ALLAN J. GOTTLIEB, '67

Canceled Checks

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later, one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to this issue's speed problems are given below. Only rarely are comments on speed problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Problems

OCT 1. The market may be speaking. This month we are scheduled to begin with a chess problem but none have been submitted and we will begin instead with a bridge problem from Tom Harriman. If you would like to see chess problems, please submit them!

NORTH
♠ A J 7 5
♥ A Q 7 4 2
♦ A J
♣ 5 4

WEST
♠ 8 6
♥ 10 8 2
♦ 10 8 7 5 3
♣ 9 8 7

EAST
♠ Q 10 9 2
♥ K 9 6
♦ K Q 6 4 2
♣ 10

SOUTH
♠ K 4 3
♥ J 5
♦ 9
♣ A K Q J 6 3 2

How does South play to make seven clubs against best defense after the opening lead of the spade 8?

OCT 2. Gordon Rice supposes that some time in the (not too distant?) future, the art of pencil-and-paper arithmetic has been forgotten. Also, your computer is giving off smoke. With no way to add, subtract, multiply, or divide except an eight-digit calculator, can you evaluate the following expressions?

$3 \cdot 180997^2 - 313496^2$

$3 \cdot 37467^2 - 64896^2$

OCT 3. John Rule has a three-digit number that, when divided by the product of its digits, yields as quotient the hundredth digit. Rule wants you to find this number and show that it is unique.

OCT 4. David Evans notes that on an 8×8 checkerboard, if two squares of the same color are removed, it is impossible to cover the remaining 62 squares with 31 1×2 tiles (since each tile covers one white and one black square). Is the converse true, i.e., if you remove 2 squares of opposite colors, can the remaining 62 squares always be covered by 31 1×2 tiles?

OCT 5. Chuck Coltharp poses the following partitioning question. Let S be a finite set of size 4n and let P be a collection of partitions of S, each of which partitions S into two disjoint sets of size 2n. Let the ith partition be the two sets A_i and B_i. We require that, for $i \neq j$, $A_i \cap B_j$ is of size n. The question is how large can P be, that is, for each n what is the largest number of partitions that can be found satisfying the above properties?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

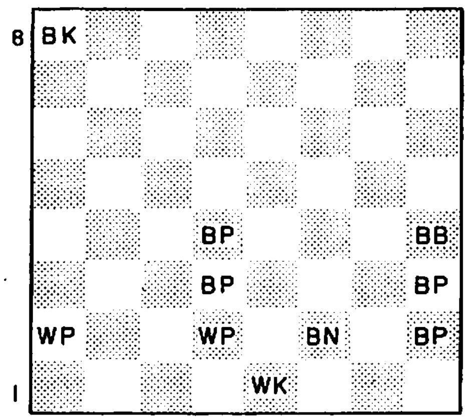
Speed Department

SD 1. Jim Landau wants two non-identical functions f(x) and g(x) such that $\int f(x)dx = g(x)$ and $\int g(x)dx = f(x)$.

SD 2. Edward Wallner notes that both William Shakespeare and Miguel Cervantes died on April 23, 1616, and asks who died first?

Solutions

M/J 1. We begin with a two-part chess problem that appeared in *The Tech* during 1984. First, in the figure below, find a helpmate in 7, i.e., black moves first and cooperates with white so that black is mated on white's 7th move. Second, solve the same problem with the bishop on H4 gone.



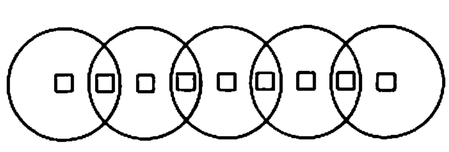
According to Richard Hess, the key to this problem is not to promote all pawns to queens. Hess's solution is:

A:	black	white
(1)	b-e7	p-a3
(2)	n-e4	k-f2
(3)	b-b4	Pxb
(4)	n-c5	Pxn
(5)	p-hl(r)	P-c6
(6)	r-al	P-c7
(7)	r-a7	P-c8(Q) mate

B:	black	white
(1)	k-b7	P-a4
(2)	k-c6	P-a5
(3)	k-d5	P-a6
(4)	k-e4	P-a7
(5)	k-f3	P-a8(R)
(6)	k-g2	R-al
(7)	k-hl	Kxn mate

Also solved by Robert Bart, Matthew Fountain, Winslow Hartford, and James Walker.

M/J 2. Nob. Yoshigahara wants you to put a unique digit from 1 to 9 in each of the nine boxes so that the sums in the circles are all equal.



Please paste in figure from M/J

Gordon Rice was able to solve this without a computer-assisted search; his solution follows:

We require that the nine numbers A, B, C, D, E, F, G, H, and I, representing a permutation of 1, 2, . . ., 9, satisfy, for a given sum S, the equations:

A + B = S
B + C + D = S
D + E + F = S
F + G + H = S
H + I = S.

We note that

A + 2B + C + 2D + E + 2F + G + 2H + I = 5S

A + B + C + D + E + F + G + H + I = 45

so

B + D + F + H = 5(S − 9).

Let us call a set of values for B, D, F, and H (ignoring the order of assignment) an "overlap set". There are 26 possible overlap sets:

S = 11	S = 12	S = 13	S = 14	S = 15
1,2,3,4	1,2,3,9 x	1,2,8,9	1,7,8,9	6,7,8,9 x
	1,2,4,8 x	1,3,7,9	2,6,8,9 x	
	1,2,5,7 x	1,4,6,9 x	3,5,8,9 x	
	1,3,4,7	1,4,7,8	3,6,7,9	
	1,3,5,6	1,5,6,8 x	4,5,7,9 x	
	2,3,4,6	2,3,6,9	4,6,7,8 x	
		2,3,7,8		
		2,4,5,9 x		
		2,4,6,8		
		2,5,6,7 x		
		3,4,5,8 x		
		3¼,6,7 x		

The 14 overlap sets marked with an "x" may be eliminated by the rule that if two members of the overlap set add up to S, no solution is possible. We establish this rule as follows.

The numbers chosen for B and H must be in the overlap set; the numbers chosen for A and I must not. If two numbers in the overlap set add up to S, then they are both unavailable for B or H. That leaves two remaining. But at least one is too small (i.e., less than S − 9). The exception is S = 15, but there both pairs of the overlap set add up to S.

That leaves 12 to try. We avoid symmetries by requiring that A be less than I. "x" marks the point at which a trial fails.

S	set	A	B	C	D	E	F	G	H	I	
11	1,2,3,4	7	4	6	1	8	2	x			
		7	4	5	2	8	1	x			
		7	4	5	2	6	3	x			
		8	3	7	1	6	4	5	2	9	solution
		8	3	6	2	5	4	x			
12	1,3,4,7	5	7	2	3	8	1	x			
		8	4	5	3	2	7	x			
	1,3,5,6	7	5	4	3	8	1	x			
	2,3,4,6	8	4	5	3	7	2	x			
13	1,2,8,9	4	9	3	1	x					
		5	8	4	1	3	9	x			
		5	8	3	2	x					
	1,3,7,9	4	9	x							
		6	7	5	1	x					
	1,4,7,8	5	8	x							
		6	7	5	1	x					
		6	7	2	4	x					
	2,3,6,9	4	9	1	3	8	2	5	6	7	solution
	2,3,7,8	5	8	x							
		6	7	4	2	x					
	2,4,6,8	5	8	1	4	7	2	x			
		5	8	1	4	3	6	x			
		5	8	3	2	7	4	x			
		7	6	5	2	3	8	1	4	9	solution
		7	6	3	4	1	8	x			
14	1,7,8,9	5	9	4	1	6	7	x			
		6	8	5	1	4	9	x			
	3,6,7,9	5	9	2	3	4	7	1	6	8	solution

Also solved by Jonathon Aronson, Robert Bart, Michael Baumann, Larry Bell, John Chandler, Walter Cluett, John Cushnie, Steve Feldman, Bridget Fitzpatrick, Matthew Fountain, Emil Frei, Jim Gawn, Thomas Harriman, Winslow Hartford, Richard Hess, Linda Kalver, T. Landale, Warren Legler, Samuel Levitin, Mary Lindenberg, Edward Martin, Robert Massard, Frank Model, Paul Ness, Walter Nissen, Gardner Perry, Steve Peters, Ron Raines, Michael Riezenman, Lorenzo Sadun, Thomas Sico, Alan Taylor, James Walker, Don Warren, Meredith Warshaw, Kelly Woods, Harry Zaremba, and the proposer.

M/J 3. Thomas Murley asks, for a random physical constant, what is the probability that its *second* digit is N?

The following solution is from John Chandler: The probability density function of the second digit of a random constant is biased in the same way as that of the first, only not so strongly. I assert that the phrase "random physical constant" means just that the log of the constant has a uniform distribution, and I believe that assertion must be considered an axiom (which might not be true—remember, Dirac's "large number hypothesis" holds that large physical constants are all the same!). Thus, just as the probability that the first digit is N is given by log N + 1/N, the probability that the second is N is given by

$$p(N) = \log\left(\frac{11+N}{10+N}\right) + \log\left(\frac{21+N}{20+N}\right) + \log\left(\frac{31+N}{30+N}\right) + \ldots + \log\left(\frac{91+N}{90+N}\right)$$

In other words, the probability that the second digit is, say, 2 is the sum of the probabilities that the first TWO digits are 12, 22, 32, 42, 52, 62, 72, 82, and 92. It is easy to show that base-ten logs give the proper normalization for the probability density, since the sum of all the individual terms of p(0) through p(9) is just log 100/10.

The following is a table of p(N).

N	p(N)
0	0.11967927
1	0.11389010
2	0.10882150
3	0.10432956
4	0.10030820
5	0.09667724
6	0.09337474
7	0.09035199
8	0.08757005
9	0.08499735

Also solved by Jonathon Aronson, Matthew Fountain, Thomas Harriman, Winslow Hartford, Richard Hess, Meredith Warshaw, and the proposer.

M/J 4. Robert Bart extends a problem posed last year and asks for the smallest positive integer A_i such that $(A_i)^{1/i}$ begins with 10 distinct digits. Note that A_1 = 1023456789 (leading zeros are not permitted) and that we established A_2 = 1362 last year ($1362^{1/2}$ = 36.90528417 . . .). Mr. Bart specifically asks for A_3 through A_{10}.

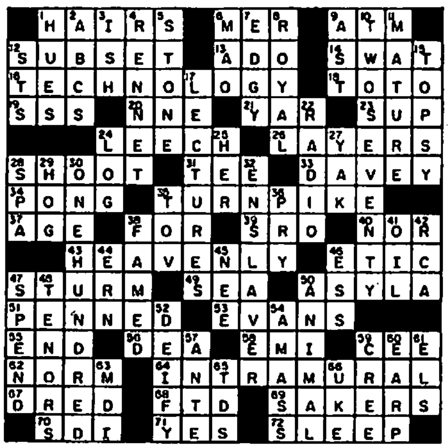
The following solution is from Michael Baumann: This clearly calls for a computer program to go about its merry way, starting at A_i = 1 and checking the result until the desired result is obtained. The puzzle asked for values for A_i for i = 3, . . ., 10, but I figured once you got the program running why stop at ten. I propose the following solutions:

i	A_i	$(A_i)^{1/i}$
2	1362	36.90528417
3	2017	12.63480793
4	6654	9.031724865
5	911	3.907542186
6	319	2.613957048
7	6702	3.520469871
8	4954	2.896473105
9	5496	2.603518974
10	9213	2.491380765
11	402	1.724836059
12	1644	1.853496702
13	1813	1.780926453
14	2059	1.724605389
15	2687	1.692847530
16	4020	1.679825340
17	679	1.467508329
18	108	1.297083654
19	540	1.392548076
20	1396	1.436297058
21	4668	1.495273086
22	128	1.246758309
23	6552	1.465309278
24	2546	1.386470925
25	1168	1.326470859
26	386	1.257430896
27	3349	1.350679824
28	6363	1.367240985
29	900	1.264359078
30	4920	1.327594680
31	980	1.248795036
32	377	1.203678954
33	4072	1.286435790
34	5825	1.290458367
35	700	1.205836749
36	1967	1.234509867
37	979	1.204569783
38	10796	1.276845930
39	4081	1.237609854
40	4653	1.235074986

Also solved by Larry Bell, Steve Feldman, Matthew Fountain, Winslow Hartford, Richard Hess, and Harry Zaremba.

M/J 5. Our last regular problem is not that regular; it is a crossword puzzle from Andrew Greene published last year in *The Tech*. I have no objection to including these kinds of problems but want to know what *you* think.

Our last solution is from Larry Bell:



As for the feedback you requested, I must say that although I don't consider myself a true crossword afficionado, I really did enjoy doing this one. The best part about it is the clues that are directed towards the "tech-minded" individual (e.g., me). Appropriate topics for crossword clues also include (my opinion): campus trivia, current "high tech" events, Star Trek, items from prior *Tech Review* articles, programming language keywords or concepts, etc.

Also solved by R. Alexander, John Chandler, Steve Feldman, Bridget Fitzpatrick, Winslow Hartford, Richard Hess, Warren Legler, Thomas Lewis, Frank Model, Gardner Perry, Thomas Sico, Alan Taylor, James Walker, Don Warren, Meredith Warshaw, Harry Zaremba.

Better Late Than Never

JAN 3. Harry Zaremba now believes that the length of the rectangle can be reduced to 165.5.

M/J SD1. Robert Bishop, Stephen Rawlinson, and Lorenzo Sadun note that 1900 was not a leap year according to the Gregorian calendar. Rawlinson suggests that perhaps the Julian calendar was being used and Bishop favors the conjecture that Frederic and W.S. Gilbert shared the widespread ignorance about the special status of 1900.

Proposers' Solutions To Speed Problems

SD 1. $-e^{-x}$ and e^{-x}

SD 2. Cervantes. Spain was using the Gregorian calendar and England the Julian.