ALLAN J. GOTTLIEB, '67

Coriolis Aloft

By the time you read this article I will be packing for Australia! The International Symposium on Computer Architecture is being held there this year. I chaired the program committee and am anxious to see if my committee did a good job in selecting papers. But of course I am even more interested in seeing the southern cross! As you can probably tell, I have never crossed the equator before. I was planning to empty a bathtub on the plane to watch the circling water get confused but, alas, the airlines would have none of it (bathtubs fit neither under a seat nor in an overhead rack).

Problems

M/J 1. We begin with a Bridge problem that Winslow Hartford sent us from the *London Sunday Observer*. In the hand shown, West missed the killing diamond opener against 7H and instead lead the spade jack. How can South now make the grand slam?

North
♠ Q 9 5
♥ 10 6 5 3
♦ A Q
♣ J 8 7 2

West
♠ J 10 8 7
♥ 4 2
♦ 10 9 6 5
♣ 9 6 4

East
♠ K 6 4 3 2
♥ 9
♦ K J 9 7 4
♣ 5 3

South
♠ A
♥ A K Q J 8 7
♦ 3 2
♣ A K Q 10

M/J 2. Gordon Rice is thinking of four positive integers

$$0 < A < B < C < D$$

that have a curious property. When numbers are written in base D

$$AB = A \text{ (modulo C)}$$
and $$BA = B \text{ (modulo C)}.$$

For what values of D do solutions exits? Are they unique? Note that AB does not represent AxB. Instead it signifies juxtaposition, e.g., if A=24 and B=345, AB is 24345.

M/J 3. Daniel Morgan wants to know the expected point count for a randomly dealt Bridge hand of 13 cards? High cards are valued as Ace=4, King=3, Queen=2, and Jack=1. In addition a void (no cards in a suit) contributes 3 points, a singleton contributes 2, and a doubleton contributes 1.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: GOTTLIEB@NYU.EDU

Speed Department

Speedy Jim Landau wants to know the smallest possible number of pitches in a complete baseball game and how many calls does the plate umpire make during this game?

Solutions

JAN 1. Our "first" problem is a computer offering from Bob High. Write the first n numbers in alphabetical (dictionary) orders as they are spelled out (i.e., one, two, three,...one million,...). To avoid ambiguity, use no "ands" or hyphens, so 837,301 would be written "eight hundred thirty-seven thousand three hundred one." 1,897 is "one thousand eight hundred ninety-seven," not "eighteen hundred ninety-seven." Define two functions, F(m,n) and G(m,n) as follows: F(m,n) is m th number in the alphabetical list of the first n numbers; G(m,n) is the position of the number m in this list. (For given n, F and G are inverses.) we ask: (1) What is F(1,000, 1,000,000)? What is G(1,000, 1,000,000)? (2) What is F(1,000,000, 1,000,000)? What is G(1,000,000, 1,000,000)? (3) For what numbers n is F(n,n) = G(n,n) = n? List the first dozen.

Speedy Jim Landau sent us a detailed solution to this problem and an extension of it. Interested readers should write to Faith Hruby at *TR* for a copy. A summary of Landau's solution follows: Consider the numbers beginning "eight." There are, in alphabetical order:

number	quantity
eight	1
eighteen	1
eighteen thousand xxx	1,000
eight hundred xxx and	100
eight hundred thousand xxx	100,000
eight thousand xxx	1,000
eighty	1
eighty x	8
eighty thousand xxx	10,000
eighty two	1
	112,112

The case of leading "one" is different because "one million" must be included and the two-digit and five-digit numbers beginning with the digit one fall alphabetically under the second digit (e.g., "eighteen").

Now we are ready to start answering questions.

(1) What is F(1000, 1M)?

F(1002, 1M) is "eighteen thousand two," which is the highest number alphabetically in the "eighteen thousand" series. Working backwards, F(1000, 1M) is "eighteen thousand twenty two" and F(1000, 1M) is "eighteen thousand twenty." [Landau's full situation also tabulates numbers with other leading digits. He then proceeds:] What is G(1000, 1M)? The "one thousand" series falls at the end of the "one" series, which means G(1xxx, 1M) runs from 549,552 to 550,551. Since 1000 falls at the beginning of the "one thousand" series, G(1000, 1M) is 549,552.

(2) What is F (1M, 1M)?

"Two" sorts highest alphabetically, and can only be followed by "hundred" and "thousand." The last number alphabetically is F(1M, 1M) = two thousand two.

What is G(1M, 1M)?

The "one" series goes

one
one hundred xxx
one hundred thousand xxx
one hundred twenty, twenty two, and two
one million
one thousand xxx

Using the answer to (1), we find G(1M, 1M) = G(1000, 1M) - 1 = 549,551.

(3) For what n does F(n,n) = G(n,n) = n?

There are exactly 64 such n. The first 4 such n are:

one
two
two hundred
two hundred two

Notice the pattern 2, 200, 202. It will repeat itself below. The next 4 n are:

two thousand
two thousand two
two thousand two hundred
two thousand two hundred two (which is F(1M, 1M) above)

There will be no new n until we find a suffix for "two" which sorts alphabetically higher than "thousand." The first such is "trillion." The next 8 n are:

two trillion
two trillion two
...
two trillion thousand two hundred two

JAN 2. Robert Bart offers the following extensions to an old problem from Nob. Yoshigahara. What is the smallest positive integer whose square root has a decimal expansion beginning with ten distinct digits. Now consider cube roots instead of square roots. Finally consider ith roots for i = 4,5,...,10.

Daniel J. Weidman not only solved this problem but, as with Landau above, extended the problem and solved the extension. Once again interested readers should contact Ms. Hruby for a copy. Weidman's solution to the original problem follows. Note that we are interpreting the "decimal expansion" to begin after the decimal point.

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The 2 th root of 143 is 11.9582607431014
The 3 th root of 939 is 9.792386145009786
The 4 th root of 633 is 5.015923768441686
The 5 th root of 8117 is 6.051723946894983
The 6 th root of 896 is 3.104926578310817
The 7 th root of 551 is 2.463729851098231
The 8 th root of 558 is 2.204597318658172
The 9 th root of 759 is 2.089425371646355
The 10 th root of 667 is 1.916075348263711

Other Responders

Responses have also been received from D. Church, D. Eckhardt S. Feldman, M. Fountain, I. Shalom,

Proposer's Solution to Speed Problem

28. One player on the home team hits the first pitch for a home run. Every one else grounds out on the first pitch. The game is called on account of rain after 4 1/2 innings. The home plate umpire makes 1/4 of a call. The ground outs are called by the first base umpire but the plate umpire participates in the decision to call the game. □