PUZZLE CORNER

ALLAN J. GOTTLIEB, '67

A Silver Jubilee

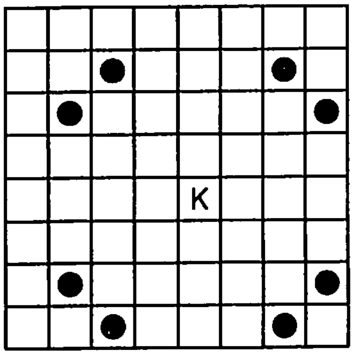
Twenty-five years! This issue marks my silver anniversary as a puzzle editor, which means among other things that I have been producing "Puzzle Corner" for more than half my life. It is hard for me to accept the fact that I started writing this column before most of my current students were born. Talk about feeling old. If I remember the Constitution correctly, "Puzzle Corner" is now eligible to run for the House of Representatives.

More seriously, I have fond memories from writing this column, mostly from many touching letters I have received. I recall two senior contributors, R. Robinson Rowe and Phelps Meaker, who, after many years of active participation, wrote to say that they would have to limit their responses since the act of writing was becoming difficult. When they could no longer correspond, members of their families wrote to me. The greatest outpouring occurred last year when, for a while, it appeared that the column would be retired. My wife, quite an accomplished biomedical researcher, reports that colleagues have asked if she was the Alice Gottlieb, the one referred to in "Puzzle Corner." Finally, the column has won a few awards, the most recent being the Lobdell award last year. Since the column is mostly reader generated, you all should feel proud.

Enough of this sentiment, let's get on with the problems so we can become eligible for the Senate and Presidency.

Problems

OCT 1. We start with a chess problem (which may well be a computer problem) from Victor Barocas. It is well known that a knight can tour the chess board, touching each square once and only once, and beginning and ending on the same square. Consider now the generalized knight K(m,n), m ≤ n, which moves m spaces along one axis of the board and n spaces along the other (the normal knight is K(1,2); also see diagram). For what values of m and n can the knight tour the board?



Moves available to K(2,3) at position e4.

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012, OR TO: gottlieb@nyu.edu

OCT 2. Gordon Rice wonders how many Pythagorean triangles you can find in which one of the three sides is 1991.

OCT 3. That famous riverboat gambler, Bob High, was inspired by 1989 JUL 5 to ask a two-part question about shuffling cards. First, in a shuffled deck, what is the average (expected) number of cards occupying their original position? (This is to ask, for n = 52, what is the average number of fixed points of a permutation of n things.) Second, which is more likely in a random shuffle (permutation) of n things: exactly one fixed point, or exactly none?

Speed Department

Given an odd number D, Temple Patton wants you to find two numbers L and S which when squared and one subtracted from the other gives D.

Solutions

M/J 1. A computer-related problem from Robert High who has a Black Box with two buttons: RED and GREEN. The Black Box can be in any of three states, one of which is known as SUCCESS. (High suggests brave souls try three non-SUCCESS states but we have space only for solutions to the original problem). When the box is in the SUCCESS state, a bell rings. Pressing either button leads to a change of state. From any initial state, there is a sequence of RED and GREEN that will ring the bell.

How many such Black Boxes are there? What is the minimal number of moves guaranteed to ring the bell at least once? How good is the strategy of choosing RED or GREEN randomly, i.e., what is the expected number of moves required?

Leonard Nissim names the three states S (the success state), 1, and 2. Since pushing a button (R or G) always leads to a change of state, there are 26 or 64 such boxes satisfying this condition only. The additional condition is that from states 1 and 2 there is some sequence (of R's and G's) that leads to state S. This eliminates four of the boxes, leaving 60 which satisfy both conditions.

If we are in the S state to begin with, the bell is ringing already. For the purposes of analyzing the sequences leading to S (at least once) from the other states, we can collect the 60 boxes into 15 equivalence classes: We ignore what happens when R or G is pushed from state S, and only keep track of the results of pushing R or G from the other two states.

The following are the results of looking at all 15 equivalence classes:

(1)No sequence of length 3 will guarantee to ring the bell. (i.e., for each sequence of length 3, there is a box and an initial state for which that sequence does not ring the bell.)

(2)The following sequences of length 4 each guarantee to ring the bell at least once: RRGG, RGGR, GRRG, and GGRR.

(3)Starting from states 1 or 2, if we push R or G randomly, the expected number of pushes needed to reach S is 31/15 or 2 1/15. Of course, a non-successful sequence of any length is possible for some boxes, but even the worst boxes have an expected number of pushes equal to 7/2 or 3 1/2.

(4)Starting from states 1 or 2, if we use any one of the length 4 sequences given above, then the expected number of pushes needed to reach S is only 26/15 or 1 11/15

The equivalent question for three non-success states is indeed very much longer, by a factor of about 100. We begin with 3^8 or 6,561 black boxes (before eliminating those for which S cannot be reached from some starting states).

M/J 2. John Rule has a right triangle with integer sides without any common factor. When each digit is replaced by a code letter, the sides are SSWTVU, PTWTS, and RRWWQ. Break the code.

The following solution is from Ken Kiesel, who writes: The hypotenuse < 99,999*sqrt (2), therefore S = 1. The hypotenuse must be between 110,234 and 119,876. Using these limits, the possible combinations of P and R can be easily limited to the following nine:

P = 5, R = 9   P = 7, R = 8   P = 9, R = 5
P = 6, R = 8   P = 8, R = 6   P = 9, R = 6
P = 6, R = 9   P = 8, R = 7   P = 9, R = 7

At this point a simple minded program (I ran it on a pocket programmable calculator) can quickly try the 9,000 combinations for P, Q, R, T, and W which result, even allowing multiple letters to take the same value. A flowgram for the program may be obtained from the editors. The only combination that worked was:

P = 7   R = 8   T = 5   V = 4
Q = 0   S = 1   U = 9   W = 6

The sides are: 75,651   88,660   116,549.

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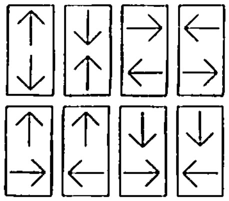
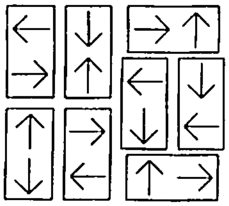
M/J 3. Nob. Yoshigahara has a strange set of 8 dominoes.



He wants you to arrange them into a 4x4 square so that four vertical, four horizontal, and two diagonal lines each contain all four kinds of arrows. The solution to this "ALHAMBRA" problem is not unique.

Nancy Barrows reports that her son Andy gave her an MIT MOM T-shirt for Christmas and, in order to earn a right to wear it, she sent us the solution drawn below. Jim McNamara, who found (essentially) the same solution, reports that the best method of attack was to first arrange the arrows correctly without the box outlines and then draw in the boxes. Frederick Furland reports having the privilege of meeting Nob. in his studio in Japan and obtaining several of his puzzles. One was ALHAMBRA, to which Furland found the Barrows solution as well as a variant in which each diagonal contains four arrows all pointing in the same direction.



Better Late Than Never

Jan 2. Apparently Winslow Hartford's declaration of solvability was just what Fred Tydeman needed for inspiration to produce the following solution. I should add that he was ably assisted by his 386SX, which worked 49 hours to help find the solution.

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                      17433
1422868652)24804869210316
            1422868652
            10576182690
             9960080564
              6161021263
              5691474608
               4695466551
               4268605956
                4268605956
                4268605956
                         0
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Other Responders

Responses have also been received from S. Feldman, G. Rice, M. Fountain, R. Hedrick, A. Silva, E. Dawson, R. Bart, H. Zaremba, R. Kinsley, Jr., J. Dorsey, W. Hartford, A. Mullin, W. Cluett, R. Hedrick, S. Berkenblit, and J. Landau.

Proposer's Solution to Speed Problem

Let $S = (D+1)/2$ and $L = S+1$. Now $L^2-S^2 = 2S+1 = D$.

Deceased

The following deaths have been reported to the Alumni/ae Association since the *Review* last went to press:

Thomas Ewing Hannah, '17; November 1, 1988; Rutherford, N.J.

Uhachi Nabeshima, '19; April 25, 1974; Tokyo, Japan.

Gerald Tattersfield, '20; May 19, 1991; Philadelphia, Pa.

Kenneth B. White, '20; March 27, 1991; Magny-en-Vexin, France.

Mrs. Laighton Evans, '21; June, 1970; Woodbury, N.J.

David Henry Harris, '22; April 24, 1991; Falmouth, Mass.

William Albert Waldschmidt, '22; December 28, 1990; Midland, Tex

Ephraim Fletcher Ingals, '23; January 5, 1990; Del Mar, Calif.

Francis La Verne Smith, '23; May 4, 1991; Seal Beach, Calif.

George Donald Fife, '24; April 15, 1991; Eastham, Mass.

Victor C. Smith, ['24; February 7, 1991; Warren, Mich.

Ronald Alexander Mitchell, '25; May 29, 1991; Stafford Springs, Conn.

Lewis Frothingham Clark, '26; March 8, 1991; Los Angeles, Calif.

Julius Friedman, '27; May 25, 1991; Lynbrook, N.Y.

Leroy Glick Miller, '27; January 30, 1991; Rockbridge, Ohio.

Edgar N. Rousseau, '27; September 11, 1978; Rockford, Ill.

Earl Hatheway Abbe, '29; May 19, 1991; Newbury, N.H.

Edwin Sheldon Worden, '31; May 15, 1991; Mount Dora, Fla.

Philip Donely, '32; May 3, 1991; Williamsburg, Va.

Jacob Millman, '32; May 22, 1991; Sarasota, Fla.

Lawrence Wilder Whitaker, '32; May 5, 1991; Stockbridge, Mass.

John W. Gaylord, '33; March 24, 1991; San Rafael, Calif.

Lawrence Chappell Kingsland, '33; April 18, 1991; Watsonville, Calif.

William Beckett, '34; July 19, 1990; Hamilton, Ohio.

Charles B. Stuart, '34; April 30, 1991; Tucson, Ariz.

Philip B. Walker, Jr., '34; April 25, 1991; Sutton, Mass.

John H. Best, '35; April 17, 1991; Urbana, Ill.

Oscar Fontaine Wiedeman, '35; March 24, 1991; New Orleans, La.

Gordon S. Donnan, '36; May 14, 1991; Newport News, Va.

Herman Brettman, '37; June 11, 1991; Salem, Mass.

John Harry Fellouris, '37; June 11, 1991; New Bedford, Mass.

Robert Treat, Jr., '38; February 11, 1991; Mesa, Ariz.

Eli M. Dannenberg, '39; April 22, 1991; Longboat Key, Fla.

Otto F.A. Arnold, '40; March 26, 1991; South Dartmouth, Mass.

Russell Thorner Werby, '40; February 13, 1991; Brookline, Mass.

Kenneth Andrew Roe, '41; June 3, 1991; Palm Beach, Fla.

Franics B. Bushey, '42; January 8, 1991; Delray Beach, Fla.

Leon W. Freeman, '42; March 15, 1991; Newton Highlands, Mass.

Trent Sumner Russell. '42; March 20, 1991; Castleton, N.Y.

Theodore Harrison White, '42; April 28, 1991; Ardmore, Pa.

James Holt Jr., '43; October 22, 1990; Littleton, Colo.

Guy L. Ottinger, '43; April 28, 1991; Sunnyvale, Calif.

Alfred J. Ehrat, '44; March 13, 1991; York, Pa.

Frederick Maxwell Griffith, '44; September 7, 1989; Charleston, S.C.

William George Martin, Jr., '45; May 15, 1991; Williamsburg, Va.

Robert Kent Schumacher, '45; May 18, 1991; Winnetka, Ill.

Melvin Wilbert Friedman, '46; May 14, 1991; Newton Center, Mass.

Arthur Linz, '46; April 3, 1991; Vero Beach, Fla.

Robert Madison Anderson, '47; November 9, 1990; Oberlin, Ohio.

Edward Theodore Clapp, '47; March 24, 1991; Columbia, Md.

Dogan H. Erokan, '47; July 13, 1988; Danville, Calif.

David Henry Frisch, '47; May 23, 1991; Cambridge, Mass.

Donald Hayward Strider, '47; February 22, 1991; Syracuse, N.Y.

James Merritt Brownlow. '48; April 11, 1991; Norwich, Vt.

Francis Xavier Crowley, '48; April 30, 1991; Wellesley, Mass.

Charlotte Meaker Davisson, '48; January 1, 1991; Oxon Hill, Md.

Cecil E. Hall, '48; March 5, 1991; Jasper, Ark.

Joseph T. Murphy, '48; April 14, 1991; Wellesley, Mass.

Alden Pugh Taber, '48; November 12, 1990; Pensacola, Fla.

John F. Elliott, '49; April 15, 1991; Winchester, Mass.

Francis Valentine McCorry, '49; June 17, 1989; Flint, Mich.

Cyril J. Brown, '51; March 20, 1991; Lexington, Mass.

Bernard Cohen, '51; March 24, 1991; Stamford, Conn.

Robert J. Pascoe, '52; April 20, 1989; South Glastonbury, Conn.

John J. Cahill, '53; March 29, 1991; Severna Park, Md.

William G. Moffatt, '53; December 21, 1990; Albuquerque, N.M.

James R. Wynne, '53; December 21, 1990; Miami, Fla.

Charles E. Loud, '54; August 13, 1990; Waterbury, Conn.

Adolph J. Hansen, '56; March 19, 1991; Hastings-on-Hudson, N.Y.

Willard L. Irwin, '57; 1987; Augusta, Ga.

Howard S. Schumacher, '57; March 2, 1991; Rochester, N.Y.

Clayne M. Yeates, '58; April 18, 1991; Montrose, Calif.

Gilbert Tukyu Chin, '59; May 6, 1991; Murray Hill, N.J.

Herman Erkku, '59; January, 1982; Burlington, Ontario, Canada.

Charles S. Ward, '59; May 2, 1991; Lexington, Mass.

Eugene A. Wailes, '60; April 19, 1991; Fremont, Calif.

Douglas Gridley Brookins, '63; April 30, 1991.

Thomas Burrell Cheek, '64; April 26, 1991; Lexington, Mass.

John R. Freeman, '64; September 26, 1990; Albuquerque, N.M.

Ian Christopher Smith, '77; June 3, 1990; Diegomartin, Trinidad.

Gainor J. Lindsey, '79; November 1, 1990; Arlington, Tex.

Burton S. Goldberg, '81; November 25, 1990; Somers, N.Y.

Victor O. DeNatale, '82; March 25, 1991; Somerville, Mass.

Edward B. Hontz, Jr., '92; June 4, 1991; Fairfax, Va.