Puzzle Corner

INTRODUCTION

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0 and 0) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1999 yearly problem is in the "Solutions" section..

PROBLEMS

Y2000. How many integers from 1 to 100 can you form using the digits 2, 0, 0 and 0 exactly once each and the operators +, -, x (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; and among solutions having a given number of operators, those using the digits in the order 2, 0, 0 and 0 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator. This year is very difficult due to the three zeros.

J/F 1. Nancy Burstein's son, Richard '02, wonders how many four-dimensional hyperspheres one can fit in a four-dimensional unit hypercube.

SPEED DEPARTMENT

Robert Bishop wants you to shrink the word FRIENDLY one letter at a time to a single letter, rearranging the remaining letters at each stage only as little as necessary to form another English word.

SOLUTIONS

Y1999. The following solution is from John Drumheller.

$1=1^{999}$	19=19+9-9	80=99-19
2=((1x9)+9)/9	20=19+(9/9)	81=((1+9)x9)-9
3=1+((9+9)/9)	26=9+9+9-1	82=((1/9)+9)x9
7=9-(9/9)-1	27=(1x9)+9+9	89=99-(1+9)
8=99-91	28=1+9+9+9	90=(19-9)x9
$9=1^{99}$x9	37=19+9+9	91=1+99-9
$10=1^{99}+9$	62=(9x9-19)	92=(9/9)+91
11=(1x99)/9	63=((9-1)x9)-9	$98=99-1^{9}$
12=1+(99/9)	71=(9x9)-(9+1)	$99=1^{9}$x99
$17=(9+9)-1^{9}$	72=(1x9x9)-9	100=19+(9x9)
18=19-(9/9)	73=91-(9+9)	

S/O 1. Larry Kells has a three-part problem entitled "War Returns" that once again involves our poor husband who keeps facing tough luck at the table. I am presenting part one in this issue of *Technology Review.*

Kells writes that, "sometime later I saw the man from the bridge club sitting on a park bench again, sobbing. I asked him what the matter was. He said his wife had just thrown him out of the house for good. During a club session that I had missed, he had three high-level doubled contracts made against him. In each case he had dealt and opened the bidding with sound values for his bid, but in the end, 'even God couldn't sink those contracts!'

"'First, I held the Q108654 of hearts, AK of spades, and 13 points overall,' the man said. 'I opened one heart, my wife responded one spade, and I rebid two hearts. Suddenly the opponents jumped in, and the bidding didn't end until it reached six hearts doubled.'

"'So you didn't have the values to warrant bidding that high.'

"'You don't understand, it was the opponents who bid six hearts, I doubled, and they made it!'

"'Wow! Your wife must have made some stupid mistake in the play.'

"'No, that contract was impregnable! My wife glowered daggers at me when it was over.'"

What deal produced this surprising result? Steve Feldman solved the bridge part of the problem, but offers no marital suggestions.

North
♠
♥ Axx
♦ xxxx
♣ XQxxxx

West
♠ QJ10xxxx
♥
♦ xxx
♣ KJ10

East
♠ AK
♥ Q108654
♦ KJx
♣ xx

South
♠ xxxx
♥ KJ97
♦ AQ10
♣ xx

West leads a diamond (the sequence of the play is only slightly different for other leads). South wins, ruffs a spade, plays a diamond to his hand, finesses the clubs, plays a diamond back to his hand, ruffs another spade, and plays a club winner from dummy.

After this trick, all East has left is his hearts. South plays a club from dummy, East ruffs and South overruffs. Then South ruffs a spade with the ace of hearts, leads a diamond from dummy and overruffs East. South then plays a spade and East wins with the 8, but he then has to lead from the Q10 of hearts into South's KJ.

S/O 2. John Prussing has a problem that I can use when I next want to go bowling with one of the blown glass balls we have around the house.

The moment of inertia of a homogeneous sphere about an axis through its center is $I={}^{2}/5\ M/R^{2}$, where M is the mass of the sphere and R is the radius. The moment of inertia of a (hollow) homogeneous spherical shell can be obtained by subtraction to be $I={}^2/5(M_2R_2 2-M_1R_1 2)$. Determine the moment of inertia of a thin homogeneous spherical shell of mass M and radius R. The thickness of the shell is negligible compared to the radius and so all the mass may be considered at a distance R from the center.

Linda Kalver makes it look pretty easy.

Consider a homogeneous sphere of a given radius R and let $R_1 < R$ be the radius of a concentric sphere. Let M_T (a variable) represent the total mass of the larger sphere. Then, $R_1{}^3/R^3$ is the fraction of the total mass M_T that is contained in the smaller sphere and $(R^3-R_1{}^3)/R^3$ is the fraction of the total mass M_T that is contained in the shell between radius R and radius R_1.

Since, by assumption, the shell always has mass M, we can express M_T as a function of R_1: $M_T(R^3-R_1{}^3)/R^3=M$, or, $MT=MR^3/(R^3-R_1{}^3)$. Hence the mass of the inner sphere, $M_1=MR_1{}^3/(R^3-R_1{}^3)$.

The moment of inertia of the shell is given by $I={}^2/5(M_TR^2-M_1R_1{}^2)$. Substituting for M_T and M_1, we get:

$I={}^2/5M(R^5-R_1{}^5)/(R^3-R_1{}^3)$

$= {}^2/5M(R^4+R^3R_1+R^2R_1{}^2+RR_1{}^3+R_1{}^4)(R^2+RR_1+R_1{}^2)$

For a shell of "negligible" thickness, the moment of inertia is given by the limit of I as R1 approaches R, which is ${}^2/3MR^2$.

S/O 3. Our last regular problem is from David Brahm, who believes it is originally from Yoshiyuki Kotani. An ant travels only along the floor, wall and ceiling of a 1x1x2 room. If he starts in a corner, what is the farthest point away the ant can reach? Measure distance as the ant travels (rather than as the crow flies).

Tim Barrows and John Marcou each avoided the trap of assuming the most distant point is the diagonally opposite corner and sent us beautiful solutions. It is hard to choose between them. I reprint Marcou's solution below; Barrows' can be obtained by request from *Technology Review.*

In Figure 1, the ant is shown to start in the corner labeled *a.* Point *f,* the corner opposite from where the ant begins, is the farthest the ant can travel on the floor and the walls. This is shown in Figure 2, a flattened view of an adjacent wall, the next wall and the ceiling. The distance to Point *f* is $2\sqrt{2}$, or roughly 2.828.

Can a more distant point be found on the ceiling? By symmetry, the farthest point on the ceiling lies directly on the diagonal *df.* The ant could reach diagonal *df* by one of two routes, climbing directly up an adjacent wall (see L_1 in Figure 3), or by starting up an adjacent wall, crossing over to the next wall, and then crossing over to the ceiling (see L2 in Figure 2).

Taking X as a point along the diagonal *df* with X=0 at point *d,* and $X=\sqrt{2}$ at point *f,* then the distance of each route is:

$L_1=\sqrt{(2+(X+\sqrt{2})^2)}$ and $L_2=\sqrt{(8+(\sqrt{(2)}-X)^2)}$

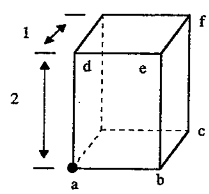

Figure 1: Ant in corner of a 1 x 1 x 2 room

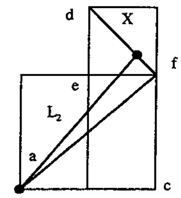

Figure 2: Most direct route to opposite corner, and Route to ceiling over adjacent and next wall

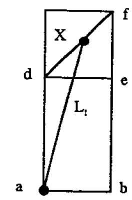

Figure 3: Route to ceiling over adjacent wall

Setting $L_1=L_2$, since neither route can be shorter if the path is most direct, the most distant point is found along the ceiling diagonal at $X = {}^3/2(\sqrt{2})$, which is ${}^3/4$ of the way from point *d* to point *f.* Substitution into the formula for either L_1 or L_2 gives a total distance traveled of about 2.850.

1999 J/F 2. We now have received many solutions to Nob Yoshigahara's "rope problem" from Alan Taylor (alan.taylor@alum.mit.edu) and Don Dechman. One will appear in a future issue, and others can be obtained, by request, from *Technology Review.*

S/O SD. Wesley Moore notes that James Thurber was a devotee of Superghosts, which is like S/O SD: One adds letters and the first to make a word loses. In "Do You Want to Make Something Out of It?", Thurber claims it was played in 1930 and he was particularly exercised by the string "sgra."

OTHER RESPONDERS

Responses have also been received from K. Bernstein, R. Bishop, S. Bragg, W. Cluett, J. Craig, C. Dale, J. Feil, S. Fetter, R. Hess, C. Hibbert, W. Himmelberger, A. Hirshberg, C. Muehe, B. Ornstein, D. Pecora, D. Porter, T. Robertson, K. Rosato, E. Sard, A. Taylor, M. Teague, A. Tracht, A. Wasserman, D. Wellington and C. Willy

PROPOSER'S SOLUTION TO SPEED PROBLEM

FRIENDLY, FLINDER, FINDER, FINER, FINE, FIN, IN, I.

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012 or to gottlieb@nyu.edu.

— Edited by Owen W. Ozier '98

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