PuzzleCorner

Alan Faller asks a question that comes up from time to time—always worded something like "Are there (or why aren't there) any books compiling TR puzzles?" My standard reply is that there are no copyright problems known to me. As far as I can tell, "all" that is needed is for a potential co-author to volunteer to select problems and deal with some publisher.

It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now. As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred, as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent via e-mail, since these produce fewer typesetting errors.

Problems

JUL 1. Somehow I forgot to include the hand with **F/M 1**. Not surprisingly, no one was able to solve the problem without the hand, so I am repeating the complete problem now and renaming it **Jul 1**.

In the hand below, submitted by Doug Van Patter, the bidding was short and sweet: South opened with 1H, West bid 2C, North bid 4H, and everyone then passed. How can South make the contract after opening lead of the King of Clubs?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO: ALLAN GOTTLIEB NEW YORK UNIVERSITY 715 BROADWAY, 10TH FLOOR NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

North
♠ J 8 6
♥ K 7 4
♦ K Q 10
♣ A 10 9 8

West
♠ K 10 3
♥ 10 3
♦ 8 4
♣ K Q J 7 5 2

East
♠ Q 7 5 4 2
♥ 9 8
♦ A J 7 6 5
♣ 3

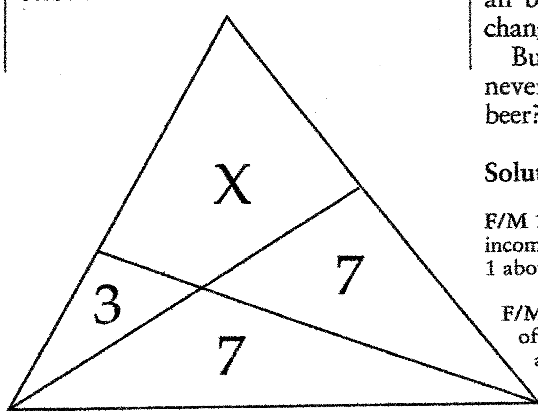
South
♠ A 9
♥ A Q J 6 5 2
♦ 9 3 2
♣ 6 4

JUL 2. Phil Bonomo seems to like "space cadet" problems, especially those involving navigation satellites. In '93 he asked about their velocity, now he questions their altitude.

Geosynchronous orbits are generally taken to be the orbits of largest radius (highest altitude) for earth-orbiting spacecraft. These circular orbits, of radius R_s (≈26,300 miles), are characterized by a 24-hour orbital period and are typically used by the equatorial, "stationary" class of communications spacecraft. Another class of communications spacecraft (the Russian "Molniya" class) employ inclined, highly eccentric orbits characterized by a 12-hour orbital period.

For what orbital conditions, if any, is the largest (apogee) radius of a Molniya orbit greater than the synchronous orbital radius R_s?

JUL 3. Nob Yoshigahara wants you to figure out the area of X in the figure below.



Speed Department

Here is one Steve Chilton gives his "methodology and statistics" class every year:

Around the turn of the century, back when you could still buy a 10-cent beer, a small logging town on the U.S.-Canadian border was experiencing a strange currency exchange situation. On the Canadian side of the border, a U.S. dollar was only worth 90 Canadian cents, while on the U.S. side, a Canadian dollar was only worth 90 U.S. cents. (In other words, the citizens of both countries discounted the other country's currency by 10 percent.)

In this particular town, the international border ran right down the center of the main street, and there were bars on both sides catering to loggers from the surrounding area. One Saturday, an American logger rolled into town with little money (only U.S. $1.00) but lots of financial cunning. He stopped at the first bar he found on the U.S. side of the street, ordered himself a10-cent beer, paid with his U.S. dollar, and asked for a Canadian dollar in change (worth only U.S. $.90, remember). After finishing his beer, he walked across the street to a Canadian bar, ordered another 10-cent beer, paid with the Canadian dollar, and asked for a U.S. dollar in change (there, worth only Canadian $.90. Back he went to the American side for another beer, then back across to the Canadian side—and so on all afternoon and evening, finally staggering back to his camp after a final drink from a Canadian bar and a U.S. one-dollar bill in change—just as he had started out with.

But what his fellow loggers could never figure out was, who paid for the beer?

Solutions

F/M 1. As indicated above this problem was incomplete and the complete version is now Jul 1 above.

F/M 2. Eugene Sard begins this month's offerings with a geometry problem. Given a triangle ABC, find a geometrical con

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Puzzle

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struction of a point X that mimimizes the sum S of the lengths AX+BX+CX. Calculate the value of S in terms of the side lengths AB, BC, and CA.

The following solution is from Matthew Fountain:

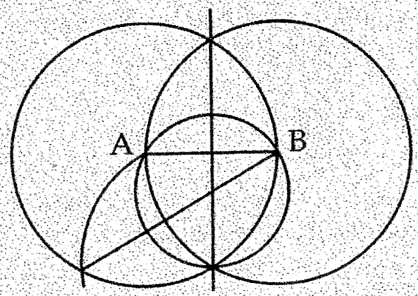
The trivial case is when one angle of the triangle equals or exceeds 120 degrees. Then S equals the sum of the two smaller sides and X is located at the vertex included between these sides.

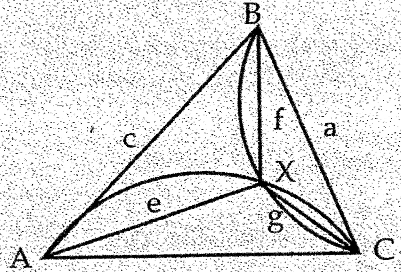
The following method applies when all angles are less than 120 degrees. Inside the triangle construct two 120-degree arcs of a circle, each on a separate side. Locate X at the point of intersection of these two arcs.

The first figure shows the construction of a 120-degree arc on line AB. The construction is similar to drawing an equalateral triangle with the given line as a side. Perpendicular bisectors to two sides meet at the center of the triangle. When a circle is drawn about this center to circumscribe the triangle, each side subtends an arc of 120 degrees. Two sides of the triangle are not shown in the diagram as they are not needed in the construction.

I conceived this construction by recalling that AX+CX would be constant if X moved on an ellipse in which AX and CX were focal radii and that BX would be minimized when it was a normal to that ellipse. If, for example, BX were not a normal to the ellipse with AX and CX as focal radii, then X can be shifted along the ellipse until it is normal to the ellipse, reducing BX while not changing AX+CX. S is minimal only when AX, BX, and CX is each normal to the ellipse having the other two as focal radii.

As $\cos(120°) = -1/2$, $a^2 = fg+f^2+g2$, $b^2 = eg+e^2+g^2$, $c^2 = ef+e^2+f^2$, and equation (1), $a^2+b^2+c^2 = fg+eg+ef+2e^2+2f^2+2g^2$. Taking the area of a triangle as half the product of two sides and the sine of the included angle and $\sin(120°) = \sqrt{3}/2$, the area A of triangle ABC equals $\sqrt{3}/4$ (fg+eg+ef). Adding 3fg+3eg+3ef to both sides of equation (1) and taking the square root yields $2(e+f+g) = \sqrt{a^2+b^2+c^2+3(fg+eg+ef)}$. The area A is also equal to $\sqrt{s(s-a)(seb)(s-c)}$ where $s = (1/2)(a+b+c)$ so that $3(fg+eg+ef) = 4\sqrt{3}A$. $AX+BX+CX = e+f+g = \sqrt{(a^2+b^2+c^2)/2+2\sqrt{3}\, s(s-a)(s-b)(s-c)}$.





FM 3. Warren Himmelberger asks a variation of the birthday problem that he first encountered in *One Hundred Mathematical Curiosities* by William Ransom. What are the odds that (at least) 2 people in a random group of 16 people have birthdays on consecutive days? Now replace 16 by 30.

This problem is harder than it might appear even after you assume 365 days in every year and that 31 Dec and 1 Jan are consecutive. For a group of two people the chances of not having consecutive birthdays is 363/365 (the second birthday can be in any of the 363 days not consecutive with the first). But for three people it is not (363/365)(361/365) since there are not always 361 days that fail to be consecutive with the first two non-consecutive birthdays (two reasons: the two birthdays could coincide leaving 363 non-consecutive days, and the two birthdays could have just one day between leaving 362). It gets worse for groups of more than 3. Tom Harriman avoided this trap. His solution follows.

Let n = number of persons in the group
c = number of persons more than one who share a given birthday, aggregated for all shared birthdays
b = n-c = number of different birthdays

We determine the totality of derangements in which there are *no* birthdays on consecutive days.

Then each birthday, signified by 1, must be followed by a non-birthday, signified by 0. Hence the 365-day year consists of b elements *10* mixed in with 365-2b elements *0*. For example, one derangement for n = 4, c = 0 could look like this:

10 10 0 10 0 0 0 0 . . . . . . 0 0 0 0 10.

There are C(365-b, b) different combinations of those elements.

When c>0, within any of those combinations the 1 in some of the 10-elements represents two, three, four, or more persons having the same birthday thus differentiating the 10-elements. Therefore there are different arrangements of the 10-elements themselves. For instance, when n = 16 and c = 5 hence b = 11, for the case where three persons have the same birthday, and three other birthdays claim two persons each, there are C(b,1) elements that could have the one set of triplets, and C(b-1,3) ways that the other elements could contain the three sets of twins. Thus there are C(b,1)C(b-1, 3) ways the 10-elements themselves can be arranged.

Further, within any of those arrangements, the first 1 represents the birthday of any one of the n individuals; the second 1, that of any one of the other n-1; the third, that of any one of the remaining n-2; etc., some 1s representing two or more individuals whose birthdays coincide. As is well know, the number of permutations is P = 16!/3!2!2!2! for the case above.

Finally, the number of derangements for each case is given by the product of the combinations, arrangements, and permutations. The aggregate of the derangements for all pertinent values of c and all significant cases, divided by the number of all possible birthday arrangements, 35^n, gives the derangement probability, p. The probability that consecutive birthdays will happen = 1-p.

For n = 16, the probability is .5046; for n = 30, .9146.

Better Late Than Never

1995 Jul 3. Mihail Ionescu and Lowell Schwartz found more direct solutions.

Other Responders

Responses have also been received from D. Eckhardt, S. Feldman, E. Hume, I. Mazin, K. Rosato, and R. Sinclair.

Proposer's Solution to Speed Problem

The central banks [ed.].