin, J. Goodman, J. Grossman, M. Haggerty, J. Harmse, T. Harriman, R. Hess, H. Hodara, E. Karaca, M. Kunitake, M. Lindenberg, B. Martin, F. Morgan, A. Ornstein, G. Perry, J. Piacente, K. Rosato, E. Sard, A. Tracht, T. Wang and D. Wellington.

PROPOSER'S SOLUTION TO SPEED PROBLEM:

$(-1)^{-((-1)^{1/2})}$

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu.

— Edited by Owen W. Ozier '98

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