INTRODUCTION

This being the first issue of a calendar year, we again offer a “yearly problem” in which you are to express small integers in terms of the digits of the new year (1, 9, 9, and 8) and the arithmetic operators. The problem is formally stated in the “Problems” section, and the solution to the 1997 yearly problem is in the “Solutions” section.

We have been running this ‘‘yearly problem’’ for quite some time and it has become somewhat of a tradition. I am aware that these next few years, especially 2000, will be tough ones but do not want to alter the problem for these few lean years. Indeed, slim pickens for our yearly problem should only be the worst problem 2000 causes.

PROBLEMS

Y1998. How many integers from 1 to 100 can you form using the digits 1, 9, 9, and 8 exactly once each and the operators +, -, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 9, and 8 are preferred. Parenthesis may be used for grouping; they do *not* count as operators. A leading minus sign *does* count as an operator.

J/F 1. Charles Wampler has a cube of marble, measuring 1 meter on a side. For an element in his newest modern masterpiece, he would like to carve from the cube a tetrahedron having the largest possible volume. How long are the edges of the tetrahedron and what is its volume? Since the grain of the stone is not uniform, the sculptor would also like to know if there is any choice in how the tetrahedron is positioned within the cube.

J/F 2. Here is one Norman Spencer found in *Better Homes and Gardens*. How can you divide a circle into *n* equal sized segments? Note that by a circle we mean the one dimensional object sometimes (in the editor's view, erroneously) called the circumference of the circle. In addition to the using the customary compass and straight-edge, you may divide a *line* into *n* equal size pieces.

SPEED DEPARTMENT

Frank Rubin wants you to move only one match so that you are left with no triangle.

Please place figure number 1 here.

SOLUTIONS

Y1997.

The following solution is from John Drumheller.

Please place figure number 2 here.

A/S 1. We begin with an unusual Bridge problem from Steve Shalom.

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S A 7
H K J 9 8 5 4 3 2
D K 4
C 7
-----------
|    N    |
|         |
|W       E|
|         |
|    S    |
-----------
S Q 8 3
H 6
D J 8 6 5 3
C A K J 2
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Only a diamond lead defeats a double-dummy declarer playing in three no-trump. The club queen, heart queen, and diamond ace are offside. No two of East’s spot cards (any card from two to ten) are of the same rank. The sum of West’s spot cards is not evenly divisible by five.

What are the precise East-West hands and what is the proper defense for beating three no-trump?

Only the proposer, Steve Shalom, sent us a solution.

Please place figure number 3 here.

A/S 2. A ‘‘jigsaw’’ puzzle from Nob. Yoshigahara. Cut out the 9 squares below and arrange them into a 3x3 square so that all the (internal) edges match. There are six solutions.

Faith, please include artwork from A/S

Don Savage found all six solutions.

Please place figure number 4 here.

A/S 3. We close with a surveying problem from Richard Mayor.

A surveyor is on a desert floor from which he can see three mountain peaks. The peaks form a triangle and are accurately located on a map?

The surveyor turns the angles between two pairs of peaks say (1,2) and (2,3)

How can he determine his location on the map.

Charles Muehe sent us a trigonometric solution while Russell Mallett employed more of a constructive geometry technique. We begin with Meuhe.

Please place figure number 5 here.

Mallett's solution follows. The locus on the map of all points from which a pair of peaks has the same angular separation consists of two circular arcs of equal radius whose endpoints are the two peaks. The two angles measured by the surveyor lead to two such loci and they intersect at peak 2 and at from one to four other points that constitute the possible positions of the surveyor.

When multiple solutions occur more information is needed from the surveyor. Suppose we are told that the measured angles don't overlap and that they were obtained by turning in a clockwise direction, proceeding from peak 1 to peak 2 to peak 3. This is enough to place the surveyor in one of the four sectors determined by the line through peaks (1,2) and the line through peaks (2,3), namely the sector that contains the triangle (1,2,3). Only one of the intersections will lie in this sector and the surveyor's position will thus be uniquely determined.

Note that we only need to draw the part of a locus that lies in the sector known to contain the surveyor's position. Also note that the point of intersection of two arcs is sometimes difficult to obtain accurately (when they are nearly tangent) and thus it will occasionally be advisable to construct a third locus, namely the locus associated with the pair of peaks (1,3).

It remains to give a method for construction of a locus. Let A denote the angle between peaks P and Q. Construct the perpendicular bisector b of the line segment PQ. Then construct a line through P that makes an angle of 90 -A/2 with PQ. The point of intersection R of these two lines is on the locus. Since we now know three points (P, Q and R) on an arc of the locus, the arc's center is easily determined (by finding the intersection of b with the perpendicular bisector of the chord PR) and the arc can be drawn. The locus is completed by drawing the mirror image in PQ of the arc just constructed.

OTHER RESPONDERS

Responses have also been received from V. Barocas, C. Dale, J. Feil, M. Fountain, R. Giovanniello, O. Helbok, R. Hess, A. Hoffman, J. McNear, B. Nolton, R. Norton, A. Ornstein, C. Rife, K. Rosato, E. Sard, E. Schenk, I. Shalom, A. Tracht, A. Ucko, T. Weiss, D. Wellington, N. Wickstrand, and D. Young.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Please place figure number 6 here.