INTRODUCTION

This being the first issue of a calendar year, we again offer a “yearly problem” in which you are to express small integers in terms of the digits of the new year (1, 9, 9, and 7) and the arithmetic operators. The problem is formally stated in the “Problems” section, and the solution to the 1996 yearly problem is in the “Solutions” section.

As you may recall, a publishing error caused the first page of the August/September column to be reprinted in October. In particular problems A/S1−A/S3 were printed in both issues and there are *no* OCT problems. As a result the solutions below are for both sets of (identical) problems and next issue there will be no solutions. At that point, things will return to normal.

PROBLEMS

Y1997. How many integers from 1 to 100 can you form using the digits 1, 9, 9, and 7 exactly once each and the operators +, -, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 9, and 7 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

JAN 1. Andre Dehnel has a square of 10×10 units and a rectangle of 1×8 units. He wants you to cut the square into two pieces and assemble the three pieces into a rectangle of 9×12 units.

JAN 2. Howard Stern needs help in packing an infinite number of disks. But don't worry, they only take up finite space.

Please place figure number 1 here.

SPEED DEPARTMENT

Thomas Harriman asks about what he calls ‘‘Slow-track Bidding’’. Without any irregularities, how many rounds of bidding can take place in bridge with the resulting contract at the level of one? How many if legally redressed irregularities occur?

SOLUTIONS

Y1996. How many integers from 1 to 100 can you form using the digits 1, 9, 9, and 6 exactly once each and the operators +, -, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 9, and 6 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

The following solution is from John Drumheller.

Please place figure number 2 here.

A/S 1. On a recent visit to the bridge club, Larry Kells took his seat kibitzing at his customary table and heard the following auction:

S	W	N	E
1D	2D	3D	4D
5D	6D	7D	Dbl*
P	P	P	

*After having found out he could not bid 8D!

Looking at the hands afterwards, it was clear that every bid was reasonable, and the contract was the best that could be reached if both sides bid optimally. Can you reconstruct the deal?

With problems of this nature, there is not a single ‘‘right answer’’. The solution below is from Len Schaider.

Please place figure number 3 here.

A/S 2. George Blondin wants to know the smallest integer whose product when multiplied by 9 is the original number with the rightmost digit rotated all the way to the left.

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abc...lmn
        9
_________
nabc...lm
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The use of the letters from a to n was just for convenience and the fact that n is the 14th letter does not imply that the answer has 14 digits.

There were many fine solutions to this problem so it was hard to pick just one. I eventually picked the joint effort from Adrian Childs and Craig Weigert for its clarity.

We presume that the smallest *positive* integer is meant, since -1 times any positive answer is also an answer. Also, 0 is a trivial answer.

Because the original and final numbers have the same number of digits, the leftmost digit of the original number must be 1, and the leftmost digit of the final number must be 9. So the rightmost digit of the original number is 9.

Given a digit x in the original number, we can calculate the next digit to the left as follows: take 9*x plus any carry from the previous step. The ones place holds the desired next digit, and the tens place becomes the carry for the next step. So the tens digit is 1 (9*9 = 81), the hundreds digit is 7 (9*1 + 8 = 17), the thousands digit is 4 (9*7 + 1 = 64), and so on.

The result is 44 digits long: 10,112,359,550,561,797,752,808,988,764,044,943,820,224,719.

A/S 3. Leonard Nissim is a fan of 9-digit numbers that contain each of the nine positive digits exactly once (there are 9 factorial such numbers). How many of these numbers are divisible by 11?

A brute force solution is possible and was done by some readers and, I confess, would have been my technique. But never again! Now I have learned the ‘‘alternating digit test’’ for divisibility by 11. Here is Pete Rauch’s solution.

A number is divisible by 11 if the difference between the sums of its alternating digits is a multiple of 11 (0 being considered a multiple of 11). For Leonard’s numbers, this means we must compare the sum of the first, third, fifth, seventh, and ninth digits to the sum of the second, fourth, sixth, and eighth. The sum of four distinct positive digits can range from 10 to 30, while the sum of five can range from 15 to 35. The possible differences between the sums ranges from +25 to -15, by increments of two. Of these possible differences, only 11 and -11 are divisible by 11. There are 2 combinations of four distinct digits that total 28 (leaving 17), and 9 combinations that total 17 (leaving 28). Each of these 11 combinations will have 4!5! permutations for digit order. 11*4!5! = 31680.

OTHER RESPONDERS

Responses have also been received from C. Bahne, R. Bradley, S. Brown, W. Burke, F. Carbin, P. Cheimets, C. Dale, J. Feil, S. Feldman, M. Fountain, J. Goldman, J. Harmse, D. Harris, R. Hess, D. Hogg, S. Kanter, M. Lindenberg, D. Lukens, R. de Marrais, P. Mock, C. Muehe, J. Norvik, A. Ornstein, E. Osman, S. Rawlinson, C. Rivers, K. Rosato, J. Ryan, E. Sard, M. Seidel, I. Shalom, R. Sinclair, M. Szymanski, A. Tracht, A. Ucko, J. Uretsky, D. Wellington, and R. Whitman.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Up to twelve without irregularities. With, infinite. Pass, Pass, Pass, One Club, Pass, Pass, Double, Pass, Pass, Redouble, Pass, Pass, One Diamond, ... gives a normal sequence of 49 bids. One Diamond, One Club, pass, One Diamond, One Club, pass, ... gives an abnormal but legal sequence, providing the pass is made before the insufficiency of the Club bid is noticed.