ALLAN J. GOTTLIEB, '67

Roving Riverward

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present three regular problems (the first of which is chess, bridge, go, or computer-related) and one "speed" problem. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers who responded. For example, solutions to the problems you see below will appear in the February/March issue and this issue contains solutions to the problems posed in May/June. Since I must submit the February/March column in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Respondents" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the January issue of each year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Problems

OCT 1. We begin with a Bridge problem from J. Harmse who notes that the highest possible declarer score is obtained by playing 1NT redoubled vulnerable making all 13 tricks. The problem is to devise a distribution of the cards in which the above occurs with "normal" bidding and play.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

OCT 2. The following problem is from Robert Sackheim. A is 73 feet from a straight river, and B is on the same side of the river but not so far from it. M and N are the points on the river nearest to A and B respectively. The length of AB, MN and BN are whole numbers of feet. Joan walks from A to B via the river (i.e., at one point she is at the river), taking the shortest possible route, and this is also a whole number of feet. How far does she walk? What is the direct distance from A to B?

OCT 3. Richard Hess entitles this one "The missing term" and writes: Given the series

...,35,45,60,x,120,180,280,450,744,1260,...

the problem is to find a simple continuous function to generate the series and from it to determine the surprise answer for x.

Speed Department

There are 13 diamond *cards* in a card deck. How many diamonds are on those 13 cards?

Solutions

M/J 1. We begin with a Bridge problem that Winslow Hartford sent us from the London *Sunday Observer*. In the hand shown, West missed the killing diamond opener against 7H and instead led the spade jack. How can South now make the grand slam?

North
♠ Q 9 5
♥ 10 6 5 3
♦ A Q
♣ J 8 7 2

West
♠ J 10 8 7
♥ 4 2
♦ 10 9 6 5
♣ 9 6 4

East
♠ K 6 4 3 2
♥ 9
♦ K J 9 7 4
♣ 5 3

South
♠ A
♥ A K Q J 8 7
♦ 3 2
♣ A K Q 10

The following solution is from Jerry Grossman. Win the spade ace, cash five hearts, pitching the diamond from dummy, and play the A, K, Q of clubs. Now with this 4-card ending, lead the last club to the Jack:

North
♠ Q 9
♥
♦ A
♣ J

West
♠ 10 8
♥ K J 6
♦ 10 9
♣

East
♠ K 6
♥
♦ K J
♣

South
♠
♥ 7
♦ 3 2
♣ 10

On the club, West must discard. If he discards a spade, then you can lead the Q of spades off dummy for a ruffing finesse against East's king, pinning West's ten. The ace of diamonds provides the entry back to the board. So assume that West discards a diamond.

Now East is squeezed. If he discards a spade, then ruff a spade and dummy is good. If he discards a diamond, then cash the ace of diamonds, dropping the opponents' remaining cards in that suit, and your hand is good. A neat trump squeeze.

M/J 2. Gordon Rice is thinking of four positive integers

$$0 < A < B < C < D$$

that have a curious property. When numbers are written in base D

$$AB \equiv A (modulo C)$$

and

$$BA \equiv B (modulo C).$$

For what values of D do solutions exits? Are they unique? Note that AB does not represent AxB. Instead it signifies juxtaposition, e.g., if A=24 and B=345, AB is 24345.

Robert High writes: Gordon Rice's relationship is far from unique; I found 7,268 solutions with D<100 and 98 with D<20. Since $0 < A < B < C < D$, the conditions boil down to the simultaneous congruences

$$A \cdot D + B \equiv A \bmod C$$
$$B \cdot D + A \equiv B \bmod C$$

(The fact that A and B are "written in base D" is really irrelevant.)

A little manipulation leads to the conclusion that these conditions are satisfied if and only if we can find $A < B < C < D$ such that

$(B+A) \cdot D$ is divisible by C

and $(B-A) \cdot (D-2)$ is divisible by C.

These conditions are satisfied by many families of quadruples; a simple three-parameter family is C=B+A; D=K·C+2, but there are many other solutions as well, such as (1,7,12,18) or (13,20,21,98).

As noted by Richard Hess, it is easy to see that no solutions exist for D<5, unique solutions exist for D=5 and D=6, and many solutions exist for every D>6

M/J 3. Daniel Morgan wants to know the expected point count for a randomly dealt Bridge hand of 13 cards? High cards are valued as Ace=4, King=3, Queen=2, and Jack=1. In addition a void (no cards in a suit) contributes 3 points, a singleton contributes 2, and a doubleton contributes 1.

Stephen Janowsky sent a solution involving fairly little calculation (i.e., a computer was not required). He uses the notation E() for expected value, P() for probability and # for "number of" and writes:

The expected high card value of a bridge hand is easily determined using the additivity of expected values:

E(ace points) = 4E(# aces) = (4)(4) E(# spade aces) = (4)(4) P(spade ace) = (4)(4)(1/4) = 4. Thus E(high card pts) = 4 + 3 + 2 + 1 = 10.

E(void pts) = 3E(# voids) = (3)(4) E(# spade voids) = (3)(4) P(spade void) = (3)(4) (39 choose 13) / (52 choose 13)

E(singleton pts) = (2)(4) P(spade singleton) =

Continued on Page MIT 54

Robert W. Barker, '21; January 21, 1992; Narbeth, Pa.
Richard P. Windisch, '21; May 4, 1992; Naples, Fla.
H. Felton Metcalf, '22; May 12, 1992; Newmarket, N.H.
Charles W. Ufford, '22; April 29, 1992; Newtown, Pa.
Edwin M. Goldsmith, Jr., '23; May 4, 1991; Wyncote, Pa.
Herbert L. Hayden, '23; May 2, 1992; Lancaster, Mass.
Max W. Tetlow, '23; May 14, 1992; New London, N.H.
Jacob Lurie, '24; April 9, 1992; Denver, Colo.
John E. Black, '25; February 2, 1992; Clearwater, Fla.
Homer S. Davis, '25; June 6, 1992; Seattle, Wash.
F. Cushing Foss, '25; May 7, 1992; Westford, Vt.
Isaac W. Gleason, '26, SM '27; May 21, 1992; Middlebury, Vt.
Albert S. Goleman, MAR '25; November 23, 1991; Houston, Tex.
Naomi C. Turner, '26; May 12, 1992; Arlington, Mass.
Winfred F. Dunklee, '27; July 25, 1991; Hamden, Conn.
Arthur Dunlevy, '27; May 1, 1992; North Falmouth, Mass.
Robert C. Wallace, '27; April 21, 1992; St. Charles, Ill.
Homer A. Burnell, '28; June 26, 1992; Chicago, Ill.
Harold L. Geiger, '28; April 18, 1992; Minneapolis, Minn.
Robert Walker Hunn, Jr., '28, MAR '29; January 1, 1992; Santa Monica, Calif.
Willis G. McGown, '28; May 6, 1992; Chicopee Falls, Mass.
James Cooper, '29; May 25, 1992.
Alfred H. Hayes, '29, SM '30; March 10, 1991; Whiting, Ind.
Laurence A. Horan, '29; May 8, 1992; North Chatham, Mass.
Robert E. Jackson, '29; May 3, 1992; Marblehead, Mass.
Armand M. Morgan, SM '29; April 25, 1992; Portland, Me.
Edward B. Papenfus, SM '29; June 5, 1992; Vancouver, BC, Canada.
Leonard C. Peskin, '29, SM '31, ScD '36; July 12, 1991; Wyncote, Pa.

Mark C. Culbreath, '30; April 12, 1992; Raymore, Md.
Charles D. May, '30; June 13, 1992.
Lloyd E. Montgomery, '30; April 29, 1991; Colorado Springs, Colo.
George D. Love, '31; April 28, 1992; Portland, Me.
Carrington Mason, '31; March 12, 1992; Houston, Tex.
Robert Sanders, '31; May 31, 1992; Annapolis, Md.
Howard L. Richardson, '31, SM '32; May 1, 1992; New Britain, Conn.
Allan L. Dunning, SM '32; April 22, 1992; Stonington, Conn.
J. Richard Rafter, '32; May 18, 1992.
Donald C. Sanford, '32; April 28, 1992; Woodbury, Conn.
Harland H. Young, Jr., PhD '32; December 31, 1991; Columbus, Ohio
Gardner Harvey, '33; February 23, 1991; Wilmington, Del.
Frederick A. Ladd, Jr., '33; November 12, 1991; Leesburg, Fla.
Hugh W. MacDonald, '33; April 29, 1992; Green Brook, N.J.
Meredith E. Morgan, '33; May 3, 1992; Kerhonkson, N.Y.
Charles E. Quick, '33; October 26, 1990; Traverse City, Mich.
Ernst W. Spannhake, '33, SM '35; April 30, 1992; Akron, Ohio
Robert C. Wellwood, '33; May 15, 1992; Saline, Mich.
Jack Delmonte, SM '34; April 15, 1992; Glendale, Calif.
Albert M. Grass, '34; May 29, 1992; Wollaston, Mass.
Louis T. Montant, Jr., '34; 1992.
Arthur O. Williams, Jr., '34; May 16, 1992; Doylestown, Pa.
John L. Fuller, PhD '35; June 8, 1992; Cambridge, Mass.
Chester H. Brown, Jr., '37; September 22, 1989; Pittsburgh, Pa.
Rolf E. Schneider, '37; January 24, 1991; Parkersburg, W.V.
F. William Brown, III, '38; May 15, 1992; Kensington, Calif.
Richard H. Koehrmann, '38; May 3, 1992; Alton, Ill.
John J. Perkins, '38; November 15, 1991; New Bern, N.C.
William S. Quigley, Jr., '39; June 6, 1992; North Scituate, Mass.
George E.B. Hill, '40; May 7, 1992; Kentfield, Calif.
Charles S. Butt II, '41; April 16, 1992; McLean, Va.
Charles Margnetti, '41; June 9, 1986; West Roxbury, Mass.
John W. Clarke, '42; April 10, 1992; Missouri City, Tex.
John Hinchman, '42; May 11, 1992; West Cornwall, Conn.
Bernard Brindis, '43; May 29, 1992; Boca Raton, Fla.
Tan Chih Lu, SM '44; June 2, 1992; Tustin, Calif.
Walter J. Loughlin, '46; May 2, 1992.
James E. Haggett, '47; May 26, 1992; Shrewsbury, Mass.
Victor H. Pomper, '48, SM '50; May 9, 1992; Weare, N.H.
Roger D. Smith, '48; March 30, 1992; New Paltz, N.Y.
James C. Buck, '49, SM '65; August 25, 1990; Coronado, Calif.
Donald P. Germeraad, '50; May 11, 1992; Underwood, Mass.
Gerard J. Griesmer, '52; May 29, 1990; Aurora, Colo.
Howard K. Larson, '52, SM '54; August 5, 1991; Saratoga, Calif.
George L. Tuer, Jr., ScD '55; March 5, 1991; Aiken, S.C.
Dorothy O. Schlag, '56; 1992; Redlands, Calif.
John E. Murray, Jr., '57; 1992; Framingham, Mass.
Amarjit Singh, '61; July 11, 1991; New Delhi, India.
George Piotrowski, '64, SM '65; May 24, 1992; Gainseville, Fla.
Dirk Berghager, '62, SM '64; January 29, 1984; Cascais, Portugal
William J. Day, '67; May 4, 1992.
Charles J. Sheehan, SM '67; May 12, 1992; Andover, Mass.
Kathleen S. Sargent, PhD '71; February 18, 1991; Winchester, Mass.
Mark E. Schaefer, PhD '77; April 13, 1992; Atlanta, Ga.
Albert B. Lester, '82; June 7, 1992; Westwood, Mass.
Paul J. Ramos, '86; May 24, 1992; Seekonk, Mass.

PUZZLE CORNER

Continued from Page MIT 55

(2)(4) (13)(39 choose 12) / (52 choose 13)

E(doubleton pts) = 4 P(spade doubleton) = (4) (13 choose 2)(39 choose 11) / (52 choose 13)

Combining the above, E(distribution pts) = (19)(29)(31)(37)(239) / [(23)(41)(43)(47)(49)] =~ 1.61748

So E(pts) =~ 11.61748

Robert High assumes that the proposer "DANIEL MORGAN is your MAIN GOREN LAD," which just goes to show what happens when you start hanging out with Nob. Yoshigahara.

Better LateThan Never

M/J 1. Darold Rorabacher and George Blondin noticed that numbers ending in one were inadvertently omitted. For example, the fourth n with F(n,n)=G(n,n)=n is "two hundred one." There are 95, not 64 solutions as previously claimed.

SD. I really do not normally include comments on speed problems but quite a ruckus has occurred regarding the minimum number of pitches in a complete baseball game and the number of calls by the first place umpire. I somehow cannot resist printing the following from Tony Carpentieri but will try hard to refrain from speed problem follow-ups in the future.

"I disagree on the number of pitches/plate umpire calls in a complete game. There are things that a pitcher can do, such as (I believe) touching his tongue to his pitching hand that result in one ball being called. So, each half inning goes like this: Pitcher licks hand (or other stuff) 12 times in a row. All calls made by field umpires. He then picks off the three runners, with calls naturally made by field umpires. Well, this goes on for a bunch of half innings, let's say 17. Then a pitcher licks his hand 16 times, and walks a run home in the bottom of any inning after 8. Pitches: 0, plate umpire calls: 0."

In addition, Joseph Gurland tells me that our problem was printed in rec.sport.baseball, a popular electronic newsgroup. Gurland sent copies of several other no-pitch solutions found by newsgroups readers.

Other Responders

Responses have also been received from M. Chellino, S. Feldman, M. Fountain, D. Grant, A. Halberstadt, M. Handel, R. Hedrick M.Herbert, B. Inadomi, R. Loesch, L. Nissim, A. Ornstein, X. Peng, K. Rosato, J. Rosenthal, G. Schwartz, and A. Wasserman.

Proposer's Solution to Speed Problem

81. Each of the 13 cards has a diamond in the upper left corner and one in the lower right corner, for a total of 26. The Jack, Queen, and King have no other diamonds on them. The Ace through10 have 1+2+3+4+6+7+8+9+10=55 additional diamonds.