ALLAN J. GOTTLIEB

Celebrating Winter With A Dogsled-Theoretic

Alan Katzenstein adds to our palindrome collection one attributed to Penelope Gilliatt in an article in *Time* magazine:
Doc, note. I dissent. A fast never prevents a fatness. I diet on cod.
and encourages palindrome fans to read "A palindromic dialogue by J.A. Lindon," which can be found in Martin Gardner's "Mathematical Games" column in *Scientific American* sometime in the early 1970s. Mr. Katzenstein kindly sent a photocopy, which I will reproduce upon request.

Ric Klass reports that a Harvard Business School classmate, Masonn Sexton, established a company called Harmonic Research that uses the Fibonacci series to aid in stock forecasting.

Finally, Meredith Warshaw has requested that "some of [our] fictitious computer experts and IEEE members be labeled 'she' instead of 'he'." Since I agree with Ms. Warshaw but prefer not to impose my will on others' words, I ask that proposers try to be evenhanded in their choice of pronouns.

Problems

F/M 1. We begin with a bridge problem from Doug Van Patter:

North
♠ A K 5
♥ Q 4 3
♦ 7 5 4 2
♣ 8 6 5

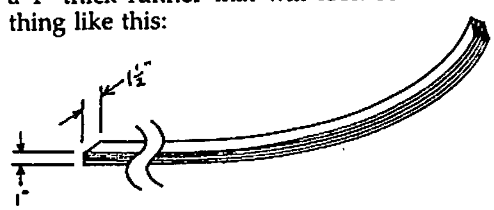
South
♠ 7
♥ A K 10 8 6 2
♦ A K 3
♣ A 7 4

Over-optimistic bidding has led to an unlikely six-heart contract. West leads the ♣K, which you take with the ♣A. When you cash the ♥A and ♥K, West shows out on the second round, leaving East with the ♥J. How do you proceed? If you were in a four-heart contract (duplicate bridge), what line of play would you adopt?

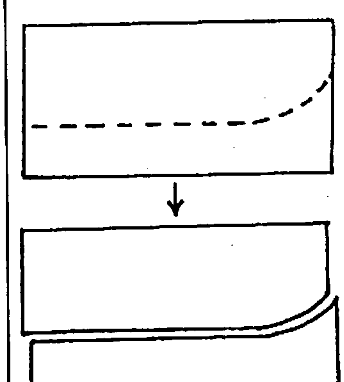
SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

F/M 2. Nob. Yoshigarhara sent us a problem from Y. Kotani:
For each positive integer n, consider writing the integers from 1 to n inclusive and let $f(n)$ be the number of times the digit 1 was used. For example $f(3) = 1$, $f(10) = 2$, and $f(12) = 5$. Clearly $f(1) = 1$; what is the smallest $n > 1$ with $f(n) = n$?

F/M 3. Clifford Cantor, from Anchorage, Alaska, asks our first-ever dogsled-theoretic problem:
My brother Jim is laminating together four thin layers of hickory to make a new runner for his dogsled. Each piece is 0.25" thick, 1.5" wide, and about 8' long. The four pieces will be laminated to form a 1" thick runner that will look something like this:



He wants to make a two-piece mold for gluing the strips together. He plans to cut an 8' sheet of plywood 1.5" thick, so the cut edges will be the surfaces of the mold.



He called me to ask whether he can produce both pieces with a single saw cut. I told him he could not. Was I right? If not, what curve would have the requisite properties?

F/M 4. Charles Piper takes us from dogsled runners to the human variety with a problem he believes dates back to a circa 1850 book, *Exercises in Algebra*, by Jones:
A offers to run three laps while *B* does two but gets only 100 yards into his third

lap when B wins. He then offers to run four laps to B's three, and quickens his pace in the ratio of 4:3. B also speeds up, in the ratio of 9:8, but in the second lap falls off to his original pace, and in the third goes only 9 yards for the 10 he went in the first race. A wins the race by 180 yards. How long is each lap?

F/M 5. Albert Mullin poses the following: By analogy with palindrome, a validrome is a sentence, formula, relation or verse that remains valid whether read forward or backward. For example, relative to (prime) factorization, 341 is a factorably validromic number, since 341 = 11·31, and when read backwards gives 13·11 = 143, which is also correct. What is the largest factorably validromic number you can find? Remember that 1 is not prime so $n = n \cdot 1$ is not valid.

Speed Department

SD 1. Jim Landau has a boxed set of books containing two volumes each containing 512 pages not counting the covers. A bookworm starts at volume I page 1 and chews his way through to volume II page 512. How many pages has he eaten through (not counting the covers)?

SD 2. Mr. Landau also wants to know how many successful parachute jumps must a paratrooper make before he can graduate from jump school?

Solutions

OCT 1. Given these hands, and the bidding shown, West leads the ♠10, and East encourages with the ♠7. How do you play this hand?

North
♠ 4 3
♥ K 7 6
♦ A Q J 5 2
♣ K Q 5

South
♠ K 6
♥ J 10 8 5 3 2
♦ K 7
♣ A 6 4

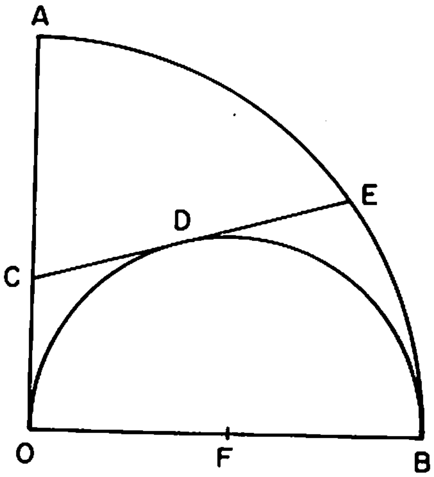
Bidding

N	E	S	W
1D	1S	2H	Pass
3H	Pass	4H	Pass
Pass	Pass		

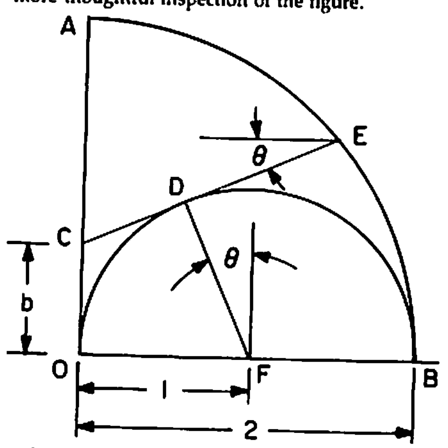
The following solution is from Robert Bart:
Trickier than it looks. If South rushes to draw trump, the play may go: heart to East, diamond return, heart to East, *low* spade to West, diamond ruff by East; down one. Why else did East duck, except to set up an entry for the ruff? South must return the Spade at trick two, killing the late entry. Subsequent play depends on how the defense continues:
If East plays a third spade, ruff low in the closed hand and then
(1) If West does not overruff, run the ♥J. Even if West is void, North can ruff the next spade and play the ♥K.
(2) If West overruffs with the ♥A, win the return and play the ♥K.
(3) If West overruffs but not with the ♥A, overruff in dummy and lead a low heart. If West follows suit and East wins to play another spade, ruff high and pull trumps. If West is now void, ruff the spade in North, come to the closed hand with a club, and play trumps.
If East shifts to a diamond at trick 3, win in the closed hand and run the ♥J.
(1) If West is out, continue trumps at every chance until East is out and then dummy is good.
(2) If West follows low, East wins with the ♥Q and returns a spade; do *not* ruff in the closed hand since this risks an overruff and a diamond ruff by East. Instead pitch a diamond, ruff in dummy, and lead the ♥K. If West ruffs with the ♥A to play a diamond, South can overruff East.

Also solved by Richard Hess, Winslow Hartford and the proposer, Doug Van Patter.

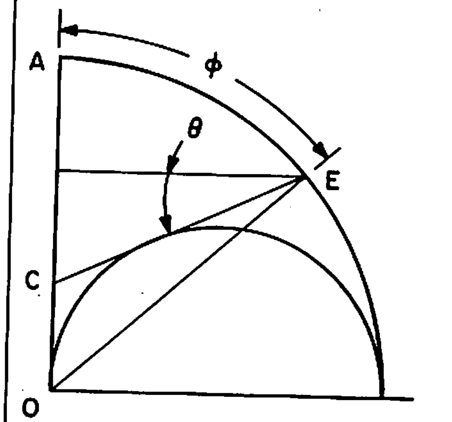
OCT 2. The figure depicts a semicircle of radius 1 in a quarter-circle of radius 2. What is the largest area that the curved region ACE may have, if CE is tangent to the semicircle?



The following solution is from John Russell:
[This problem] requires us to find the largest possible area for region ACE in the figure below. My first attempts to solve it bent my hammer and tongs and left me with cumbersome polynomials. I was happy, then, to find that the problem yielded to a more thoughtful inspection of the figure.



Three observations are useful. First, OC is always equal in length to CD. Both radiate from point C and are tangent to circular arc ODB, so one must be as long as the other. Call their length "b" for ease of reference. Next, since CE and DF are perpendicular to one another, CE's slope angle, labeled "θ," must equal the angle between DF and the vertical. Therefore, the horizontal and vertical coordinates of point D are $(1 - \sin\theta, \cos\theta)$, and the length of CD, b, must be $(1 - \sin\theta)/\cos\theta$. It follows that $\sin\theta = (1 - b^2)/(1 + b^2)$ and $\cos\theta = 2b/(1 + b^2)$. Finally, when region ACE has maximum area, CE is 2b units long. Suppose we start generating the area by moving point E up to A. Initially, OA and CE will be colinear, and the area between them will be zero. Then let point E descend towards B. DE, being longer, will generate area far more rapidly than CD can eliminate it. Net area will continue to increase until CD and DE are equal, after which CD will begin to deduct more than DE can add. Therefore, area is maximum when CD equals DE, and CE = 2b. By the law of cosines, $OE^2 = OC^2 + CE^2 - 2OC \cdot CE \cos OCE$. In terms of b and θ, $4 = 5b^2 + 4b^2\sin\theta$, or in terms of b alone, $b^4 + 5b^2 - 4 = 0$. It follows that
$b^2 = [(41)^{1/2} - 5]/2$, and
$\sin\theta = [7 - (41)^{1/2}]/[(41)^{1/2} - 3]$;
$\theta = 10°6'5''$.



We can now note that the horizontal projections of OE and CE are identical. That is, identifying angle AOE as φ, $CE \cos\theta = OE \sin\varphi$. Since the length of OE is 2, $2\sin\theta = 2b \cdot 2b/(1 + b^2)$, and $\sin\theta = 2b^2/(1 + b^2)$, $\varphi = 55°32'56''$. The area of sector AOE is therefore $OE^2\varphi/2 = 1.939$. From that we must deduct the area of triangle OCE. Letting s stand for the semiperimeter, $s = (3b + 2)/2$, and the triangle's area, $[s(s - b)(s - 2b)(s - 2)]^{1/2} = 0.6907$. The maximum area will therefore be $1.939 - 0.6907 = 1.2483$.

This solution may be too late to qualify. Mail takes a while to reach here (Scotland), and pondering the problem took more time than I like to admit. Nonetheless, it's been fun, and typing the solution has been a brand new challenge.

Also solved by John Wrench, Hy Tran, Harry Zaremba, Norman Wickstrand, Steven Feldman, Robert Bart, Richard Hess, Fred Furland, W. Kelly Woods, Turker Oktay, Dave Mohr, Robert Light, Harry Garber, and the proposer, Matthew Fountain.

OCT 3. On a train, Smith, Robinson, and Jones are the fireman, brakeman and engineer, but NOT NECESSARILY respectively. Also aboard the train are three businessmen who also have the same names: a Mr. Smith, a Mr. Robinson, and a Mr. Jones.
(1) Mr. Robinson lives in Detroit.
(2) The brakeman lives exactly halfway between Chicago and Detroit.
(3) Mr. Jones earns exactly $20,000 per year.
(4) The brakeman's nearest neighbor, one of the passengers, earns exactly three times as much as the brakeman.
(5) Smith beats the fireman at billiards.
(6) The passenger whose name is the same as the brakeman's lives in Chicago.
Who is the engineer?

Tom Campbell notes that the brakeman's nearest neighbor can't be Mr. Jones since $20K isn't divisible by 3 [rules 3 and 4]. The nearest neighbor can't live in Chicago or Detroit (Mr. Robinson) since they are equidistant [1, 2, and 6]. The nearest neighbor must therefore be Mr. Smith. Mr. Jones must live in Chicago. The brakeman must be Jones [6]. The fireman can't be Smith [5], so the engineer must be.

Also solved by Larry Marden, Ken Rosato, Gordon Rice, Matthew Fountain, Harry Garber, Harry Zaremba, Dave Spiewak, Robert Bart, Raymond Gaillard, Richard Hess, Fred Furland, Mac Zimmerberg, W. Kelly Woods, Avi Ornstein, Walter

Cluett, Walter Hicks, Martin Weinstock, and the proposer, Ron Raines.

OCT 4. Three of you in a room are told each of you has a prime number written on your forehead and that the numbers form the sides of a triangle with prime perimeter. Each person is asked in turn if he can deduce his number.
(a) You see a 5 and 7 and have heard "don't know" from the other two. What is your number?
(b) You see a 5 and 11 and have heard "don't know" from the others on each of their first two turns. You have stated "don't know" on your first turn. It is now your second turn; what is your number?

The proposer wrote to say that the problem was flawed—more conditions were needed to answer part (a). As often occurs, however, the flaw did not destroy the problem. Instead, it just changed the character. The following solution is from Matthew Fountain:

Part (a) has no solution. Part (b) has 13 as my number. I presume part (a) was posed to encourage interest in part (b) without being too obvious a clue. I do not discuss part (a) further as its features are incorporated in part (b). To make my solution easier to follow I present the following pairs of primes that might be visible to one participant, followed by the primes that could be on his forehead. When A and B are the visible primes the restrictions on C, the third prime, are $|A - B| < C < A + B$ and $A + B + C$ is prime.

3-3 (5) 3-5 (3,5) 3-7 (7) 5-7 (5,7,11)
5-11 (7,13) 5-11 (7-13) 7-7 (3,5)

I am the third responder and see 5-11. I temporarily assume it to be 7. Mr. 11, the participant I see with 11, then will see 5-7 and then will test if his number is 5. He reasons, "When I have 5, the third responder, Mr. 7, will see 5-5 and wonder if his number is 3, which would cause the first two responders to see 3-5. But then the first response of 'I can't decide' would inform the second responder that the first did not see 3-3. The second responder, seeing 3-5, could then decide that his number could not be 3 and would announce that it was 5. As he does not do this, Mr. 7, seeing 5-5, can decide his number is 7." But when he doesn't do so, Mr. 11 eliminates the possibility that his number is 5. Mr. 11 will next test if his number is 7. He reasons, "When I see 5-7 and my number is 7 then Mr. 5 sees 7-7 and will decide that his number is 5 and not 3 as soon as he hears someone else respond 'I can't decide.' For had the responder seen 3-7, the responder would have announced his number to be 7." When Mr. 5 does not announce his number his first turn after hearing someone else, Mr. 11 concludes his own number is not 7. Mr. 11 then will realize that the only possibility left is that it is 11, and announce his number to be 11. I check to make certain that it does not matter if Mr. 11 is either the first or second responder, he will still be able to decide on his second turn if he sees me with 7. Because he is unable to decide on his second turn, I announce my number is 13.

Also solved by Robert High (who developed an algorithm for a class of such problems).

OCT 5. What is the largest integer that is less than $1 + 1/2 + 1/3 + \ldots + 1/162{,}754$?

Kent Boklan writes: "It was nice to see that my old MIT friend Scott Berkenblit is still alive. To show that I am the same, I'm taking the liberty of sending a brief solution to his [problem]:

In general, call

$$S_n = \sum_{k=1}^{n} \frac{1}{k},\ n$$

a positive integer. The desired value is $[S_{162,754}]$, which is easily seen to be 12:

For, by the Euler summation formula, indeed a handy tool,

$$S_n = \log n + Y + \int_n^\infty \frac{\{t\}}{t^2}\,dt$$

where log n is base e, Y is Euler's constant:

$$1 - \int_1^\infty \frac{\{t\}}{t^2}\,dt = .57721 \ldots,$$

and $\{\alpha\}$ is the fractional part of $\alpha = \alpha - [\alpha]$. Thus

$$0 < S_n - \log n - Y = \int_n^\infty \frac{\{t\}}{t^2}\,dt < \int_n^\infty \frac{1}{t^2}\,dt = \frac{1}{n},$$

with some sharpening possible. And, for any n,

$$\log n + Y < S_n < \log n + Y + \frac{1}{n},$$

and consequently for $n = 162{,}754$, $12.577211 < S_{162,754} < 12.577217$, and we are done."

Also solved by Bill Cain, Robert Bart, Kent Boklan, John Wrench, Fred Furland, Ken Rosato, Gordon Rice, Matthew Fountain, Harry Zaremba, Jeff Abrahamson, Steven Feldman, Richard Hess, Gerald Leibowitz (who refers us to Boas, "Convergence, Divergence, and the Computer," in *Mathematical Plums* by Honsberger to see how sums of tiny numbers are calculated), and the proposer, Scott Berkenblit.

Better Late Than Never

M/J 2. Several comments have been received on the published solution, the consensus being that the problem is rather more difficult than it appears. Bob High comments that any "pure" strategy cannot be optimal; instead, some probabilistic mix of pure strategies will be needed. High supports this claim by arguing that Bart's diagram is generic, so that for any pure strategy there will be non-shaded points. High asserts that any optimal strategy for a two-person zero-sum game must remain optimal even if disclosed to the opponent, but if the opponent knows or can deduce which pure strategy is being used she can "just pick a point nearby in the non-shaded area, sit back, and wait to count [her] money." Dave Mohr recommends J.D. Williams, *The Complete Strategist*, to learn about these problems. Mohr has also proposed a discrete version of the problem, which will appear in a future issue of "Puzzle Corner."

JUL. Robert Bart responded on time to the first three problems but I inadvertantly misfiled his letter with the October solutions.

Proposers' Solutions To Speed Problems

SD 1. Zero. Volume I page 1 is adjacent to volume II page 512.

SD 2. All of them!