PUZZLE CORNER

ALLAN J. GOTTLIEB, '67

A One-Cup-of-Coffee Solution: the Coffee's Still Warm

This being the first issue of another year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (1, 9, 8, and 8) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1987 yearly problem is in the "Solutions" section.

Bridge problems are in short supply.

Problems

Y 1988. Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 8, and 8 exactly once each and the operators +, −, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1, 9, 8, and 8 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

JAN 1. We next offer a bridge problem that Douglas Van Patter noticed in the 1986 summer nationals:

Dummy
♠ 5
♥ A K 8
♦ Q 9 3
♣ A K 8 6 5 3

Declarer
♠ A K 10 9 8 7 2
♥ 6
♦ J 10 8 6
♣ Q

You are in a four-spade contract and take the opening lead of ♥2 in dummy. Assuming rubber bridge with neither side vulnerable, how do you play the trumps? What if match points are being used?

JAN 2. Jim Landau asks us a *non*-computer problem. Compute the natural logarithms of the integers from 2 to 10, using pencil and paper *only* (no calculators, computers, or numeric tables),to four decimal places of accuracy.

JAN 3. The following problem, from Ahmad Khan and Rao Yelamarty, first appeared in the October 1986 issue of *IEEE Potentials*. It is possible to obtain all the integers from 1 to 40 by adding and subtracting various combinations of only four different integers. What are these four integers?

JAN 4. Our last regular problem, entitled"The 20th Anniversary Party," is from Richard Hess:
The hostess told me the youngest of her three children likes her to pose this problem, and proceeded to explain. "I normally ask guests to determine the ages of my three children, given the sum and product of their ages. Since Smith missed the problem tonight and Jones missed it at the party two years ago, I'll let you off the hook." Your response is "No need to tell me more, their ages are. . ."

Speed Department

SD 1. Our first speed problem is from Bonnie Dalzell and Jim Saklad. The "17-year cicada" (*Magicicada Septendecim*) is so-called because it spends 17 years underground before emerging as a winged adult. What survival-of-the-fittest advantage does this unusual life span offer?

SD 2. Greg Huber wants to know the next number in the sequence
10^3, 10^9, 10^{27}, 10^2, 0 . . .

Solutions

Y1987. This is the problem with respect to 1987 that is posed in Y1988 for the new year. It was solved by John Drumheller who, when a fully-in-order so-

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lution was impossible, tried to get three digits in order, etc.

1 1^{987}
2 81 − 79
3 1 + [(9 + 7)/8]
4 91 − 87
5 (91/7) − 8
6 7 − 1^{98}
7 1^{98} × 7
8 1^{98} + 7
9 1^{97} + 8
10 (71 + 9)/8
11 1 + 9 + 8 − 7
12 (97 − 1)/8
13 91 − 78
14 (1 × 98)/7
15 1 + (98/7)
16 97 − 81
17 17 × (9 − 8)
18 89 − 71
19 19 × (8 − 7)
20 19 + 8 − 7
21 (91/7) + 8
22 —
23 9 + 8 + 7 − 1
24 (1 × 9) + 8 + 7
25 1 + 9 + 8 + 7
26 —
27 98 − 71
28 8/[(9/7) − 1]
29 —
30 —
31 —
32 —
33 —
34 19 + 8 + 7
35 91 − (8 × 7)
36 (9 − 7) × 18
37 (8 × 7) − 19
38 —
39 [8 × (7 − 1)] − 9
40 [7 × (8 − 1)] − 9
41 —
42 —
43 —
44 —
45 (9 × 7) − 18
46 (8 × 7) − 1 − 9
47 (8 × 7 × 1) − 9
48 1 − 9 + (8 × 7)
49 $7^{(18/9)}$
50 —
51 —
52 —
53 —
54 71 − 9 + 8
55 (9 × 8) − 17
56 1^9 × 8 × 7
57 1^9 + (8 × 7)
58 [7 × (8 − 1)] + 9
59 78 − 19
60 —
61 79 − 18
62 [(7 − 1) × 9] + 8
63 (81/9) × 7
64 (17 − 9) × 8
65 81 − 9 + 7
66 1 + 9 + (8 × 7)
67 —
68 87 − 19
69 (78 × 1) − 9
70 71 − 9 + 8
71 (9 − 8) × 71
72 89 − 17
73 [(1 + 9) × 8] − 7
74 [(8 + 1) × 9] − 7
75 19 + (8 × 7)
76 91 − 8 + 7
77 (19 − 8) × 7
78 (87 × 1) − 9
79 97 − 18
80 1^8 + 79
81 98 − 17
82 (1 × 89) − 7
83 81 + 9 − 7
84 —
85 —
86 87 − 1^9
87 1^9 × 87
88 1^9 + 87
89 17 + (9 × 8)
90 1 + 97 − 8
91 (1 × 98) − 7
92 1 + 98 − 7
93 —
94 —
95 87 − 1 + 9
96 (1 × 9) + 87
97 19 + 78
98 1^7 × 98
99 (18 − 7) × 9
100 —

Also solved by Allen Tracht, A. Holt, Naomi Markovitz, Mark Johnson, Greg Spradlin, Raymond Kinsley, Steve Feldman, Avi Ornstein, Harry Zaremba, and Alan Foonberg.

A/S 1. How can West make a contract of 6 hearts against any defense?

♠ J 9 5 3
♥ J 8 2
♦ 3
♣ A J 8 6 4

♠ A K 2
♥ A K Q 10 9 7
♦ A K 7 4
♣ —

♠ Q 10 6
♥ 6 5 3
♦ J 6 5 2
♣ K Q 7

♠ 8 7 4
♥ 4
♦ Q 10 9 8
♣ 10 9 5 3 2

The following solution is from Edgar Rose: If North leads ♣A, draw trumps, and using ♠Q as entry, discard the closed hand's losing diamonds on the two high clubs in dummy. This will provide declarer with an over-trick. With any other lead, win in hand playing low from dummy and play all trumps, discarding low diamonds and a low club from dummy. Next play remaining high cards from the closed hand except ♠A, always playing low from dummy. At this point the closed hand has ♠A, ♠2, and two low diamonds. North has black suit cards only. Dummy has ♠Q, ♠10, ♣Q, and ♣K. Declarer now leads the ♠2, finessing from the ♠J. Dummy wins as cheaply as possible, thus establishing the remaining spade as a high card. Next, dummy leads the ♣Q, discarding the ♠A in the closed hand! North's ♣A can win one trick, but dummy will win any return.

Timothy Maloney and Neil Cohen remark that Charles Goren made essentially the same contract 50 years ago.

Also solved by Alan Berger, Arthur Cowen, Bill Huntington, Bowman Cutter, Douglas Van Patter, Dudley Church, Eric Youngdale, Garabed Zartarian, Kenneth Bernstein, Mark Foster, Michael Fu, Richard Hess, Robert Bart, Robert Briselli, Robert Lax, Tom Harriman, William Strauss, Winslow Hartford, and the proposer, Warren Himmelberger.

A/S 2. Is it true that every prime will divide evenly into at least one member of the following sequence: 10, 110, 1110, 11110, . . .?

The following solution is from Kenneth Bernstein: That statement is correct. Using the notation (a)(b) to denote the decimal integer formed by writing a 1's followed by b 0's. The given sequence consists of the numbers (n)(1) for n = 1,2, . . . Obviously 2 and 5 divide every number in the sequence. Let p be prime, p ≠ 2 or 5, and consider the p + 1 numbers (m)(1)m = 1,2, . . ., p + 1. Two of these numbers must be equal modulo p. Denote them by (n)(1) and (n')(1) with n' > n. Their difference, (n' − n)(n + 1) = 10^n(n' − n)(1), is then divisible by p. But p does not divide 10^n. Thus p divides(n' − n)(1).

Also solved by Bill Huntington, Charles Sutton, Dennis White, Edward Dawson, Neil Cohen Harry Zaremba, Richard Hess, Robert Bart, Scott Berkenblit, Steven Feldman, Tom Harriman, and Winslow Hartford.

A/S 3. Show that, using only the operations of addition, subtraction, and multiplication, any three integers can combine to produce a multiple of 12.

The following solution is from Edward Dawson: Let the integers be represented by L, M, and N. For each of them a congruence can be written as follows: L ≡ l(MOD 12), M ≡ m(MOD 12), N ≡ n(MOD 12), where l, m, and n can have values of 0, ∓1, ∓2, ∓3, ∓4, ∓5. When l, m, and n are unequal, one of the operation sequences in the following table will yield a multiple of 12. When the same operations are performed on the corresponding values L, M, and N, the resulting integer will also be a multiple of 12.

l	m	n	Operations	Result
1	2	3	1 + 2 − 3	0
1	2	4	(1 + 2) 4	12
1	2	5	(1 + 5) 2	12
1	3	4	1 + 3 − 4	0
1	3	5	(5 − 1) 3	12
1	4	5	1 + 4 − 5	0
2	3	4	(2) (3) (4)	24
2	3	5	2 + 3 − 5	0
2	4	5	(5 − 2) 4	12
3	4	5	3 + 4 + 5	12

Similar operations on mixtures of positive and negative values of l, m, and n, will also provide multiples of 12. When any two, or all three, values of l, m, and n are equal, a computation of the form (l − m)n, where l = m, will give a result of zero, so that (L − M)N will be a multiple of 12. If l, m, or n is zero, the product (L)(M)(N) is a multiple of 12.

Also solved by Bill Huntington, Harry Zaremba, Richard Hess, Robert Bart, Steven Feldman, William Strauss, Winslow Hartford, and the proposer, Michael Gennert.

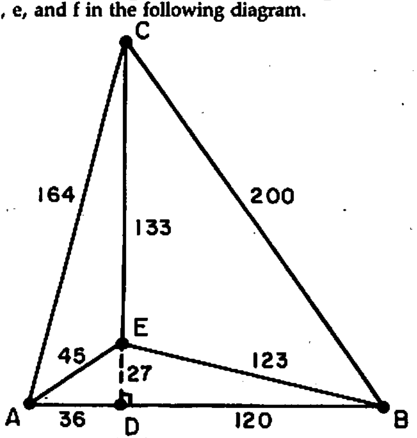
A/S 4. A fly is sitting on the positive x-axis a distance h from the origin. A man with a fly swatter is sitting at the origin and spies the fly. At t = 0 the fly is pursued by the man, who has a maximum acceleration of unity. The fly evades the swatter by flying out along the x-axis with an acceleration 0.6t. What is the largest initial separation h for which the swatter can intercept the fly in a finite (positive) time?

Tom Kelley felt that this problem, which he classifies as "a simple one-cup-of-coffee problem," made his day.

His solution is:
$a(fly) = 0.6 \times t$
$v(fly) = 0.3 \times t^2$ (initial $v = 0$)
$d(fly) = 0.1 \times t^3 + h$ (h = initial separation)
$a(man) = 1$ (max.)
$v(man) = t$ (max., initial $v = 0$)
$d(man) = 0.5 \times t^2$ (max., initial $d = 0$)
Note 1: Use maximum acceleration to catch fly at largest initial separation.
Note 2: Man and fly have the same velocity at $t = 0$ (both at rest) and $t = 10/3$ ($v = 10/3$). Between $t = 0$ and $t = 10/3$ the man is moving faster than (catching up to) the fly; after $t = 10/3$ the fly is moving faster and the man can never catch it.
Note 3: At $t = 10/3$, $d(man) = 0.5 \times (10/3)^2 = 0.5 \times 100/9 = 50/9 = 150/27$, and $d(fly) = 0.1 \times (10/3)^3 + h = 0.1 \times (1000/27) + h = (100/27) + h$. Equating the two (splat!): $h = 50/27$. Coffee's still warm.

Also solved by Antony Merz, Bill Huntington, David DeLeeuw, Dennis White, Duncan Allen, Edward Dawson, Harry Zaremba, John Rulnick, Kelly Woods, Ken Rosato, Kenneth Bernstein, Mark Foster, Michael Auerbach, Michael Jung, Peter Silverberg, Ray Kinsley, Richard Hess, Richard Marks, Robert Bart, Robert Moeser, Ruben Cohen, Scott Berkenblit, Steven Feldman, Tom Harriman, Winslow Hartford, and the proposer, John Prussing.

A/S 5. Find integral values for the lengths a, b, c, d, e, and f in the following diagram.



Harry Zaremba found a solution that depends on two assumptions:
It is assumed all the lengths are to be unequal, and that the extension of line CE = e is perpendicular to the line AB = c at the point D.
One solution involving Pythagorean triplets is indicated in the figure in which the required integral lengths are:
AC = a = 164
BC = b = 200
AB = c = 156
AE = d = 45
BE = f = 123
CE = e = 133

Also solved by Avi Ornstein, Bill Huntington, Dudley Church, Edward Dawson, Garabed Zartarian, Joe Neuendorfer, Ken Rosato, Mark Foster, Mary Lindenberg, Robert Bart, Scott Berkenblit, Tom Harriman, Yamaji, William Strauss, and Winslow Hartford.

Better Late Than Never

APR 1. Robert Bart does not consider the solution "an entirely different approach."

APR 2. Unfortunately, either I misplaced Robert Bart's solution or it was lost in transit. Mr. Bart had recognized that APR 2 is the famous "Problem of Apollonius," which admits eight exact solutions. Stanley Zisk also found eight solutions and writes: One way to describe the eight solutions would be as follows. With your numbers 1, 2, and 3 as the original circles, imagine a circle "0" drawn through their centers. Then let us define that a solution circle "contains" one of the original circles when its tangent point lies outside the circle "0." Now we see that your number 4 is the solution that "contains" none of the originals; number 5 contains all three of the originals; and numbers 6, 7, and 8 each contain two of the originals. It is easy to see that there are three more circles that each contain only one of the originals. This definition for "contain" will not always be strictly accurate for all combinations of sizes and arrangements. Another way to describe the solutions would be to start with the inscribed circle, number 4. Keeping it tangent to numbers 1 and 2, allow it to enlarge and intersect with number 3. Continue to enlarge it until it has entirely swallowed and is again tangent to number 3, on the other side. This could also be done for numbers 1 and 2, yielding a total of three new solutions. Now going back to the first of the new solutions, keeping it tangent to number 2 (on the outside) and to number 3, allow it to enlarge and intersect number 1 until it is again tangent to number 1 on the outside. This results in your number 8, and so forth. So, I suggest that the general number of solutions is, in fact, eight: three each containing one and two circles, and one each containing zero and all three circles.

APR 5. William Strauss found an alternative solution that he believes is easier to do in your head.

M/J 1, M/J 4. Tom Harriman has responded.

JUL 2. Naomi Markovitz and Tom Harriman have responded.

JUL 3. Michael Jung, Dave McNally, Tom Harriman, and Alan Taylor have responded.

JUL 4. Naomi Markovitz, Tom Harriman, and Norman Wickstrand have responded.

JUL 5. Michael Jung and Tom Harriman have responded.

A/S SD1. Dudley Church notes that it takes training to tie a bowline with one hand.

Proposers' Solutions to Speed Problems

SD 1. All predators of the cicada must have a generation length that is relatively prime to the cicada's. Hence if the predator has a population increase due to a good supply of cicadas, the next swarm of cicadas will emerge out of sync with the predators, reproductive cycle. (If the predator has a one-year cycle it will need to wait 16 generations for cicadas, and no predators have life spans of 34, 51, etc., years.)

SD 2. The ith term of the sequence is the smallest non-negative integer whose name contains the ith letter of the alphabet, i.e., thousAnd, Billion, oCtillion, . . .