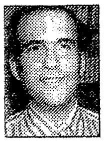
PuzzleCorner

Editor's Note: In the October issue, we accidentally published a repeat of the first page of the A/S Puzzle Corner, along with the second half of the solutions for M/J. The real puzzle was how to rectify this with the least disruption. Therefore: the Oct. problems now become N/D; here also are the first half of the M/J solutions and all of the July solutions. In the February/March '97 issue, there will be no October solutions, but all will then be back on track.

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present three regular problems (the first of which is chess, bridge, go, or computer-related) and one "speed" problem. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers who responded. For example, solutions to the problems you see below will appear in the February/March issue and the current issue contains solutions to the problems posed in May/June. Since I must submit the February/March column in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Respondents" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.



SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB NEW YORK UNIVERSITY 715 BROADWAY, 10TH FLOOR NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

There is also an annual problem, published in the January issue of each year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Problems

N/D 1. We begin with a bridge problem from Dudley Church, who wants to know if South can make six spades against an opening lead of the heart king?

North
♠ K Q 3
♥ Void
♦ J 6 5
♣ A K 10 8 5 4 2

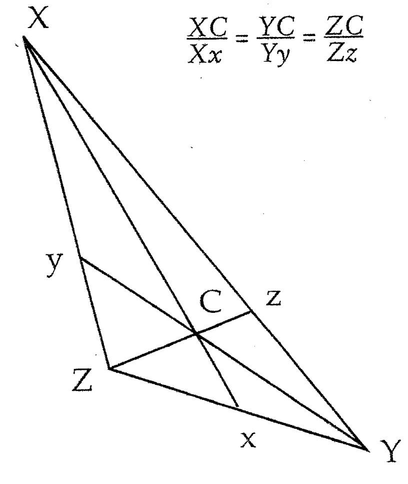
West
♠ J 8 4
♥ A K 6 3
♦ K 10 9 3
♣ 9 6

East
♠ 2
♥ 10 9 8 7 4 2
♦ 7 4 2
♣ Q J 7

South
♠ A 10 9 7 6 5
♥ Q J 5
♦ A Q 8
♣ 3

N/D 2. In the diagram below C is a point inside an arbitrary triangle XYZ. Nicholas Strauss has drawn a line segment from X through C and ending on the opposite side at a point we call x. Similarly, he has drawn lines starting at Y and Z and ending at y and z. Show that

$$\frac{XC}{Xx} = \frac{YC}{Yy} = \frac{ZC}{Zz}$$



N/D 3. Joseph and Charles Horton have inscribed a hexagon with side lengths 2,2,2,11,11,11 in a circle of radius r. What is the value of r?

Speed Department

Ken Rosato and his calculator have found a function f such that for x={0,1, 2,3,4,5,6,7,8,9}, f(x)={6,1,3,2,0,0,0,4, 1,3}. What is this function?

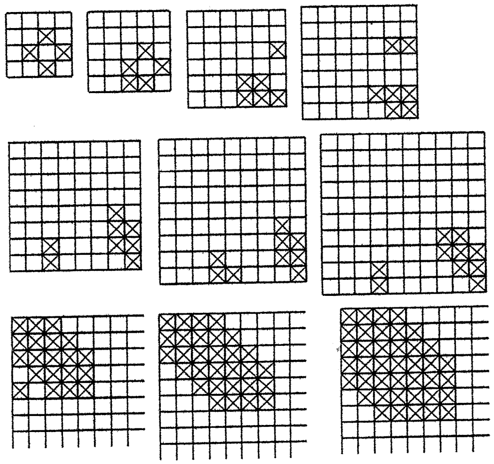
Solution for M/J 1

M/J 1. Mario Velucci wants you to place N queens on an N by N chess board so that the maximum number of vacant squares are Unattacked. For N=1, there are no vacant squares, and, for N=2 and N=3, all vacant squares are attacked. But, for N=4, you can leave 1 vacant square un-attacked and, for N=5, you can leave 3. How about N=10 and N=20 or possibly even N=30?

Robert Bart notes that by bunching the queens in a corner, two triangular regions near the opposite corner are free, i.e. un-attacked. As N gets large, so does Free(N), the number of free squares. Your editor wonders if one can show that Free(N) grows at least linearly with N, i.e., Free(N) = Ω(N).

The diagrams and explanations below are from Matthew Fountain. The proposer supplies values for Free(N) for all N up to 30. His only disagreement with Fountain is Free(N)=22, but no diagram was supplied.

The 10 diagrams show the position of the N



queens that allows the following number of free squares: 1 when N=4, 2 when N=5, 4 when N=6, 7 when N=7, 11 when N=8, 15 when N=9, 21 when N=10, 145 when N=20, 420 when N=30, and 841 when N=40. In cases of N<7 I

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Puzzle

Continued from Page MIT 55

found it best to arrange the free squares first. For example, when N=4 the free squares can be placed anywhere on the board. When placed on the edge, there are six squares available to place the 4 queens. In cases of N>6 I arranged the position of the queens first. I started by placing the queens in a compact group in one corner of the board. I selected the corner since then I had only one diagonal width of the group to minimize while I was also attempting to minimize the row and column widths of the group. Tight packing of the queens assured that certain squares would be attacked many times while leaving others free. As each queen attacks 3N-2+2E squares, where E is the distance in squares of the queen from the edge squares of the board, a few widely distributed queens can attack large numbers of squares. Five queens, properly placed, can attack every square of a N=11 board. Upon examining my placement of queens in the corner, I found, in some cases, their grouping could be made improved by relocating certain queens on the edge of the group so the group would have a slimmer appearance viewed along a row, column, or long diagonal of the group.

Solutions for July

Jul 1. In the hand below, submitted by Doug Van Patter, the bidding was short and sweet: South opened with 1H, West bid 2C, North bid 4H, and everyone then passed. How can South make the contract after opening lead of the King of Clubs?

North
♠ J 8 6
♥ K 7 4
♦ K Q 10
♣ A 10 9 8

West	East
♠ K 10 3	♠ Q 7 5 4 2
♥ 10 3	♥ 9 8
♦ 8 4	♦ A J 7 6 5
♣ K Q J 7 5 2	♣ 3

South
♠ A 9
♥ A Q J 6 5 2
♦ 9 3 2
♣ 6 4

The solution to the problem lies in establishing a club as the 10th trick.
North wins the 1st trick with CA. South wins the next two tricks with HA and HQ leaving HK in the dummy. On trick-4, South plays its last club and losing to West's CJ (North plays C8).

If West leads a spade on trick-5, South wins with SA, crosses over to North's HK and discards S9 on North's C9 losing to West's CQ. South then concedes North's DK to East's DA, and uses North's DQ as the entry to use the now good CT for a diamond discard.

If West leads a diamond on trick-5, North plays DK:

- If East wins with DA and returns a spade (best), South wins with SA. North's HK is then used as the entry to discard South's S9 on North's C9. North's established CT is then used for a diamond discard using DQ as the entry.
- If East refuses to win trick-5 with DA, North then plays C9 and South discards a diamond on trick-6. The defense will get one more trick in diamonds. South's losing S9 will be discarded on North's CT using HK as the entry.

In all cases, N-S loses 2 club tricks and one diamond trick.

Jul 2. Phil Bonomo seems to like "space cadet" problems, especially those involving navigation satellites. In '93 he asked about their velocity, now he questions their altitude.

Geosynchronous orbits are generally taken to be the orbits of largest radius (highest altitude) for earth-orbiting spacecraft. These circular orbits, of radius R_s (≈26,300 miles), are characterized by a 24-hour orbital period and are typically used by the equatorial, "stationary" class of communications spacecraft. Another class of communications spacecraft (the Russian "Molniya" class) employ inclined, highly eccentric orbits characterized by a 12-hour orbital period.

For what orbital conditions, if any, is the largest (apogee) radius of a Molniya orbit greater than the synchronous orbital radius R_s?

The solution below is from Andrew Mazzella. Note that it satisfies the constraint that the perigee radius must exceed the earth's radius. John Prussing, a professor of aero and astro at Illinois, tells me that this constraint "is a prime recommendation of NASA."

By Kepler's Law: $\left(\frac{R_s}{R_M}\right)^3 = \left(\frac{T_s}{T_M}\right)^2$
where
R_s=geosynchronous orbital radius
R_M=Molniya orbit semi-major axis
T_s=geosynchronous orbital period (24 hours)
T_M=Molniya orbital period (12 hours)
Therefore

$$\left(\frac{R_s}{R_M}\right)^3 = \left(\frac{T_s}{T_M}\right)^2 \quad \text{or} \quad R_M = R_s\left(\frac{T_M}{T_s}\right)^{\frac{2}{3}} = \frac{R_s}{4^{\frac{1}{3}}}$$

giving (approximately)

$$R_M = 16570 \text{ miles}$$

For a Molniya apogee greater than the geosynchronous orbital radius, the Molniya orbit eccentricity (e) must satisfy:

$$R_M \cdot (1+e) > R_s \quad \text{or} \quad 1+e > 4^{1/3}$$

giving (approximately)

$$e > 0.587$$

However, to be an allowable orbit, perigee must be greater than the radius of the earth (R_e)(and should exceed it by a reasonable margin to allow for the atmosphere), so:

$$R_M \cdot (1-e) > R_e$$

where
$R_M \cdot (1-e)$=16570 miles(1-0.587)=6843 miles
and R_e=3995 miles
so this orbit is attainable.

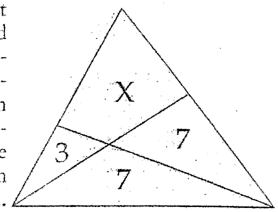
Allowing 100 miles for atmosphere, the limiting eccentricity for a feasible orbit is given by:

$$R_M(1-e_x) = R_a \quad \text{or} \quad e_x = 1 - \frac{R_a}{R_M}$$

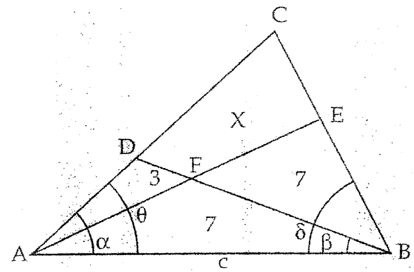
giving

$$e_x = 1 - \frac{4055 \text{ miles}}{16570 \text{ miles}} = 0.755$$

Jul 3. Nob Yoshigahara wants you to figure out the area of X in the figure below.



An embarrassment of riches: I received a number of beautifully drawn solutions, all of which deserved publication. After some thought I decided on Kenneth Bernstein's.



Label the figure as shown. The length of AB is c. The altitude from D in triangle ABD is c/(cotθ + cotβ); the altitude from F in triangle ABF is c/(cotα + cotβ); and the altitude from E in triangle ABE is c/(cotα + cotδ). Calculating the areas of the triangles:

ABD: $c^2 = 20(\cot\theta + \cot\beta)$;
ABF: $c^2 = 14(\cot\alpha + \cot\beta)$;
ABE: $c^2 = 28(\cot\alpha + \cot\delta)$.

From these equations we have:

$\cot\beta = (5/2)\cot\theta + (7/2)\cot\delta$; and
$\cot\alpha = (5/2)\cot\theta + (3/2)\cot\delta$.

Finally, from triangle ABC: $34 + 2X = c^2/(\cot\theta+\cot\delta) = 14(5\cot\theta+5\cot\delta)/(\cot\theta+\cot\delta) = 70$, or X = 18.

Other Responders

Responses have also been received from D. Alexander, C. Bahne, R. Bart, A. Beris, G. Blondin, J. Bush, X. Dai, E. Dawson, D. Detlefs, R. Dreselly, B. Durie, M. Egerton, E. Field, M. Fineman, M. Fountain, J. Grossman, J. Harmse, R. Hess, S. Hsu, P. Jung, L. Krakauer, J. Kusters, M. Lindenberg, G. Lum, N. Markovitz, D. Marron, J. Miller, C. Muehe, A. Peralta, J. Prussing, C. Rife, K. Rosato, E. Sard, L. Schaider, R. Scheidenhelm, E. Shung, R. Sinclair, N. Spencer, H. Stern, B. Thorburn, M. Veall, D. Wellington, J. Wright, and R. Yaseen.

Solution to Speed Problem

f(x) = g(x)-x where g(x) is the number of LCD elements in x. [I was personally baffled by this answer until I realized that LCD does NOT mean least common denominator (think calculator)—ed.]