ALLAN J. GOTTLIEB, '67

Kissing Cosines

Hello from the land of boxes. My wife, Alice, has accepted a position as associate medical director of Roche Dermatologics (the dermatology division of the Hoffman LaRoche drug company) and we have moved to New Jersey! As many of you can well imagine, the local recycling center will be well supplied with cardboard and packing paper.

Problems

N/D 1. Lester Steffens wonders what is the highest score a Bridge pair can obtain on a single hand (excluding illegalities and penalties for reneging, etc.) when neither of them has a card higher than a ten.

N/D 2. Nob. Yoshigahara wants you to substitute the digits 1–9 once each in the following equation.

$$\frac{AB}{CDE} + \frac{FG}{HI} = 7$$

N/D 3. John Rule has a point P situated inside a square ABCD so that PA=1, PB=2, PC=3. He wants you to calculate angle APB "using only the methods of Euclid."

Speed Department

Here is a "mental creativity challenge" from my NYU colleague Ron Bianchini. Each item contains the initials of words that make it correct and you are to fill in the words. For example, given "16 = O. in a P." the answer is "Ounces in a Pound." Now try the following five examples: "9 = P. in the S. S."; "88 = P. K."; 13 = S. on the A. F."; "32 = D. F. at which W. F.," "18 = H. on a G. C."

Solutions

JUL 1. We begin with a well-known computer problem suggested by the late Robert High:

In your favorite programming language (C, Lisp, Apl, etc.) write a program that, when run, produces output that is an exact copy of its own source code. Calls to system functions to "echo" the source from a file are not in the spirit of the problem!

Although I personally use C more than Lisp, my conversion to the Emacs "editor" has somewhat rekindled a love of Lisp that I had as an MIT undergraduate. The Lisp solution from Walter Hamscher also appeals to me. Hamscher writes:

I've never sent a response to any of the puzzles in your column till now—this problem concerning self-duplicating code was just too easy an opportunity to advertise my favorite language, Lisp:

```
((lambda (x) (list x (list (quote quote) x)))
(quote (lambda (x) (list x (list (quote quote) x)))))
```

Here is a C solution from Scott Brown:

```
char a[]="char a[]=;main()
{printf(a+48,a,34,a,34,10,a+9);}
%.9s%c%s%c%c%.39s"
;main(){printf(a+48,a,34,a,34,10,a+9);}
```

JUL 2. A "classic" from Gordon Rice:

While cleaning out my office for retirement, I came across my freshman physics text, *Introduction to Mechanics and Heat*, 2nd edition, 1939, by N.H. Frank. On page 204 is the following gem:

A slender homogeneous rod of length 60 cm., resting on a perfectly smooth horizontal surface, is struck a blow at right angles to the length of the rod at one end of the rod. Find the distance through which the center of the rod moves while it makes one complete revolution.

The following solution is from Matthew Fountain:

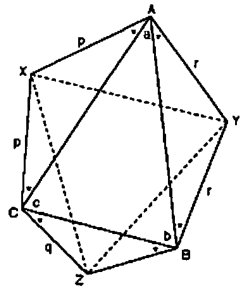
The center of the rod moves 62.8 cm. The force F on the end of the rod normal to its length imparts an acceleration a to the center of the rod equal to F/M, m being the rod's mass. As F is 30 cm. out of line with the center of gravity of the rod, F produces a torque $T = 30F = 30\,ma$ on the rod. Recalling that the definition of "moment of inertia" is "the ratio of the torque applied to a rigid body free to rotate about a given axis to the angular acceleration thus produced about that axis and equal to the sum of the products of each element of mass by the square of its distance from the given axis," we may write: $(30ma)/(d\omega/dt) = \int_0^{30} mx^2dx = 300m$. Therefore $d\omega/dt = 30ma/300m = a/10$.

Although a sharp blow exerts a varying force, at every instance the ratio of rotational acceleration to the acceleration of the center of gravity remains constant, and consequently the velocity of rotation in radians is at each instant equal to one-tenth the velocity of the center of the rod in centimeters. In the time it takes the rod to rotate 2π radians the center of the rod advances $20\pi = 62.8$ centimeters.

JUL 3. Consider an arbitrary triangle ABC. From each of the vertices, extend two lines on the exterior of the triangle, each at a 30-degree angle from the sides. These lines intersect at the points X, Y, and Z opposite sides AB, BC, and CA. Show that triangle XYZ is equilateral.

Howard Stern sent us a nicely done solution, reprinted below.

Consider the accompanying diagram:



where $\triangle ABC$ has angles {a,b,c}, angles denoted by a "*" are 30°, and we are to show $\triangle XYZ$ is equilateral.

Lengths {p,q,r} are labelled as such because $\triangle$s AXC, BCZ, and AYB are isosceles. In addition, by the Law of Cosines we have:

$$p = \frac{AC}{\sqrt{3}} \qquad q = \frac{BC}{\sqrt{3}} \qquad r = \frac{AB}{\sqrt{3}}$$

Applying the Law of Cosines again we have:

$$XY^2 = p^2+r^2-2pr\text{Cos}(60°+a)=$$

$$\frac{AC^2}{3}+\frac{AB^2}{3}-\frac{2(AC)(AB)}{3}\left\{\frac{1}{2}\text{Cos}(a)-\frac{\sqrt{3}}{2}\text{Sin}(a)\right\}$$

But $\text{Cos}(a) = \dfrac{AC^2+AB^2-BC^2}{2(AC)(AB)}$ using the Law of Cosines.

Also $\dfrac{\text{Sin}(a)}{BC} = \dfrac{\text{Sin}(b)}{AC} = \dfrac{\text{Sin}(c)}{AB} = Q$ (some constant) by the law of Sines.

Substituting and simplifying we get:

$$XY^2 = \frac{AC^2}{6}+\frac{AB^2}{6}+\frac{BC^2}{6}+\frac{\sqrt{3}}{2}Q(AC)(AB)(BC)$$

Due to the symmetry of the problem, solving for the other sides: XZ or YZ yields exactly the same expression. Thus the three sides are of equal length, implying $\triangle XYZ$ is equilateral.

Better Late Than Never

F/M 3. Harold Boas has located references to a variant of this problem that appeared on the Cambridge Math Tripos Exam in 1871 so problems like this have been around for over a century.

JUL SD. Tim Johnson was able to generalize this problem to the case where an unknown number of the M jars have weight A and the remaining jars have weight B for arbitrary M, A, and B.

Other Responders

Responses have also been received from H. Boas, E. Dawson, W. Hartford, R. Hess, H. Hodara, R. Hoffman, T. Johnson, N. Megill, D. Miller, R. Moeser, G. Neben, S. Ponzio, J. Prussing, K. Rosato, E. Sard, A. Ucko, and C. Wampler.

Proposer's Solution to Speed Problem

"Planets in the Solar System," "Piano Keys," "Stripes on the American Flag," "Degrees Farenheit at which Water Freezes," "Holes on a Golf Course."

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU