INTRODUCTION

I have in the past recommended listening to books on tape during long car commutes; my wife and I are now thoroughly addicted. I tend to alternate between classics such as *A Tale of Two Cities* and less serious books like *The Runaway Jury*. Norman Spencer's problem below reminds me that I should mention another kind of audio book namely anything by Feymann. I particularly enjoyed *Six Easy Pieces* and *Six not so Easy Pieces*, both of which are taken from Feynman's freshman and sophmore lectures that led to the *Lectures on Physics* three volume set. Trivia question: what was the nickname of the competing MIT course. Answer: ‘‘PANIC’’, which was the acronym of the text, Physics: A New Introductory Course.

PROBLEMS

M/A 1. Larry Kells finds pleasure in seeing well intentioned doublers with good hands punished, both in points and verbally. He writes.

I was once again out kibitzing at my local bridge club. You remember that couple, where the man keeps doubling his opponents' contracts without enough cards to beat them, then gets berated by his wife? Well, it happened again! I wasn't at their table at the time, but from across the room I suddenly heard this loud exchange:

Wife: Well you've done it again! You double 7NT, they redouble and make it, and get over 700 points more than if you had left well enough alone!

Husband: But I had 22 points and 4-3-3-3 distribution. I was sure they would go down plenty! How could I have imagined what happened?

Wife: We had no defense! Wasn't it obvious from the bidding that they had to have a running suit? I told you never to double 7NT with an unstopped suit!

Husband: Unstopped suit? You call my clubs an unstopped suit?!

Wife: (sneering) You sure didn't stop them from running their clubs now, did you?

By past experience, I knew they were telling the truth. But I didn't see what their hands were. Can you reconstruct them?

M/A 2. Jerry Grossman reports that in 1989 the largest number known to be prime was $2^{216091} - 1$. Imagine writing out this number in full (but don't do it!). How many digits does it have, what are the three leftmost digits, and what are the three rightmost digits?

M/A 3. Norman Spencer sends us a problem he found in Glick's *Genius* (the life of Richard Feynman). Evaluate

$$(\cos 20)(\cos 40)(\cos 80)$$

SPEED DEPARTMENT

My former NYU colleague Ron Bianchini notes that each item below contains the initials of words that will make it correct. You are to complete the missing words. For example, given ‘‘16 = O. in a P.’’, the answer is ‘‘Ounces in a Pound’’.

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24 = H. in a D.
 1 = W. on a U.
 5 = D. in a Z. C.
57 = H. V.
11 = P. on a F. T.
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SOLUTIONS

Oct 1. Tom Harriman wonders how you can set the contract sitting East after the Heart 2 has been led. Harriman emphasizes that you are expected to find the solution ‘‘at the table’’ so you should not assume that all hands are known to all players.

There were some minor typos in the card distribution, but that did not cause our readers to quit. The consensus is that one must trick declarer into an error. Jorgen Harmse writes

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                S T87
                H J87
                D A2
                C KQJx3
S AJ2                           S Q96
H KT32                          H A65
D T86                           D 9754
C 9xx                           C Txx
                S K543
                H Q96
                D KQJ3
                C A2
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The club distribution is shown as it must be for the deal to be correct. Double dummy, the contract is unstoppable. There is only one play which gives a competent declarer a chance to misguess.

With so many tricks for declarer in the minors, East begins by winning the ace of hearts. East knows from the lead that West has only 4 hearts, so something more is needed to defeat the contract. If West has KQ in hearts the something else will provide the entry for the defence to run the suit, while if the queen is in the closed hand a heart return establishes a trick for declarer. The defence’s minor suit winners will certainly make since declarer needs the minors to bring home nine tricks. The danger is that spade winners will not be established in time. East should lead a spade in the hope of finding West with two honors. Which spade is led makes a difference when declarer has the king and the defence does not have 4 heart tricks. (If South has the ace, spades and hearts cannot both be developed in time, so West needs a minor suit winner to beat the contract.) Any lead except the queen allows South to play low, and West cannot profitably continue spades. Therefore East should lead the queen. (I am not saying that I would find this at the table, but that an expert could.) South must cover and West naturally wins with the ace. West should see the importance of establishing spades, and should also see that the lead of the jack merely establishes dummy’s ten. West therefore returns the 2 of spades, and South must guess. The lead of an unsupported queen is sufficiently unusual that South might assume East holds the jack. In this case the 8 is played from dummy, and the defence takes 3 spades and 2 hearts. [A similar variation, pointed out by the proposer, is that, fearing East holds the jack, declarer might not cover the spade Queen earlier−ed.]

Oct 2. Howard Stern is a coin flipper from way back who likes to see heads. He flips K coins and then picks up all those that show tails and reflips them. He continues reflipping the coins showing tails until all the coins show heads. Given K coins how many (rounds of) flips are needed to have at least a 50% probability of all heads showing? Conversely, if you are going to use N (rounds of) flips (1 flip of K coins and N-1 reflips, with each reflip including all the tails showing), what is the largest number of coins you can start with and still have at least a 50% probability of all heads showing?

Mark Perkins liked this one. He writes: Thanks for the fun probability problem. After a few false starts, I finally saw the light−a blinding flash in this case. We are looking for n(K), the minimum number of rounds required to have at least a 50% probability of all heads showing; and k(N), the maximum number of coins to start with to have at least a 50% probability of all heads showing after N rounds. It is not stated the coins are fair, so I will assume each coin comes up heads with probability p and tails with probability $q = 1 - p$. If the coins are fair then $p = q = 0.5$.

The first step is to find the probability that K coins will all be heads after N rounds, denoted p(N,K). Consider first a single coin. The probablity it will be tails after N rounds is the probability it comes up tails on every flip or qˆN. Thus, $p(N, 1) = 1 - q^N$. When additional coins are added, they may each be considered independently, so the probability that all coins end up heads is the product of the probabilities of each coin ending up heads. Thus,

$$p(N, K) = (1 - q^N)^K.$$

From here it is straightforward algebra to solve

$$0.5 = (1 - q^N)^K$$

for K and then for N. This yields the desired answers:

$$k(N) = \left\lfloor \ln(0.5)/\ln(1 - q^N) \right\rfloor$$

$$N(k) = \left\lceil \ln(1 - 0.5^N)/\ln(q) \right\rceil$$

As a final note, it is fun to check one's intuition on this type of problem. For fair coins, it seems that k(N+1) should be approximately double k(N). That is, for each additional round you can have twice as many coins. And sure enough, when q=0.5, the limit of k(N+1)/k(N) as N goes to infinity is indeed 2.

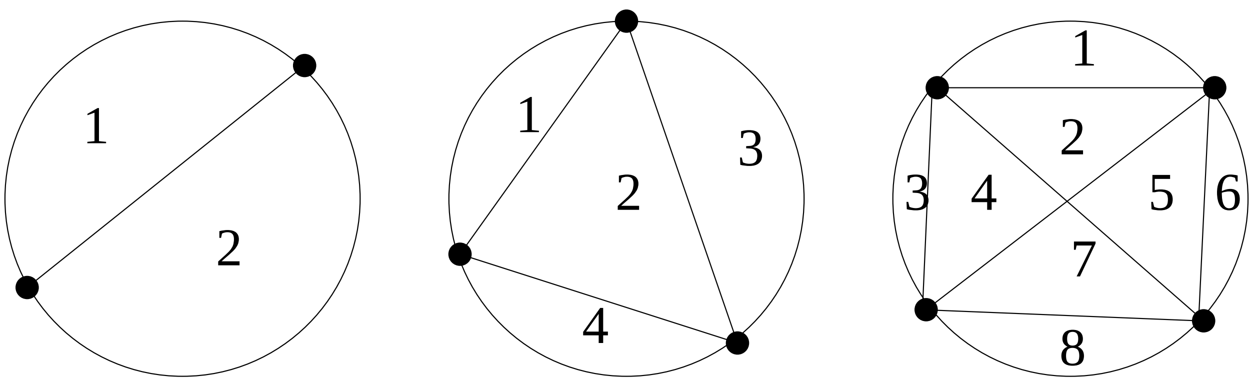
Oct 3. Here is one from Nob Yoshigahara that I confess I would never be able to solve. I have added a hint (forgive me, Nob) but still would have trouble. You have a function defined on the integers greater than 1. The first few values are f(2)=2, f(3)=4, f(4)=8, f(5)=16, and f(6)=31. You are to find f(7) and explain f. Editor's hint: there is some geometry involved.

Richard Hess found the geometrical solution. The value f(n) is the number of regions a circle can be divided into by connecting n points on its boundary with straight lines. There are also algebraic formulas for this function. For example, Hess notes that

$$f(n) = \frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24} \quad .$$

Hess also notes that Sloan's book on integer sequences gives several formulas that match Nob's sequence for the few values given. Sloan's formula that matches the geometry is

$$M1119:\ \ 1, 2, 4, 8, 16, 31, 57, 99, \cdots \ = \binom{n}{4}+\binom{n}{3}+\binom{n}{2}+\binom{n}{1}+\binom{n}{0} \ .$$



BETTER LATE THAN NEVER

M/J 2. Eugene Sard and Matthew Fountain sent detailed remarks on the solution printed.

OTHER RESPONDERS

Responses have also been received from K. Bernstein, R. Breed, Z. Charon, K. Duisenberg, M. Fountain, R. Giovanniello, T. Harriman, W. Himmelberger, S. Hsu, T. Lewis, A. Ritter, K. Rosato, F. Rubin, E. Sard, R. Schweiker, and M. Thompson.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Hours in a Day
Wheels on a Unicycle
Digits in a Zip Code
Heinz Varieties
Players on a Football Team