Puzzle Corner

INTRODUCTION

I don't often give travel recommendations, but my family and I really enjoyed our cruise along the fabled Alaskan "inside passage." The scenery was spectacular, the whales put on a real show, the glaciers were awe-inspiring, and the stops at Juneau, Skagway and Sitka were worthwhile.

In Juneau, my younger son and I took a helicopter ride to a dogsled camp on a glacier and enjoyed about a 30-minute (three-mile) dogsled ride. Bring on the Iditarod!

PROBLEMS

N/D 1. Robert Bart offers us the following Bridge endgame study in which South, on lead, needs six tricks for his no-trump contract. Are they available?

North
♠ J3
♥ 4
♦
♣ Q642

West
♠ 65
♥
♦ 86432
♣

East
♠
♥ Q
♦ KJ7
♣ J98

South
♠ 92
♥ J
♦ AQ9
♣ 5

N/D 2. Frank Rubin wants you to find a positive integer solution to the equations:

$$a^2 + b^3 = c^4$$

$$a^4 + b^6 = c^7$$

N/D 3. Robert Bishop, MIT professor emeritus of economics, wants you to find the longest English word such that if it is reduced by one letter at a time, another English word can be formed of the remaining letters—down to a final one-letter word. For example: galleons, galleon, gallon, along, long, log, go, o.

SPEED DEPARTMENT

John Mattil HM sent us the following problem, which he attributes to Alan Guth. Arrange these types of electromagnetic radiation in the order of increasing frequency: red light, green light, gamma rays and the broadcast of a rerun of "Cheers," the popular television program from the 1980s that was based in a Boston pub.

SOLUTIONS

J/A 1. A two-part bridge problem from Larry Kells: What is the fewest number of points a pair can hold and still make a game against best defense? What is the most points a pair can hold and still be unable to make a game against best defense?

Richard Hess found that the one-point hand below will make four spades, using a cross-ruff, if played by South (can anyone find a no-point solution?). Also, Hess found that the 38-point hand on the bottom cannot make a game in any suit or no-trump.

North
♠ 65432
♥
♦ 5432
♣ 5432

West
♠
♥
♦ J109876
♣ QJ109876

East
♠ AKQ
♥ AKQJ10
♦ AKQ
♣ AK

South
♠ J10987
♥ 98765432
♦
♣

North
♠ AKQJ32
♥ AKQ5432
♦
♣

West
♠
♥
♦ 10987654
♣ J109876

East
♠ 10987654
♥ J109876
♦
♣

South
♠
♥
♦ AKQJ32
♣ AKQ5432

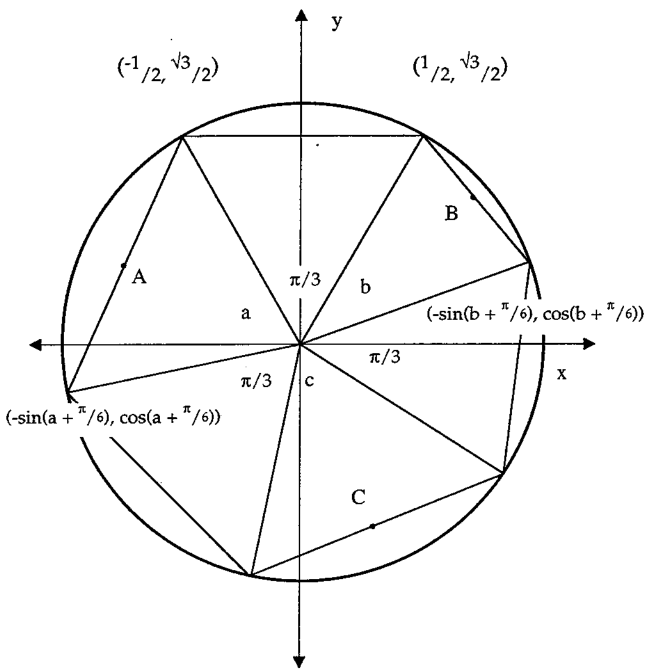
J/A 2. The next problem, which Mike Moritz attributes to Leon Greitzer, appears to be a non-trivial geometry challenge. But I've been wrong so don't be discouraged.

In a circle of unit radius, inscribe a hexagon such that three alternating sides are of unit length. The other (also alternating) sides may be of any length. Connect the midpoints of these (nonequal) sides to make a triangle. Prove that the resulting triangle is equilateral.

Naturally, the problem I flag as potentially difficult resulted in several beautiful solutions from readers. Either I am a poor judge of difficulty/interest, or my declaration was "inspiring." The following solution is from Scott Brown:

Let A, B and C be the midpoints of the nonequal sides and let a, b and c be angles subtended by the corresponding nonequal sides. The sides of length 1 tend angles of size p/3.

Establish a coordinate system whose origin is at the center of the circle and whose y-axis bisects the unit length side between A and B (see figure below).



We see from the figure above that the coordinates of points A and B are

A = 1/2 (-sin(a+p/6)-1/2,cos(a+p/6)+(√3)/2)
B = 1/2 (sin(b+p/6)+1/2,cos(b+p/6)+(√3)/2)

So

4|A-B|2 = [sin(a+p/6)+sin(b+p/6)+1]2
+ [cos(a+p/6)-cos(b+p/6)]2

= sin2(a+p/6) + sin2(b+p/6) + 1 +
2sin(a+p/6)sin(b+p/6) + 2sin(a+p/6)
+ 2sin(b+p/6) + cos2(a+p/6) +
cos2(b+p/6) - 2cos(a+p/6)cos(b+p/6)

Using the identities sin2(x) + cos2(x) = 1 and cos(x+y) = cos(x)cos(y) - sin(x)sin(y) this expression becomes

4|A-B|2 = 3 - 2cos(a+b+p/3) + 2sin(a+p/6)
+ 2sin(b+p/6)

This formula can be applied to points A and C:

4|A-C|2 = 3 - 2cos(a+c+p/3) + 2sin(a+p/6)
+ 2sin(c+p/6)

Since

a+p/3+b+p/3+c+p/3 = 2p, c = p - a - b

and

4|A-C|2 = 3 - 2cos(4p/3-b)
+ 2sin(a+p/6) + 2sin(7p/6-a-b)

The identity sin(x+3p2) = -cos(x) shows sin(7p/6-a-b) = -cos(a+b+p/3) and sin(b+p/6) = -cos(4p/3-b) so 4|A-C|2 = 4|A-B|2. By symmetry we also have 4|A-B|2 = 4|B-C|2, so the triangle ABC is equilateral.

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     PP
  UUUUU
  ZZZZZ
  ZZZZZ
+  LLLL
 PUZZLE
```

J/A 3. We end up with an additional puzzle Nob Yoshigahara attributes to Kyoko Ohnishi:

Matthew Fountain sent us the following solution:

It is reasonable to assume all the carries involved in the addition are 2 as none of the carries can be more or less than the carry from the second column to the first. The first column must receive a carry of 2 for U+Z+Z+Carry=PU. P=1 and P=2 would make the sum of the last column equal to the sum of the third column plus a carry of 2. P=1 and therefore Z=4. With P=1, comparison of the third and fourth column shows that L=Z+1=5. Comparison of the third column with a carry of 2 and the last column shows that E=Z-1=3. In the third column, 2+U+4+4+5=24, so U=9.

```
     11
  99999
  44444
  44444
+  5555
 194453
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OTHER RESPONDERS

Responses have also been received from T. Barrows, D. Dechman, S. Harlan, H. Ingraham, N. Markovitz, D. McIlroy, C. Muehe and E. Sard.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The rerun of "Cheers," red light, green light and the gamma rays.

Send problems, solutions and comments to: Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu.

— Edited by Owen W. Ozier '98

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