ALLAN J. GOTTLIEB, '67

# What Happened 'Ere I Saw You Last?

Let me begin with several contributions from readers.

Albert Mullin notes that James Joyce anticipated important parts of modern physics—more precisely the vocabulary of modern physics. For example, "gauge symmetry" ( . . . "gauging the symmetry . . ." uttered by Bloom in *Ulysses*) and "quarks" ("Three quarks for Muster Mark!" in *Finnegans Wake*). Mullin adds that less well known is Joyce's mathematical puzzle appearing in *Ulysses*, in which the hero Bloom, a forerunner of modern graph theorists, ponders the problem of crossing Dublin without passing a pub, only to abandon the attempt as impossible.

A local newspaper noted that the New Bedford (Mass.) Great Books Discussion Group founded by our Mary Lindenberg 40 years ago is one of the oldest such groups in the country. Indeed, that is twice the age of "Puzzle Corner."

Matthew Fountain has written about Captain Waterman, who founded the city of Fairfield, Calif.:

After setting several speed records in clipper ships under his command, Waterman had to leave the sea due to the notoriety he gained as a hard master. He had earlier bought a one-third interest in 30 square miles of land northeast of San Francisco for $16,666. I'll bet it's worth a *lot* more now.

Readers interested in handmade wooden jigsaw-puzzles, tangrams, and casse tete should write to Ateliers de la Petite Pree, Pierre Jouan, La Godiniere, 49150 Cheviere-le-Rouge, France.

Stephen Cheng, a colleague of mine at Baker House (and especially the Baker House ping-pong tables), has just completed a book entitled *MOS Digital Electronics*. Finally, Neil Cohen notes that after 39 years of bachelorhood he married Lea Linsk. Best wishes to you both.

*SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.*

## Problems

**F/M 1.** Our first challenge is a computer-related problem from Jim Landau, which essentially asks for a nonrecursive version of Oren Cheyette's solution to **1986 N/D 2**. Landau writes:

Define an ordering of the $n!$ permutations of $n$ objects by considering the objects as the first $n$ letters of the alphabet and arranging the $n!$ permutations in alphabetical order. For n = 3 this gives

| | |
|---|---|
| 1. A B C | 4. B C A |
| 2. A C B | 5. C A B |
| 3. B A C | 6. C B A |

The $m$th permutation is now defined as the permutation that is number $m$ in alphabetical order. Devise a nonrecursive algorithm and/or write a program which will find the $m$th permutation of $n$ objects. By nonrecursive I mean that the algorithm can find the $m$th permutation without knowing (or computing) the $(m-1)$th or any other permutation.

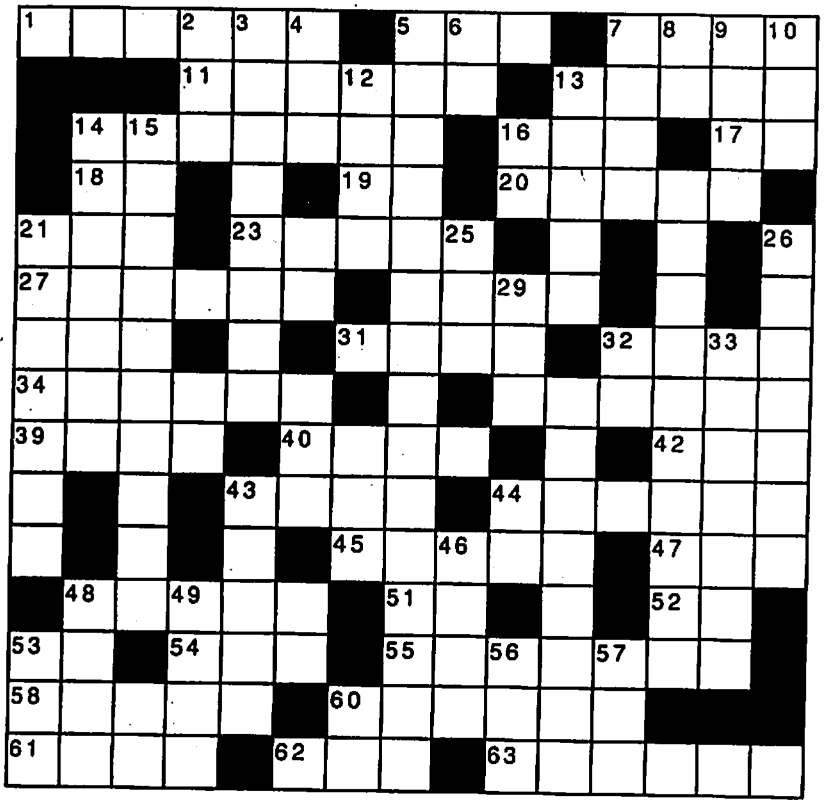
**F/M 2.** Frank Rubin has positive integers x and y and now needs to find positive integers a, b, c, and z satisfying
$a^x + b^y = c^z$.

**F/M 3.** Nob Yoshigahara wants you to find the smallest integer A such that the first ten digits of the square root of A are distinct.

**F/M 4.** Here is a cute problem originally contributed by Solomon Golomb to the *Johns Hopkins Magazine*:

Form palindromes of the type "—— was I ere I saw ——," where the *last* word of the sentence is a place name, and where the cleverness of the composition depends on to whom you attribute it. The classic is "able was i ere i saw elba," attributed to Napoleon.

**F/M 5.** Our last regular problem is a crossword puzzle by Dave Wagger that first appeared in the *Graduate Student News*, a publication of the M.I.T. Graduate Student Council. No solution was supplied, and the editor suspects that no complete solution is possible. Can our readers best the best that graduate students have to offer?



**ACROSS**

1. Better than brand A
5. This place
7. — and crafts
11. 5A's expense
14. What a naive grad can get if not too careful
16. Winter hazard
17. Not down
18. Medial suffix
19. Bismark's state
20. Research tool
22. A level (abbr.)
23. Famous campus lunch vendor
27. Survey again
28. Chicken shop?
30. M—A (jock institution)
31. "R" on a schedule
32. Nephew "Fijian"
34. Common grad affliction
39. Common transportation
40. 43560 sq ft
42. —ence
43. It takes forever here
44. Across the river
45. First performance
47. Anglo-Saxon god of peace
48. 100% person, if you want an apartment
51. Opposite of NW
52. Outside diameter
53. Not DC
54. Gazelle
55. Graduate dean
58. Possessive
60. Associate graduate dean
61. Course VI
62. See 44 across
63. Something to beat

**DOWN**

2. Type of register
3. Current M.I.T. experiment
4. Yuppie car
5. Popular watering hole
6. Form of "to be"
7. M.I.T. street
8. Week before classes
9. Given to prospective students
10. 273k, 1 atm
12. Await a decision
13. What to do with extra tickets
14. Editor
15. Very hard to find
16. Springfield is the capital
21. Wham-o
25. Old French coin
26. Student problem (nonhealth)
29. Boundary (comb. form)
32. Against (abbr.)
33. You might not want this from the commons
43. Some departments require this
44. Can cross-register here
46. Go to 50 for this
48. Another grad complaint
49. A computer system's protection
50. Some grads are/have this
53. What one did at 23A
56. Found in E23
57. Latest graduate foe
60. "Et —, Brutus"

## Speed Department

**SD 1.** Timothy Maloney sends us a "speed" problem based on a real-life incident; but it is not quite so simple as that: beware, trivia knowledge is required:
Last summer my cousin Jeff and I discovered that we went to the same summer camp at the same age, but at different times. "That was a third of my life ago," said Jeff. I replied, "That was two thirds of my life ago." I went to this camp when we all first heard Neil Sedaka sing "Breaking Up is Hard to Do." If it is now 1986, how old is Jeff?

**SD 2.** Howard Sard wants us to construct a bridge deal such that North-South can make 7NT against best defense holding the minimum number of high-card points.

## Solutions

**OCT 1.** A bridge player holding the following hand took five tricks in his own hand. Nobody made any illegal plays. What was the deal and how did the play go?

♠ 4 3 2
♥ 4 3 2
♦ 4 3 2
♣ 5 4 3 2

Charles Blair found a solution in which *six* tricks are taken. As a fringe benefit I now have have a set of $\TeX$ subroutines for printing bridge hands. These are available upon request. Blair writes:
I enjoyed the problem and thought you might like to see a deal in which a hand containing no cards higher than 5 (see West, below) wins *six* tricks:

North:
♠ A K Q J 10
♥ A K Q J 10
♦ 5
♣ K 9

West:
♠ 4 3 2
♥ 4 3 2
♦ 4 3 2
♣ 5 4 3 2

East:
♠ —
♥ —
♦ A K Q J 10 9 8 7 6
♣ A 8 7 6

South:
♠ 9 8 7 6 5
♥ 9 8 7 6 5
♦ —
♣ Q J 10

With clubs trump, West leads a diamond, and East plays four rounds while North and South throw hearts. West trumps the fourth round (first trick he wins) and leads a trump to East. West trumps the fifth round and leads a spade trumped by East. West trumps the sixth round of diamonds as North throws his last heart. Now West is just able to cash three hearts without anybody trumping. Maybe there is even a way to win seven tricks, although I doubt it.

Also solved by Matthew Fountain, Stephen Callaghen, Richard Hess, Charlie Larson, Gerald Roskes and Stefan Ralescu, Steven Feldman, Robert Bart, Gerald Lippey, and the proposer, Lawrence Kells.

**OCT 2.** Show that for k, r, and positive integers the expression

$$\sum_{k=1}^{r-1} \frac{r!n^k}{k!(r-k)!}$$

is always even—i.e., an integral multiple of 2.
Linda Kalver begins by noting that

$$(1+n)^r = \sum_{k=0}^{r} \frac{r!n^k}{k!(r-k)!}.$$

Hence

$$\sum_{k=1}^{r-1} \frac{r!n^k}{k!(r-k)!} = (1+n)^r - (n^r+1).$$

Since any integer has the same parity (odd or even) when raised to any positive power, the two terms on the right hand side have the same parity and hence their difference is even, as required.

Also solved by James Landau, Scott Berkenblit, John Chandler, Winslow Hartford, Chip Whiting, Matthew Fountain, Harry Zaremba, Richard Hess, Steven Feldman, Robert Bart, and the proposer, Hy Tran.

**OCT 3.** A "numble" is an arithmetical problem in which digits have been replaced by capital letters; there are two messages, one which can be read right away and a second one in the digit cipher. The problem is to solve for the digits. Each capital letter in the arithmetical problem stands for just one digit 0 to 9. A digit may be represented by more than one letter. The second message, expressed in numerical digits, is to be translated (using the same key) into letters so that it may be read; but the spelling may use puns or deliberate (but evident) misspellings, or may be otherwise irregular, to discourage cryptanalytic methods of deciphering.

```
          T H E
      x W O L F
      ---------
        K T L O
      E W T F
    S W E W
  H H W L
  -------------
  W L I W O O O
```

Many readers questioned the meaning of "Each . . . letter . . . stands for just one digit . . . [but] a digit may be represented by more than one letter." Frankly, I just do not see the problem. Were there 13 different letters in the problem (remember, the instructions were describing a class of problems), one would always have a digit represented by more than one letter even if each letter stands for just one digit. Perhaps even simpler is to consider a problem with five letters all standing for zero. Certainly the wording about the second message *was* confusing; however, Matthew Fountain figured out what should have been said (possibly the most challenging part of the problem) and writes:
The arithmetic problem is

```
          9 7 3
      x 8 6 4 2
      ---------
        1 9 4 6
      3 8 9 2
    5 8 3 8
  7 7 8 4
  -------------
  8 4 0 8 6 6 6
```

and the message is

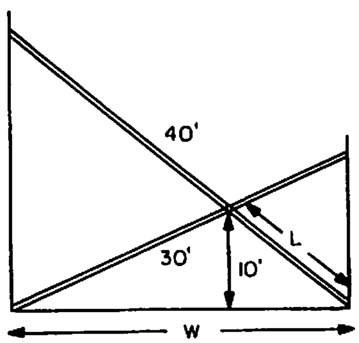
```
0 1 2 3 4 5 6 7 8 9
I K F E L S O H W T
```

or IKE FELL, SO WAIT.

I began by observing that E × F = O, E × O = W, E × W = L, and E × L = F, when only the last digit of each product is considered. This shows that O is not an odd digit, as an odd O would force E, W, L, and F to be odd also. But none of these five letters equals 1, and the set of five odd digits must include 1. Therefore O is even, and consequently so are W, L, and F. Six letters begin a number and three others yield four-digit products when they multiply THE. This leaves only the letter I as the letter equal to 0. This accounts for all even numbers. T, H, E, K, and S are odd. Since KTLO and the other three partial products must not exceed 8 × 975 = 7800, K, E, S, and H are not 9. This leaves T to be 9. Scanning the columns of additions, I saw that L + F = O without a carry, for O is even and T + T + W = O, i.e., 9 + 9 + (even digit) = O plus a carry of 2. I then saw W − 2 = O, making W larger than O which in turn is equal to the sum of L and F. W = 8, O = 6, and L and F are the set of 2 and 4. I now could write THE × W = HHWL = 9HE × 8 = HH8L. H, being odd, is 7. Making this substitution, 97E × 8 = 778L. E cannot be 1 or 5; therefore it must be 3. 973 × 8 = 7 784, making L = 4, and consequently F = 2. From THE × F = KTLO = 973 × 2, it is seen that K = 1. This leaves S, the last remaining digit, to be 5.

Also solved by Richard Hess, Robert Bart, Frederick Furland, Winslow Hartford, and John Chandler.

**OCT 4.** Given an alley of width W. Two ladders of length 40 ft. and 30 ft. are laid against opposite walls. They intersect 10 ft. above the ground. Find W and L, the width and one of the lengths of intersection.




The following solution is from Edward Dawson:
Given: AD = 40, BC = 30, and EF = 10.
From similar triangles, CF/EF = CB/BD = $W/\sqrt{30^2 - W^2}$. Therefore
$CF = 10W/\sqrt{30^2 - W^2}$.
In like manner, $DF = 10W/\sqrt{40^2 - W^2}$, and
$CF + DF = W = 10W/\sqrt{30^2 - W^2} + 10W/\sqrt{40^2 - W^2}$. Let $x = \sqrt{9 - (W/10)^2}$; then
$[x/(x - 1)]^2 - x^2 = 7$ and $x = 1.49094$.
Hence $W = \sqrt{9 - x^2} = 26.033$.
$L = ED = (EF/AC)AD = (10)(40)/\sqrt{40^2 - W^2} = 400/\sqrt{922.283} = 13.171$.

Also solved by John Chandler, Sidney Williams, Harry Garber, Mary Lindenberg (who notes that my feeling that the problem might have appeared here ten or fifteen years ago was too conservative; it appeared in 1970), Kelly Woods, Winslow Hartford, Richard Marlowe, Chip Whiting, Matthew Fountain, Harry Zaremba, Ken Rosato, Williams Evans, Richard Marlowe, Frederick Furland, Robert Bart, Steve Feldman, Edward Martin, Richard Hess, and the proposers, Joseph Molitoris and George Butwin.

**OCT 5.** Suppose you take a table of physical constants expressed in scientific notation (e.g., the speed of light is $2.998 \times 10^8$ m/s) and construct a histogram of the first digits of their mantissas (e.g., 2, for the speed of light). In other words, count how many times each of the digits 1 to 9 occurs as the leading digit. What functions *a priori* do you expect to best fit this histogram? That is, for a random physical constant, what is the probability that its leading digit is n?

Many readers realized that the distribution is logarithmic, with some noting that this is an outgrowth of the "independence of scale" (see below). Richard Hess points out that Martin Gardner discussed this problem, and Leo Sartori confirmed the logarithmic distribution by inspecting an up-to-date table of 133 physical and chemical constants. I am reprinting the proposer's (Oren Cheyette) solution since it gives a rigorous proof of the distribution:
The probability that the leading digit is $n$ is
$p(n) = \log_{10}[(n + 1)/n]$.
i.e., $p(1) = \log_{10} 2 \approx .30$, etc.
Proof: Let $q(x)dx$ be the probability of the mantissa of a physical constant being in the range of $x$ to $x + dx$. This distribution must be the same regardless of the units used to express the numbers (since the units are fundamentally arbitrary). Hence
$q(x)dx = q(\lambda x)d\lambda x$
where $\lambda$ is an arbitrary scale factor. Differentiate with respect to $\lambda$, and set $\lambda = 1$ to obtain
$q(x) = -xq'(x)$
or
$q'/q = -1/x$.
Integrating both sides, and normalizing the total probability to 1 gives
$q(x) = 1/(x\ln 10)$.
The probability function $p(n)$ is then given by:

$$p(n) = \int_n^{n+1} q(x)dx = \ln[(n + 1)/n]/\ln 10$$

$= \log_{10}[(n + 1)/n]$.

Also solved by Harry Zaremba, Matthew Fountain, Winslow Hartford, John Chandler, Scott Berkenblit, Gerald Lippey, and Alain Hanover.

**Better Late Than Never**

**1987 F/M 3.** Frank Rubin argues (rather convincingly, I believe) that "the answer 'anything' [you were to find the eleventh terms in certain sequences—ed.] is a fine surprise ending, sort of like finding out the murderer was Perry Mason himself."

**M/J 1.** Jim Landau notes that he did not solve the problem and therefore Winslow Hartford was the *only* one to solve it, a very rare occurrence.

**A/S 1.** William Hart and Lawrence Kalman have responded.

**A/S 2, 3, 4.** Naomi Markovitz has responded.

**A/S 5.** George Parks has responded.

**Proposers' Solutions to Speed Problems**

**SD 1.** Jeff is 18. We each went to camp at age 12, and Neil Sedaka first sang his song in 1962. (Without the trivia question, we know only that I am now twice as old as Jeff.)

**SD 2.** Eleven points suffices.

North
♠ —
♥ —
♦ 9 8 7 6 5 4
♣ 8 7 6 5 4 3 2

| West | East |
|---|---|
| ♠ K | ♠ — |
| ♥ K Q J 10 9 8 7 6 5 4 3 | ♥ — |
| ♦ — | ♦ A K Q J 10 3 2 |
| ♣ — | ♣ A K Q J 10 9 |

South
♠ A Q J 10 9 8 7 6 5 4 3 2
♥ A
♦ —
♣ —