PuzzleCorner

This next academic year will be unusual for me. For the first time since 1963, when I went off to MIT as an undergraduate, I will not be primarily at a university. Instead, I will be on leave from NYU in order to spend the year at NEC research. My NYU e-mail and U.S. mail addresses will still work, but you can also reach me at <gottlieb@research.nj.nec.com> and NEC Research Institute, 4 Independence Way, Princeton NJ 08540.

Problems

A/S 1. On a recent visit to the bridge club, Larry Kells took his seat kibitzing at his customary table and heard the following auction:

S	W	N	E
1D	2D	3D	4D
5D	6D	7D	Dbl*
P	P	P	

*After having found out he could not bid 8D!

Looking at the hands afterwards, it was clear that every bid was reasonable, and the contract was the best that could be reached if both sides bid optimally. Can you reconstruct the deal?

A/S 2. George Blondin wants to know the smallest integer whose product when multiplied by 9 is the original number with the rightmost digit rotated all the way to the left.

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abc...lmn
         9
nabc...lm
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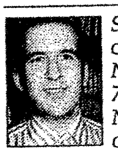
The use of the letters from a to n was just for convenience and the fact that n is the 14th letter does not imply that the answer has 14 digits.

A/S 3. Leonard Nissim is a fan of nine-digit numbers that contain each of the nine positive digits exactly once (there are nine factorial such numbers). How many of these numbers are divisible by 11?



Send problems, solutions, and comments to: Allan Gottlieb, '67
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Speed Department

Jon Sass has some more quickies where each capital letter represents a word beginning with that letter. For example, the answer to the first problem below is 16 ounces in a pound.

16 O in a P
90 D in a R A
1 W on a U
5 D in a Z C
11 P on a F T
1,000 W that a P is W
29 D in F in a L Y
64 S on a C
40 D and N of the G F

Solutions

Apr 1. We start with a bridge problem from Doug Van Patter:

North
♠ K J 10 4
♥ K Q 6 5 2
♦ A Q 8
♣ 4

South
♠ A Q 9 3
♥ 7 4
♦ 7
♣ A K Q 9 7 6

South dealt and the bidding went as follows with East-West silent: 1C 1S 4NT 5D 6S. What is South's best line of play?

The following solution is from Thomas Harriman.

The toughest opening lead is a diamond. Since finessing for the king would present a 50 percent chance of being set immediately on the obvious return lead of a heart, dummy must play the ace. Anticipating a 4-1 trump split, lead the spade 3 to the ace and then lead a small heart toward the king.

If West has the ace but holds off, the king wins. The spade king exposes the bad trump split, but the club ace followed by a club ruff high should set up the clubs: pull trump and run clubs for 12 tricks.

If West grabs the ace and leads a diamond to force declarer to ruff, a small heart to the king followed by a heart ruff high establishes the 12th trick: pull trump, take the heart queen, then three top clubs.

When East holds the heart ace, best defense is to take the king and lead a diamond to force a ruff. Declarer leads to the heart queen, next ruffs a heart high, and pulls trump. If hearts broke 3-3, dummy leads two good hearts and declarer takes the rest with high clubs. Otherwise he plays top clubs to win if *they* break 3-3. This should win about 2/3 of the time.

If the first trump lead shows a 5-0 split, declarer survives with some distributional luck. Again lead a low heart: if the holder of the ace is void of trump and declarer can guess the distribution, he can take toppers and cross-ruff, opponents' five spaces falling under higher ones.

The danger in not pulling trump right away is of course a heart ruff when the ace is opposite a singleton, about a 1/6 chance (even less: if West had a singleton, he would love to lead it). But a 4-1 trump split probability is about twice that.

Apr 2. Ermanno Signorelli wonders if there is a right triangle with integer sides such that both legs are odd integers.

Robert Barnes shows us that no such triangle exists.

Supposing that there is a right triangle with all sides integers, the legs being odd, this gives:

$(2m+1)^2+(2n+1)^2=r^2$, so
$(4m^2+4m+1)+(4n^2+4n+1)=r^2$, so
$4(m^2+n^2)+4(m+n)+2=r^2$.

LHS is obviously even, so RHS is even; since r^2 is even, so is r.

$4(m^2+n^2)+4(m+n)+2=(2k)^2=4k^2$, so
$2(m^2+n^2)+2(m+n)+1=2k^2$.

But now LHS is odd, and RHS is even.

Hence, there is no such triangle.

Apr 3. An illuminating question from Chuck Livingston

Lamp posts are to be installed on the equator of a perfectly spherical planet in such a way that they illuminate the entire equator. A few very tall lamps could be used—three is the minimum—or many short lamps. In what way should this be done so that the total height of the posts is as small as possible.

Mike Gennert notes that infinitely many infinitesimally small lamps can bring the total height down to zero.

Let the planet have radius 1. If there are N lamps, each lamp must illuminate $2\pi/N$ of the equator. A right triangle going from the lamp to the center of the earth to the edge of the region illuminated by that lamp has hypotenuse $1+H$, where H is the lamp post height, and angle π/N at the center of the earth. Therefore $(1+H)\cos(\pi/N)=1$ so $H=(\cos(\pi/N))^{-1}-1$. The total height of all lamp posts is just $T=NH$. T is a monotonically decreasing function of N approaching zero (using l'Hôpital's rule) as N goes to infinity. This can be checked by computing the derivative of T w.r.t. N.

$$\frac{dT}{dN}=\frac{1}{\cos(\pi/N)}-1-\frac{\pi\sin(\pi/N)}{N\cos^2(\pi/2N)}$$

Also, as N gets large, T behaves as $\pi^2/2N$.

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Puzzle

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placed anywhere on the board. When placed on the edge, there are six squares available to place the 4 queens. In cases of N>6 I arranged the position of the queens first. I started by placing the queens in a compact group in one corner of the board. I selected the corner since then I had only one diagonal width of the group to minimize while I was also attempting to minimize the row and column widths of the group. Tight packing of the queens assured that certain squares would be attacked many times while leaving others free. As each queen attacks 3N-2+2E squares, where E is the distance in squares of the queen from the edge squares of the board, a few widely distributed queens can attack large numbers of squares. Five queens, properly placed, can attack every square of a N=11 board. Upon examining my placement of queens in the corner, I found, in some cases, their grouping could be made improved by relocating certain queens on the edge of the group so the group would have a slimmer appearance viewed along a row, column, or long diagonal of the group.

M/J 2. Just before mailing this problem to me in March '93, Eugene Sard purchased a bunch of 29-cent and 23-cent postage stamps and was surprised to note that the total was a (non-zero) whole number of dollars. What is the smallest number of stamps that Sard could have been purchasing?

The following solution is from Mark Seidel. The general equation to solve is 29n + 23m = 100k, for some integers k,m and n, such that n+m is minimized. Taking the general equation modulo 23 and then modulo 3 yields 2k mod 3 = 0 so k = 3r (for some positive integer r). Taking the general equation modulo 23 and then modulo 8 yields 6n mod 8 = 0, so n = 4p. Taking the general equation modulo 4 yields n+3m mod 4 = 0, so (because n=4p) m = 4q. Plugging these relations into the general equation yields the reduced equation 29p + 23q = 75r, which when taken modulo 23 yields 6(p-r) mod 23 = 0, i.e. p = r + 23s. Plugging this equation back into the reduced equation yields q = 2r -29s. Minimizing n+m means minimizing (n+m)/4 = p+q = 3(r-2s), which must be positive (and therefore 3 or greater). One obvious solution is s=0 leading to (p,q,r) = (1,2,1) making p+q=3, so this must be a minimal solution. The solution to the original problem is therefore four 29-cent stamps and eight 23-cent stamps costing 3 dollars.

M/J 3. Nob Yoshigahara wants you to factor 123456789 into two five digit numbers.

The following solution is from Leonard Nissim. The two factors must both be fairly close to the square root of the original (about 11111.111), since both are over 10000. Clearly 123456789 is divisible by 9, as the sum of its digits is 45. Since 123456789/9 = 13717421, we are reduced to finding two factors of 13717421 which are close to its square root (about 3703.704). Dividing it by primes close to that value yields 1371721 = 3607 x 3803, and the solution is three times each of these primes, namely 123456789 = 10821 x 11409.

Better Late Than Never

1995 A/S 2. James Datesh and Bob Sackhein each found two errors. The entry for 16A should be 3419 and the entry for 25D should be 1815.

1995 Jan 1. This is quite amusing. We asked if there are infinitely many numbers that can be formed using their own digits in a non trivial way. Dave Pecora and Ethan Rappaport each responded with a family of solutions. The amusing part is that the base of Rappaport's solution is 117,648 and the base of Pecora's is 117,649.

Rapport notes that $117648 = (7^6 - 1^{48}) \times 1$, and hence $1176480 = (7^6 - 1^{48}) \times 10$, etc. Pecora starts with $7^6 \times 1^{149} = 117649$ and then adds six zeros to get $70^6 \times 1^{1490000} = 117649000000$, etc. Can anyone find a family where the number of nonzero digits grow without bound?

Other Responders

Responses have also been received from J. Abbott, H. Amster, R. Ball, L. Bell, J. Bush, F. Carbin, D. Diamond, S. Feldmen, E. Friedman, A. Guttag, J. Harmse, T. Hartford, R. Hess, H. Huang, H. Ingraham Jr., J. Kenton, M. Lindenberg, C. Muehe, R. Nelson, A. Ornstein, M. Perkins, G. Perry III, D. Plass, R. Ruiz, D. Savage, A. Shagen, P. Silvenberg, K. Stahl, A. Taylor, D. Thresher, and C. Whittle.

Solution to Speed Problem

f(x) = g(x)-x where g(x) is the number of LCD elements in x. [I was personally baffled by this answer until I realized that LCD does NOT mean least common denominator (think calculator)—ed.]