

# PuzzleCorner

This next academic year will be unusual for me. For the first time since 1963, when I went off to MIT as an undergraduate, I will not be primarily at a university. Instead, I will be on leave from NYU in order to spend the year at NEC research. My NYU e-mail and U.S. mail addresses will still work, but you can also reach me at <gottlieb@research.nj.nec.com> and NEC Research Institute, 4 Independence Way, Princeton NJ 08540.

## Problems

A/S 1. On a recent visit to the bridge club, Larry Kells took his seat kibitzing at his customary table and heard the following auction:

S	W	N	E
1D	2D	3D	4D
5D	6D	7D	Dbi*
P	P	P	

\*After having found out he could not bid 8D!

Looking at the hands afterwards, it was clear that every bid was reasonable, and the contract was the best that could be reached if both sides bid optimally. Can you reconstruct the deal?

A/S 2. George Blondin wants to know the smallest integer whose product when multiplied by 9 is the original number with the rightmost digit rotated all the way to the left.

$abc\dots lmn$

9

$nabc\dots lm$

The use of the letters from a to n is just for convenience and the fact that n is the 14th letter does not imply that the answer has 14 digits.

A/S 3. Leonard Nissim is a fan of nine-digit numbers that contain each of the nine positive digits exactly once



SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB, '67  
NEW YORK UNIVERSITY  
715 BROADWAY, 10TH FLOOR  
NEW YORK, NY 10012,  
OR TO: <GOTTLIEB@NYU.EDU>

(there are nine factorial such numbers). How many of these numbers are divisible by 11?

## Speed Department

Jon Sass has some more quickies where each capital letter represents a word beginning with that letter. For example, the answer to the first problem below is 16 ounces in a pound.

16 O in a P  
90 D in a R A  
1 W on a U  
5 D in a Z C  
11 P on a F T  
1,000 W that a P is W  
29 D in F in a L Y  
64 S on a C  
40 D and N of the G F

## Solutions

Apr 1. We start with a bridge problem from Doug Van Patter:

North  
 ♠ K J 10 4  
 ♥ K Q 6 5 2  
 ♦ A Q 8  
 ♣ 4  
 South  
 ♠ A Q 9 3  
 ♥ 7 4  
 ♦ 7  
 ♣ A K Q 9 7 6

South dealt and the bidding went as follows with East-West silent: 1C 1S 4NT 5D 6S. What is South's best line of play?

The following solution is from Thomas Hariman.

The toughest opening lead is a diamond. Since finessing for the king would present a 50 percent chance of being set immediately on the obvious return lead of a heart, dummy must play the ace. Anticipating a 4-1 trump split, lead the spade 3 to the ace and then lead a small heart toward the king.

If West has the ace but holds off, the king wins. The spade king exposes the bad trump split, but the club ace followed by a club ruff high should set up the clubs: pull trump and run clubs for 12 tricks.

If West grabs the ace and leads a diamond to force declarer to ruff, a small heart to the king followed by a heart ruff high establishes the 12th trick: pull trump, take the heart queen, then three top clubs.

When East holds the heart ace, best defense is to take the king and lead a diamond to force

a ruff. Declarer leads to the heart queen, next ruffs a heart high, and pulls trump. If hearts broke 3-3, dummy leads two good hearts and declarer takes the rest with high clubs. Otherwise he plays top clubs to win if they break 3-3. This should win about 2/3 of the time.

If the first trump lead shows a 5-0 split, declarer survives with some distributional luck. Again lead a low heart: if the holder of the ace is void of trump and declarer can guess the distribution, he can take topplers and cross-ruff, opponents' five spaces falling under higher ones.

The danger in not pulling trump right away is of course a heart ruff when the ace is opposite a singleton, about a 1/6 chance (even less: if West had a singleton, he would love to lead it). But a 4-1 trump split probability is about twice that.

Apr 2. Ermanno Signorelli wonders if there is a right triangle with integer sides such that both legs are odd integers.

Robert Barnes shows us that no such triangle exists.

Supposing that there is a right triangle with all sides integers, the legs being odd, this gives:

$$(2m+1)^2 + (2n+1)^2 = r^2, \text{ so}$$

$$(4m^2 + 4m + 1) + (4n^2 + 4n + 1) = r^2, \text{ so}$$

$$4(m^2 + n^2) + 4(m+n) + 2 = r^2.$$

LHS is obviously even, so RHS is even; since  $r^2$  is even, so is  $r$ .

$$4(m^2 + n^2) + 4(m+n) + 2 = (2k)^2 = 4k^2, \text{ so}$$

$$2(m^2 + n^2) + 2(m+n) + 1 = 2k^2.$$

But now LHS is odd, and RHS is even.

Hence, there is no such triangle.

Apr 3. An illuminating question from Chuck Livingston

Lamp posts are to be installed on the equator of a perfectly spherical planet in such a way that they illuminate the entire equator. A few very tall lamps could be used—three is the minimum—or many short lamps. In what way should this be done so that the total height of the posts is as small as possible.

Mike Gennert notes that infinitely many infinitesimally small lamps can bring the total height down to zero.

Let the planet have radius 1. If there are  $N$  lamps, each lamp must illuminate  $2\pi/N$  of the equator. A right triangle going from the lamp to the center of the earth to the edge of the region illuminated by that lamp has hypotenuse  $1+H$ , where  $H$  is the lamp post height, and angle  $\pi/N$  at the center of the earth. Therefore  $(1+H)\cos(\pi/N) = 1$  so  $H = (\cos(\pi/N))^{-1} - 1$ . The total height of all lamp posts is just  $T = NH$ .  $T$  is a monotonically decreasing function of  $N$ , approaching zero (using l'Hôpital's rule) as  $N$  goes to infinity. This can be checked by computing the derivative of  $T$  w.r.t.  $N$ .

$$\frac{dT}{dN} = \frac{1}{\cos(\pi/N)} - 1 - \frac{\pi \sin(\pi/N)}{N \cos^2(\pi/2N)}$$

Also, as  $N$  gets large,  $T$  behaves as  $\pi^2/2N$ .

Continued on Page MIT 35

# Puzzle

*Continued from Page MIT 55*

## Better Late Than Never

1995 N/D 2. Harvey Amster sent us some observations on the round-off errors that arise. A copy of Amster's remarks can be obtained from the editors of *Technology Review*.

## Other Responders

Responses have also been received from R. Bart, B. Cain, F. Cardin, J. Chandler, W. DeHart, D. Eckhardt, M. Egerton, S. Feldman, R. Ferguson, M. Foster, M. Fountain, W. Hartford, R. Hess, M. Ionescu, J. Landau, E. Levy, L. Nissim, A. Ornstein, D. Pecora, F. Pownser, E. Rappaport, K. Rosato, E. Sard, A. Schuchat, M. Seidel, R. Shapiro, R. Sinclair, and R. Spencer.

## Proposer's Solution to Speed Problem

16 ounces in a pound; 90 degrees in a right angle; 1 wheel on a unicycle; 5 digits in a zip code; 11 players on a football team; 1,000 words that a picture is worth (I know, that stretches it); 29 days in February in a leap year; 64 squares on a chessboard; 40 days and nights of the Great Flood.