# Puzzle Corner

## INTRODUCTION

It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred, as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed or sent via e-mail, since these produce fewer typesetting errors.

## PROBLEMS

**J/A 1.** A two-part bridge problem from Larry Kells. What is the fewest number of points a pair can hold and still make a game against best defense? What is the most points a pair can hold and still be unable to make a game against best defense?

**J/A 2.** This problem, which Mike Moritz attributes to Leon Greitzer, appears to be a non-trivial geometry challenge. But I've been wrong so don't be discouraged.

In a circle of unit radius, inscribe a hexagon such that three alternating sides are of unit length. The other (also alternating) sides may be of any length. Connect the midpoints of these (nonequal) sides to make a triangle. Prove that this resulting triangle is equilateral.

## SPEED DEPARTMENT

Ken Rosato wants to know what gets twice as long when you cut it in half.

## SOLUTIONS

**M/A 1.** Larry Kells continues the saga of the fighting bridge partnership.

I saw the husband who had been henpecked by his wife for making unsound doubles, until she was victimized herself. I asked him why they weren't coming to the club anymore. He told me they had decided to try duplicate bridge instead. They figured, if they were cursed to be continually dealt powerful hands, which then couldn't defeat opposing contracts, at least they would share their misery with other players playing the same deals. But then a hand came up that renewed all the old tensions.

They had bid to a cold, vulnerable six-spades contract that no other pair reached. But then the opponents sacrificed—nonvulnerable—at seven spades (!) and, despite a 12-trick set, left our couple with a bottom on the board.

"We got 600 points, but everyone else with our cards either bid four spades and made six, or 3NT and made four or five, for 630-680 points," he said.

"Why on earth didn't you double them?"

"We had made a compact in blood that neither of us would ever again double for penalties unless the doubler held the cards in his or her own hand that would give us an ironclad guarantee of defeating them. Neither of us could single-handedly guarantee that they wouldn't make seven spades. So we couldn't double."

"Didn't one of you have the ace of spades?"

"No, that was our only loser."

"But why under heaven did they sacrifice in your suit?"

"Apparently one of them overheard us discussing our blood compact and knew that we would not double seven spades. If they had sacrificed at seven in some other suit, we would have doubled because one of us had the ace of that suit, and if they had sacrificed at 6NT, either of us could take several winners off the top, so we could have safely doubled them."

Can you reconstruct the deal?

The following solution is from the proposer:

The play for E-W to make 12 tricks in spades, on offense or defense, is straightforward: by drawing trumps and setting up the diamonds. But East, by himself, cannot assure defeating seven spades, for example, if the rest of the deal were as follows (see Deal A on next page):

| | North | |
|---|---|---|
| | ♠ 87 | |
| | ♥ Jxxx | |
| | ♦ J10xx | |
| **West** | ♣ Qxx | **East** |
| ♠ K10xx | | ♠ QJxx |
| ♥ AKx | | ♥ xx |
| ♦ xx | | ♦ AKQxx |
| ♣ AKxx | **South** | ♣ xx |
| | ♠ A9x | |
| | ♥ Q10xx | |
| | ♦ xx | |
| | ♣ J10xx | |

It takes three finesses to pick up East's trumps. Dummy has enough trumps to do this (even if forced to ruff the opening lead), and enough entries to do this and still get back to cash its winners after drawing trumps. And the actual West, by herself, could not assure defeat of seven spades, if the deal were as in Deal B (on the next page).

South only needs the lead twice to pick up West's spades. (He starts by leading the queen.) His high diamonds are the entries to do so, and North can safely ruff the opening lead if necessary. The lead will always be in dummy after drawing trumps, so he can run the diamonds.

| West | North | East |
|---|---|---|
| | ♠ 7xxx | |
| | ♥ AKQJ10 | |
| | ♦ Void | |
| ♠ Void | ♣ AKQJ | ♠ QJxx |
| ♥ xxxx | | ♥ xx |
| ♦ J10xxx | **Deal A** | ♦ AKQxx |
| ♣ 10xxx | | ♣ xx |
| | South | |
| | ♠ AK1098 | |
| | ♥ xx | |
| | ♦ xxx | |
| | ♣ xxx | |

| West | North | East |
|---|---|---|
| | ♠ AJ87x | |
| | ♥ Void | |
| | ♦ KJ10xxxxx | |
| ♠ K10xx | ♣ Void | ♠ Void |
| ♥ AKx | | ♥ QJ10xxx |
| ♦ xx | **Deal B** | ♦ x |
| ♣ AKxx | | ♣ QJ10xxx |
| | South | |
| | ♠ Q9xx | |
| | ♥ xxxx | |
| | ♦ AQ | |
| | ♣ xxx | |

**M/A 2.** A seasonal problem from Larry Kells: We know that as the seasons progress, first one hemisphere, then the other receives a larger amount of daily solar radiation. But clearly, if the sun is just barely north of the equator, then the North Pole—with the sun barely above the horizon all 24 hours—will not receive as much energy per day as a point just south of the equator, where the sun still climbs nearly overhead. The problem is this: How far north of the equator does the sun have to be in order for *every* point in the northern hemisphere to receive more energy per day than *every* point in the southern hemisphere? (Note: the amount of energy received from the sun per unit of time is proportional to the sine of the altitude of the sun above the horizon when it is up, zero when it is down.)

This was a fine problem, as it inspired a number of beautiful solutions. I especially liked the ones from Tim Barrows and Howard Stern. Both are somewhat lengthy so space consideration forbids me from printing both. The coin came up heads and Stern's solution follows. Readers desiring Barrows' solution should contact me with either a postal address or a fax number.

Imagine the Earth centered at the origin with a point, P, on its surface; let the position of the Sun be fixed. In spherical coordinates:

| | |
|---|---|
| point P on Earth: | $\{1, L_{Earth}, T\}$ |
| the Sun: | $\{R, L_{Sun}, 0\}$ |

where 1 is the normalized radius of the Earth, $L_{Earth}$ is the latitude of P and T is the time. Let T=0 be considered Noon and $-\pi$ Midnight. Thus there are $2\pi$ "Hours" in a day. R is the distance from the Sun to the center of the Earth, $L_{Sun}$ is the angle of the Sun above the equator and 0 is the fixed time angle, thought of as Noon.

The Cartesian coordinates of the Earth and Sun are:

Earth: $\{\cos(L_{Earth})\cos(T), \cos(L_{Earth})\sin(T), \sin(L_{Earth})\}$
Sun: $\{R\cos(L_{Sun}), 0, R\sin(L_{Sun})\}$

The cosine of the angle between the Sun and Earth, $\alpha$, is the dot product of the two vectors' Cartesian coordinates divided by R:

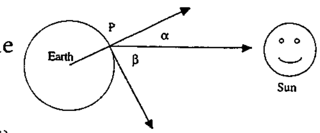
(1) $\cos(\alpha) = \cos(L_{Sun})\cos(L_{Earth})\cos(T) + \sin(L_{Sun})\sin(L_{Earth})$

The energy reaching P on the Earth's surface is proportional to the sine of the angle of the sun with the horzon, $\beta$. From the picture below, this is also the cosine of $\alpha$.

Therefore, the daily energy reaching P is the integral of (1) from sunrise to sunset. Sunset occurs when the angle $\alpha$ is 90°, or the cosine is 0. From (1), this occurs when:

$$T_{Sunset} = \cos^{-1}(-\sin(L_{Sun})\sin(L_{Earth})/\cos(L_{Sun})\cos(L_{Earth}))$$
$$T_{Sunrise} = -T_{Sunset}$$

The daily energy reaching the point on the Earth is:

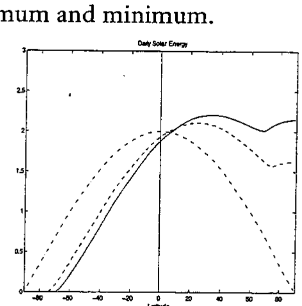



(2) $\cos(L_{Sun})\cos(L_{Earth})\cos(T) + \sin(L_{Sun})\sin(L_{Earth})\ dT$

This is easily integrated:

(3) $2\cos(L_{Sun})\cos(L_{Earth})\sin(T_{Sunset}) + 2\sin(L_{Sun})\sin(L_{Earth})T_{Sunset}$

When the sun is on the equator, (3) is symmetric and has one local maximum. However, when the sun rises above the equator the function assumes an unusual shape. The function is graphed here for 0° (dashed and dotted), 15° (dashed) and 20° (solid) above the equator. For latitudes greater than 0°, the curve has both a local maximum and minimum.



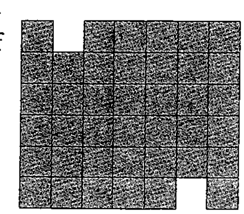

It can be seen that when the sun is 15° above the equator there are points north of the equator that do not receive as much energy as points just south of the equator. However when the sun is 20° above the equator, all ponts north of the equator receive more energy than all points south of it.

The solution to the problem is the critical value for the sun when the local minumum of the energy curve equals the value at the equator. This value can be solved for numerically and is approximately 18.6°.

**M/A 3.** (From "Golomb's Gambits.") How can you dissect this figure into four congruent pieces?

Several readers sent me correct solutions. Particularly interesting was one from J. Walker, which contained the remark, "Not bad for someone who has spent his post MIT career being an actor, director and playwright, eh?" Indeed, Mr. Walker, not bad.

The following solution from Donald Savage includes a description of the computer search performed:

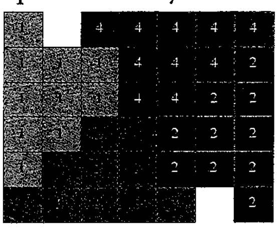


I noted that Golomb's notched rectangle is made up of 40 squares, so I (arbitrarily) made these assumptions: (1)

Each (congruent) piece is made up of 10 (uncut) squares; (2) each square can be laid down in one of the positions shown in Golomb's diagram; (3) the 10 squares making up a piece are "together": each square is connected to a neighbor via a full edge boundary, and no subset of the 10 is unconnected to the others. At the start, I didn't know whether making these assumptions eliminated the possibility of a solution.

Next I noted that the upper left square and its neighbor below must be members of the same piece. (Ditto for the lower right pair.) I fooled around a bit with pencil and paper, got nowhere, so I fired up the computer.

My algorithm is in two parts: (1) generate all patterns of 10 squares that obey the above assumptions, and which include the upper left square and its neighbor below. (The computer found 1705 of them.) Each of these is a "piece number 1." (2) For each of these 1705 patterns, make an identical copy as "piece number 2," and try to place it in Golomb's diagram without overlapping #1. Systematically go through all possible translations, rotations and flippings. If successful, try to place #3 without overlap, and ditto for #4. Any time no placement is possible, back up to the previous step, and increment the translation (or rotation or flipping).



At pattern #103, the computer got an answer, at right. With the exceptions of labeling permutations, from #104 to #1705, no more solutions were found.

### BETTER LATE THAN NEVER

**1999 N/D 2.** Eugene Sard believes the solution given is incomplete and does not show that the third circle described in the problem has the same diameter as the circle circumscribing pentagon ADB in the solution.

### OTHER RESPONDERS

Responses have also been received from M. Fountain, T. Harriman, R. Hess, R. Kinsley, M. Lindenberg, E. Sard, M. Seidel, R. Sinclair, A. Taylor and J. Wouk.

### PROPOSER'S SOLUTION TO SPEED PROBLEM

A Mobius strip.

*Send problems, solutions and comments to: Allan Gottlieb, New York University, 715 Broadway, 10th floor, New York, NY 10012, or to gottlieb@nyu.edu.*

*Edited by Owen W. Ozier '98*