Puzzle Corner

INTRODUCTION

Since this is the first issue of a new academic year, let me once again review the ground rules under which this column is conducted.

In each issue I present three regular problems (the first of which is often chess, bridge, go, or computer-related) and one "speed" problem. Readers are invited to submit solutions to the regular problems and two issues later, one submitted solution is printed for each regular problem; I also list other readers who responded. For example, solutions to the problems you see below will appear in the January/February 2001 issue and the current issue contains solutions to the problems posed in the May/June 2000 issue.

I am writing this column in late June and hence anticipate that the January/February 2001 column will be due in late October. Please try to send your solutions early to ensure that they arrive before my submission deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the January/February issue of each year, and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

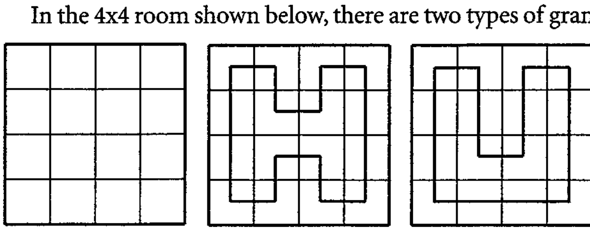
PROBLEMS

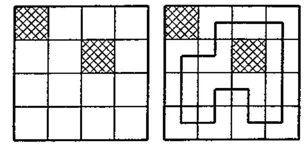
S/O 1. Here is a rather weird bridge problem from Larry Kells.

North-South have arrived in a contract that is makeable with best play and defense for almost all distribution of cards between their opponents. Indeed, the probability of defeat (again with best play and defense) is greater than 0 but less than 1 in 10 million. What sort of hands can they hold?

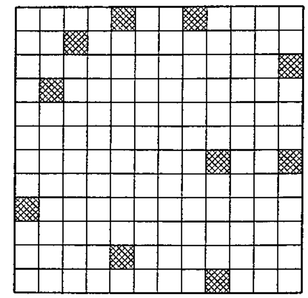
S/O 2. Nob Yoshigahara attributes the following problem to Professor Kotani:

In the 4x4 room shown below, there are two types of grand tour routes that visit each room exactly once. Including their rotation, six routes are possible. But if two rooms are closed, as in the figure at right, only one route remains.





In the 12×12 room figure at bottom right, 10 rooms are closed. Find its grand tour route. The solution is unique!



S/O 3. Ernest Steel wants to know the resistance between diagonal nodes of a four-resistor loop in a two-dimensional infinite lattice of identical resistors.

SPEED DEPARTMENT

Robert Bishop wants you to shrink the word "DRAGOONS" one letter at a time to a single letter, rearranging the remaining letters at each stage to form another English word. You are asked to find two solutions that share no common words except for DRAGOONS.

SOLUTIONS

M/J 1. Jerry Grossman sent us the following bridge problem that he reports actually occurred:

West	North	East
	♠ Ax	
♠ xx	♥ AJxx	♠ xx
♥ Qxxx	♦ xxx	♥ Kxx
♦ Jxx	♣ A10xx	♦ Qxxx
♣ Jxxx		♣ Qxxx
	South	
	♠ KQJxxxx	
	♥ xx	
	♦ AKx	
	♣ K	

The contract is six spades with South declarer, and the opening trick is low club, low from dummy, queen from East, king. This enables you to make the hand. How?

I am sorry to say that the intention was to make seven spades. Fortunately, a number of readers figured this out. The following grand-slam solution is from Robert Bishop.

After cashing dummy's trump ace and club ace, discarding a heart, declarer runs all of his trumps. His remaining cards are as follows:

North	South
♠ -	♠ -
♥ AJ	♥ x
♦ x	♦ AKx
♣ 10	♣ -

As one possible defense, East can keep K x of hearts, thereby giving up a third diamond but allowing West to keep J x x of diamonds and the club jack. This fails when declarer leads his small heart, squeezing either a fatal diamond or club discard from West.

Alternatively, East can keep just the heart king along with three diamonds. West then keeps the Q x of hearts, the club jack and one other card. But this also fails when declarer leads the diamond ace and king, again fatally squeezing West to discard either a heart or the club jack.

M/J 2. I became lactose intolerant a few years ago and so the following problem from Ken Rosato is painful for me to type.

Recently, a pizza company advertised a special deal offering two pizzas, each with up to five toppings, for one low price. They added that the pizzas need not have the same toppings on them, and that there were 1,048,576 possible different combinations of two pizzas. Assuming a topping can be used only once per pizza, how many different toppings does the pizza company have?

Seth McGinnis appears to be a real pizza aficionado as his solution below will demonstrate. Howard Stern remembers well when Little Caesars ran this promotion and sent me copies of his correspondence with them, in which he explained that they should be including repetitions, i.e. double, triple, etc., portions of the same topping. He corresponded with "a freshly-minted MBA/Marketing type from Corporate Communications with no mathematical ability...[who] didn't even give me a coupon for a free pizza for all the work I did!!!" Readers wishing a copy of the Stern correspondence should contact Jon Paul Potts, senior editor of the *MIT News.* McGinnis' solution follows:

The answer is 11, but the pizzas must be distinguishable. With 11 toppings, there are:

$$\sum_{i=0}^{5} \binom{11}{i} = 1+11+55+165+330+462 = 1024 = 2^{10}$$

different topping combinations possible. (Be sure not to forget that "none" is a valid topping choice, or it's only 1023 combos.) If you purchase two pizzas with up to five toppings each, that's $2^{10} \times 2^{10} = 2^{20} = 1{,}048{,}576$ possible orders. But unless you buy your pizzas from someplace like Little Caesar's, where they'll give you one square one and one round one, an order of "one plain pepperoni, and one bacon-avocado-prawn-squid-muskmelon" is indistinguishable from an order of "one bacon-avocado-prawn-squid-muskmelon and one plain pepperoni", so instead of 1024 x 1024 total orders, there would only be $(1024+1) \times (1024/2) = 524{,}800$ different possible orders.

M/J 3. We end with a dissection problem from Richard Hess that, as far as I can tell, has nothing to do with gross anatomy.

For a unit-sided equilateral triangle, what is the minimum cut-length to dissect it into four parts of equal area? For a unit square, what is the minimum cut-length to dissect it into five parts of equal area? What about an equilateral triangle dissected into five parts?

I received no proofs of minimality so better answers are possible. We assume the given triangles and squares have side-length of 1. For the first part, Ken Rosato chose circular arcs centered at the three vertices each cutting off pieces of area 1/4. Their total cut length is 1.429. For the second part, Rosato and Charles Muehe cut out a square of area 1/4 from the middle of the given square and rotated 45 degrees. They then cut from each vertex of the removed square to the midpoint of the nearby side of the original square. The total cut length is 2.524.

For the last part, Muehe first chose a vertex of the original triangle and cut off a triangle of area 1/5 having the chosen vertex and having base parallel to the base of the original triangle. He then cut off an additional 2/5 of the initial triangle's area with a cut parallel to its base, and bisected the two areas thus produced.

BETTER LATE THAN NEVER

J/F 1. We should have made clear in the problem itself that the hyperspheres have radius 1/2.

M/J SD. Timothy Rueger points out that this puzzle first appeared in a 1981 issue of *Games Magazine* and was created by Will Shortz, currently the crossword editor for *The New York Times.*

OTHER RESPONDERS

Responses have also been received from S. Avgoustiniatos, A. Curtis, J. Grossman, J. Harmse, T. Harriman, B. Huntington, H. Ingraham, Jr., L. Iori, P. Latham, M. Lindenberg, B. Margolin, L. Nissim, E. Sard, R. Schweiker, T. Terwilliger and Y. Zussman.

PROPOSER'S SOLUTION TO SPEED PROBLEM

DRAGOONS, DRAGOON, DRAGON, RADON, ROAD, ADO, AD, A.

DRAGOONS, DRAGONS, GROANS, GROAN, ROAN, OAR, OR, O.

Send problems, solutions and comments to: Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu.

— Edited by Owen W. Ozier '98

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