ALLAN J. GOTTLIEB, '67

And One for the Monkey

It has been at least a year since I specified the size of the backlogs for the various kinds of problems that are printed. Let me do so now. I have over a year's supply of regular and speed problems and a half-year's supply of bridge problems—but computer and chess problems are in short supply.

My classmate John Rudy reminds me that our 25th reunion will occur next year and asks if some special event could be run in "Puzzle Corner" to acknowledge the event. Although the anniversary of my graduation may not be a noteworthy event for the column, a related anniversary is. November 1966 was the first appearance of "Puzzle Corner" in *Technology Review* after a year in the (now defunct) student-run *Tech Engineering News*. Hence we are just a half year away from *our* 25th anniversary. Any suggestions on a possible anniversary event?

Problems

APR 1. Robert Bart offers the following six-card problem that is thought to have been invented to test the powers of the legendary Oswald Jacoby. South is on lead with spades trump and is to make 5 of the 6 tricks against best defense.

NORTH
♠ A J 4
♥ J 6 2
♦ -
♣ -

WEST
♠ Q 8 7
♥ -
♦ -
♣ Q 8 7

EAST
♠ -
♥ Q 8 7
♦ Q 8 7
♣ -

SOUTH
♠ K 9 6 5
♥ J 6
♦ -
♣ -

APR 2. Warren Himmelberger sent us an old chestnut (really an old coconut) involving a monkey and some monkey business. In addition to the original problem, which is given below, Himmelberger suggests some interesting generalizations. Anyone interested in these extensions should write to the editors of *Technology Review* and ask for a copy of Himmelberger's February 1973 article from the *Mathematics Teacher*.

Four men are going to divide a pile of coconuts equally the following day. But, during the night, the first man decides to take his share secretly. He divides the coconuts into four piles, finds there is one nut left over and gives it to the monkey. He takes his share and puts the other three shares back. The second, third, and fourth man, in turn, proceed to do the same thing, each giving his share, and returning the other three shares to the pile. Then, in the morning, the four men meet to divide the remaining nuts into equal shares, and find there is again one nut left for the monkey. The puzzle is to find the least number of nuts in the original pile.

APR 3. Stephen Callaghan proposes the following problem. It looks difficult to me but I have been proven wrong in this regard before! You are to find the number of "distinct" $m \times m$ matrices with $n<m$ 1s and $m-n$ 0s in each row and column. Two matrices are considered equivalent (i.e. not distinct) if one can be converted to the other by permuting the rows and the columns. As an example, for $m=4$ and $n=2$, the following two matrices are distinct.

$$\begin{matrix}1&1&0&0\\1&1&0&0\\0&0&1&1\\0&0&1&1\end{matrix} \qquad \begin{matrix}1&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&1\end{matrix}$$

Speed Department

Phelps Meaker wants to know at what time between two o'clock and three o'clock will the minute hand be exactly sixteen minute spaces counterclockwise of the hour hand?

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PUZZLE CORNER

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show no way to play and 2,266 show only one way to play. Of the remaining 1,774, 1,222 can be optimally played by selecting the roll of the dice from the markers remaining using as few markers as possible. (Take "8" if you can, not "5 + 3" or "4 + 3 + 1," for example.)

That leaves 552 real problems. These are characterized by multiple ways to play using the smallest number of markers. And it matters—if you have (1,2,4,5,8,9) as the set of markers and roll 10, the choice between 9+1 and 8+2 makes a difference of 4.55 in the expected value! In fact, 239 of the 552 make a difference > 1.0 in the expected value.

I found no good way to determine the right way to play the "club of 552" except to have a list . . . then I wondered how just *picking at random* from those ways with the least markers would do in practice. Monte Carlo simulation with this simple rule resulted in an expected value of 33.13—good enough for casual barroom play, and easy to remember.

F/M 2. Robert High has apparently studied the art of dueling from a mathematical viewpoint. In this he clearly outclasses me since the only mathematical statement I can make about dueling is that it robbed us of much of Galois's expected lifetime.

Two duelists take turns firing at each other. They continue until one of them is hit. As is only fair, the weaker (less accurate) duelist goes first. If his accuracy is 1/3, how accurate must his opponent be for the match to be fair?

Now consider three duelists who take turns firing. Assume each knows the accuracy of his opponents. Is it always optimal to fire at the strongest opponent? Can the weakest duelist ever have an advantage, even if he fires last?

Finally, consider four or more duelists, again with perfect knowledge. Is it always optimal to fire at your strongest opponent? Is it necessary to know one's *own* accuracy? Can it ever *improve* one's chances to become *less* accurate, even if the overall ordering of the duelists' accuracies remains the same?

For two duelists, Kelley Woods shows that if the first duelist has accuracy a, the second needs accuracy $a/(1 - a)$ for a fair match. For three duelists, Woods notes that it pays to aim at the stronger (if you hit, it is better to face the weaker than the stronger and if you miss it doesn't matter who you tried to hit). An interesting remark from Matthew Fountain is that you might well be better off deliberately missing. Indeed, the proposer's program, when allowed to "pass," suggests the globally optimal strategy of everyone continually passing and hence everyone having a 1.0 survival probability. An interesting anti-war argument. If passing is outlawed, it is possible for the least accurate shooter to have the highest survival probability. Woods illustrates this phenomenon with the first, second, and third participants (in order of shooting, not accuracy) having accuracy .4, .5, and .3 respectively.

The situation with four or more duelists is complicated. Readers interested in this problem are invited to write to Faith Hruby at *Technology Review* for the proposer's comments (including computer programs), which are too lengthy to be reproduced here.

F/M 3. The following problem is from Gordon Rice. Lay out the A, 2, 3, 4, and 6 of spades in that order. Now roll one of the dice from your backgammon set. For each roll of the die, exchange positions between the ace and the indicated card. (If a 1 is rolled, do nothing.) By repeated rolls, we can generate "random" permutations of the six cards.

For a "trial" of N rolls, a certain set of permutations are possible outcomes. How big does N need to be for every permutation to be a possible outcome? Are there any N such that all possible outcomes have equal probability?

The following is an abbreviated version of Matthew Fountain's solution (write to Faith Hruby at *TR* for the full text):

All permutations are possible at $N = 7$ when N is the number of rolls, including those in which the 1 turns up. For all permutations to be equally probable N must be so large that the parity of N is independent of the parity of M, the count of those rolls which switch cards. Half the permutations are possible when M equals six or any larger even number, and the remaining half are possible when M equals seven or any larger odd number. But as N increases, the probability that M is odd approaches 1/2 and at large N the permutations of both halves are equally probable. The probability that $N - M$ is odd is the sum of the even terms in the expansion of $(5/6 + 1/6)^N$. The probability is 0.5057 that M is odd when $N = 11$.

To find out how fast the permutations become equally probable requires much doing. The most probable always remains the starting permutation, and the least probable remains those that had the last chance to appear. The starting permutation has the probability of 2.068/720 at $M = 22$, while the least probable have the probability of 1.979/720 at $M = 19$.

Robert High notes that for any N the probability of any given outcome will be a sum of powers of 1/6. Since 1/720 cannot be expressed as such a sum, there can be no N for which all permutations are equal (although this could be the limiting value as N increases).

OTHER RESPONDERS

Responses have also been received from W. Hartford, K. Rosato, and J. Rulnick.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Play the club ace and lead the 8. If West plays the king, you have 12 tricks (three spades, one heart, five diamonds, and three clubs). If West plays the 9, you win with the jack and now give East the king of hearts making two clubs and two hearts.

I did not hear from **Arthur Haskins** with his usual Christmas letter bringing us up to date with his and his son's sailing exploits, so I telephoned him a few days ago to learn how he was. His wife Dorothy died January 12 after a long illness and I passed along the deepest sympathy of his classmates to him. He has been keeping busy with work at his church and doing things with his son Dan. Art says he has stopped cutting down trees and is no longer doing any consulting at Bath Iron Works, where he worked most of his life. I am sure he would enjoy hearing from his friends: call him at (207) 443-2780.

Our class president emeritus and friend **Bernard H. Nelson** died suddenly of heart failure on March 22 while visiting in Bonita Springs, Fla. Bernie, a member of Beta Theta Pi, served as our class president from our 35th to our 55th Reunions, and he was deeply involved in our class activities as well as MIT. He was on MIT's Awards Committee and was presented with the Bronze Beaver. He was on MIT's National Nominating Committee and president of the MIT Cape Cod Club. During World War II, he was chief signal officer, AMET, Command and General Staff School, and was awarded the Legion of Honor. He spent all his business life with the New York Telephone Co. and retired as assistant vice-president. He is survived by two daughters, Deborah N. Aylsworth of Wellesley, Mass., and Elizabeth N. Duenas of Holmdel, N.J., and three granddaughters.

I had a phone call from **Les Brooks** telling me that **Michael Kelakos** died February 27. He had heard from Mike's wife Theresa. I also received a note from her enclosing a copy of his obituary. Mike died of complications from Parkinson's disease. In previous Class Notes I wrote of Mike and Les Brooks working together for a number of years. Mike joined the State Department in 1951 and as a member of the Foreign Service served in Greece and Germany. He subsequently was assigned to UNESCO in Paris as deputy U.S. representative, then became scientific attache in Rome, moved to Tel Aviv in 1966, and served there until he retired in 1973. He is survived by his wife; a son, George, of Wollaston; a daughter, Eleni, of Los Angeles; a brother, Thomas, of Lowell; and three sisters, Ann, Ourania, and Dorothy, all of Lowell.

Alfred Johnson died at his home in Naples, Fla., on February 4 after a sudden and brief bout with cancer. He graduated from Boston College Law School in 1944, joining Arthur D. Little, Inc., in 1942. He became the director of Invention Management and oversaw the negotiations and legal proceedings on behalf of an ADL client John C. Sheehan, an MIT chemistry professor whose patented process for synthesizing pencillin has produced $2 million in royalties from pharmaceutical houses since 1964. Al enjoyed golf and played in our class tournaments from 1960 to 1985. He retired from ADL in 1977 and became a consultant for a Boston law firm. He is survived by his wife Ruth (Hanson); two daughters, Priscilla of Nottingham, N.H., and Deborah McCanne of Aurura, Colo.; a brother, Horace L. of West Yarmouth; and three grandchildren.

Dexter J. Clough, II, M.D., died December 18, 1990, at a Bangor, Maine, hospital. He graduated from the University of Pennsylvania School of Medicine in 1939. After three years of internship, he returned to Bangor and practiced medicine with his father in 1942. Following World War II he limited his practice to ophthalmology and in 1948 was certified by the American Board of Ophthalmology. He was a member and past president of Maine Medical Association, member of the American Medical Association, and life member of the American Academy of Ophthalmology. He enjoyed golf and played in the class tournaments for 25 years. In 1984 he received the George B. Morgan award from MIT for serving with merit for 30 years on the MIT Educational Council. His survivors include a daughter, Frances C. Butler of Mount Desert; two sons, Peter J. of Aurora, Colo., and David R. of South Freeport; one brother, Dr. Herbert T., Jr., of Orrington; three sisters; and four grandchildren. Dexter was predeceased by his wife, E. Frances Palmer Clough, on February 8, 1990.

Patrick J. Mahoney died February 5, 1991, in Natick, Mass., after a long illness. Prior to his retirement in 1985, he had been employed as a chemist for the U.S. Army Laboratories for 25 years. He is survived by his wife, Vivian (Jones); one son, John M. of Salt Lake City; two daughters, Patricia A. Garcher of Valparaiso, Ind., and Lisa M. of San Francisco; and two sisters, Elizabeth Williams and Mary Mahoney, both of Norwich, Conn. . . . I have sent our condolences to the surviving widows, to Dexter Clough's daughter, and to Bernie Nelson's two daughters.

I am sure you are all aware of California's fifth year of drought. For San Diego County it has been very serious since all of our water comes from outside the county. However, the jet stream