ALLAN J. GOTTLIEB

I Spy the Fly; But Will Physics Let Me Swat It?

After last month's long introduction, I find myself surprisingly speechless this time. So let's get right to the problems.

Problems

A/S 1. We begin with a bridge problem from Warren Himmelberger in which West must make a contract of 6 hearts against any defense:

North:
♠ J 9 5 3
♥ J 8 2
♦ 3
♣ A J 8 6 4

West:
♠ A K 2
♥ A K Q 10 9 7
♦ A K 7 4
♣

East:
♠ Q 10 6
♥ 6 5 3
♦ J 6 5 2
♣ K Q 7

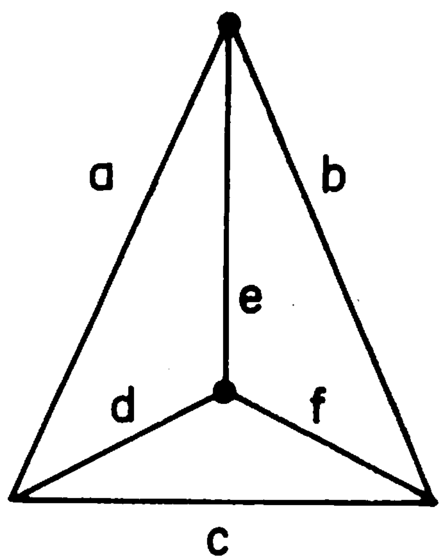
South:
♠ 8 7 4
♥ 4
♦ Q 10 9 8
♣ 10 9 5 3 2

A/S 2. Howard Sard feels that every prime *will* divide evenly into at least one member of the following sequence: 10, 110, 1110, 11110, ... Is he right?

A/S 3. Michael Gennert wants you to show that using only the operations of addition, subtraction, and multiplication, any three integers can be combined to produce a multiple of 12. Parentheses are permitted. For example, given 9, 11, and 52, we can form
$9 \times 11 \times 52 = 12 \times 429$
and given 15, 17, and 23, we can form
$(23 + 17) \times 15 = 12 \times 50$.

A/S 4. John Prussing's swatter can out-accelerate his flies (at least initially); but is that good enough? A fly is sitting on the positive x-axis a distance h from the origin. A man with a fly swatter is sitting at the origin and spies the fly. At $t = 0$ the fly is pursued by the man, who has a maximum acceleration of unity. The fly evades the swatter by flying out along the x-axis with an acceleration 0.6t. What is the largest initial separation h for which the swatter can intercept the fly in a finite (positive) time?

A/S 5. Richard Hess and Nob. Yoshigahara want you to find integral values for the lengths a, b, c, d, e, and f in the following diagram.



Speed Department

SD 1. Jim Landau asks what you should do if you are hanging onto a rope for dear life and are almost (but *knot* quite) at the end of your rope.

SD 2. Phelps Meaker has a regular hexagonal basket whose vertical sides are equal in area to $\sqrt{3}$ times that of the bottom. How deep is the basket?

Solutions

APR 1. South is declarer at a six-heart contract. Opening lead: ♠5. Make 12 tricks against best defense:

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

	North	
	♠ A K 10 4 ♥ A J 10 ♦ A 7 5 ♣ A 5 3	
♠ Q 9 8 5 ♥ 2 ♦ J 9 4 3 2 ♣ Q 9 2		♠ J 3 ♥ Q 9 4 3 ♦ 10 8 6 ♣ J 8 6 5
	♠ 7 6 2 ♥ K 8 7 6 5 ♦ K Q ♣ K 10 4	

This hand occurred at a duplicate bridge club. South was the only declarer to make 12 tricks. In her successful line of play, East made the ♥9 at the end of the hand. After looking at this double-dummy problem, another friend suggested an entirely different approach. In his line, East makes the ♥9 earlier in the play. What are these two successful lines of play?

The following solution is from Timothy Maloney:

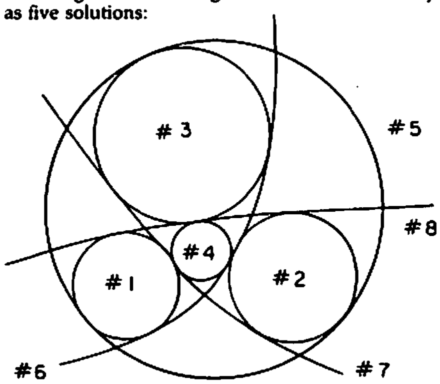
Declarer takes the ♠A in dummy, then leads the J and 10 of trump from dummy. East must cover with the ♥Q, which is covered by the ♥K. South leads out the ♦K and ♦Q, then another heart to the ♥A leaves East with the master ♥9. Declarer discards a spade on the ♦A, wins the ♠K, and when he leads the ♠10 at trick 9, East decides the course of play:

1. East takes the ♥9 and South discards a club. East must return a club and declarer takes two clubs and two remaining trumps for 12 tricks: 5 hearts, 2 spades, 3 diamonds, and 2 clubs.
2. East discards a club and South ruffs with ♥7 or ♥8. He returns to dummy with ♣A and leads his last spade. East can win with his ♥9 and return a club to South's ♣K and last trump, or he can duck again. If East ducks, South ruffs again, wins his ♣K and concedes the last trick to East's ♥9. Again, 5 hearts, 2 spades, 3 diamonds, 2 clubs to declarer.

Also solved by Matthew Fountain and the proposer, Doug Van Patter.

APR 2. Given the circles of different radii, no circle contained within another, construct a fourth circle tangent to the other three.

No one found an exact geometric solution. James Landau enclosed an analytic solution from Coxeter's book *Introduction to Geometry* published by John Wiley and Sons, and Norman Wickstrand sent one from *Tips and Techniques—Engineering Aids*, Volume 2 (1956). This last reference also gives geometric solutions, which are exact if the three circles are of equal radii. In addition, Mr. Wickstrand drew the following sketch showing that there can be as many as five solutions:



1, 2, and 3 are the given circles; 4 is the inscribed circle; 5 is the circumscribed circle; 6, 7, and 8 are approximate arcs indicating how three more circles can be drawn. Circle 8 could be a circle of infinite radius or could be concave up or concave down, as shown.

Finally, the following geometric approximation technique is from Matthew Fountain:

The center of the fourth circle is at distance of q + t, r + t, and s + t from centers of the other three, where q, r, s, and t are the radii of the respective circles. Therefore the center of the fourth circle q − r farther from the center of the first circle than from the center of the second circle and must lie on a hyperbola with foci at these last two centers. Sections of this hyperbola may be plotted to any required degree of accuracy by plotting the locus of the points of intersection of arcs drawn about the centers of the first two circles, the first of each pair of intersecting arcs being q − r greater in radius than the other. A second hyperbola through the center of the fourth circle may be obtained by plotting the points of intersection of arcs drawn about centers of the second and third circles, the first arc of each pair of intersecting arcs being r − s greater in radius than the other. The intersection of these two hyperbolas is the desired center of the fourth circle. It is not necessary to find many points in practice, as one can guide one's progress by assuming that over short sections the hyperbolas are straight lines. I do not believe there is any theoretical way of locating the center of the fourth circle exactly with just straightedge and compass.

Also solved by John Cushnie, John Woolston, Mark Markish, Mary Lindenberg, and the proposer, J. Ruoff.

APR 3. The diagram represents the product of three-digit numbers by two-digit numbers. What were the original problems?

The spacing was not done very well on the first part of this problem. Larry Bell, however, was not deterred and solved the problem that was intended:

We need to make some assumptions about liquid crystal displays. First, any number appearing as three horizontal lines (on a calculator that cannot draw vertical lines) must be either 2, 3, 5, 6, 8, or 9. A number appearing as a blank must be a 1. In the problems given there can be no 4s, 7s, or 0s. Label the digits

```
    A B C
  ×   D E
---------
V W X Y Z
```

In each answer, Z is blank and is therefore 1. So, C = E = 9. The last two problems are

```
    5 2 9     and       5 6 9
  ×   5 9             ×   3 9
---------           ---------
3 1 2 1 1           2 2 1 9 1
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In the first problem, there may be a "typo," since there is no match for X = Z = 1 and V = either 0 or 1. The closest match I could find is

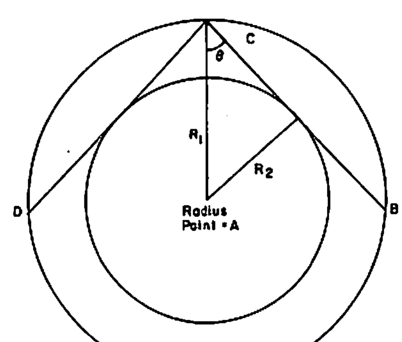
```
    3 5 9
  ×   5 9
---------
2 1 1 8 1
```

which could be correct if the width of the first space in the first answer was not drawn to scale.

Also solved by Alan Taylor, Dick Robnett, James Landau, John Woolston, Matthew Fountain, Steve Feldman, and the proposer, Nob. Yoshigahara.

APR 4. A star-shaped figure can be drawn with two concentric circles, drawing chords tangent to the inner circle, successively around the outer one until they meet the starting point. How many points, blunt as they may be, result if the inner circle is .66913 of the outer? Compass and straightedge not allowed.

The following solution is from Ken Rosato.



By observation R_1/R_2 = sin θ. So sin θ = $\sin^{-1}(.66913)$ = 42°00′. By symmetry, angle $\overline{ACB}$ = angle ACD = θ, so angle BCD = 2θ = 84°. $\overline{BC}$ and $\overline{CD}$ are chords on the star in question, and point C is a typical apex of the star. The supplement of angle BCD will be the deflection angle for each point of the star, and will have a value of 180° − 84° = 96°. Now for a star to be properly formed, the successive chords must be formed until the first time a new chord falls exactly on an old one. Since a circle has 360°, this condition will be fulfilled when the running total of the deflection angles reaches an integral multiple of 360°. We thus have n(96°) = k(360°), where n is the sought-after number of sides of the star, and k is an integer. Re-arranging gives n = (360°/96°)k, which reduces to n = (15/4)k. For k = 1, 2, or 3, n will not be an integer and the star will not have its proper form. But when k = 4, n = 15, giving a 15-sided star whose chords circumnavigate the larger circle exactly 4 times.

Also solved by Alan Hodgkinson, Alan Taylor, Daniel Morgan, Dick Robnett, Frederick Furland, James Landau, John Prussing, John Woolston, Larry Bell, Matthew Fountain, Steve Feldman, and the proposer, Phelps Meaker.

APR 5. Mr. Brown asked Mr. Smith to perform the following operations in the order named, without Brown's being able to see Smith's work:
(a) Write any positive integer, preferably of two digits to save labor on the part of Smith.
(b) Multiply this number by the next highest integer.
(c) Multiply the result of (b) by 225.
(d) Add 56 to the result of (c).
(e) Tell Brown all the result of (d) except the two right-hand digits.
Smith gave 4064 in response to the request in (e), whereupon Brown, after a moment's computation, informed Smith that his result after step (d) was 406406 and that the number he originally chose was 42. Smith confirmed these statements. How did Brown reach his conclusion?

Our last solution is from Sidney Shapiro:
For integer values of x, the computed number (x)(x+1)(225) + 56 ends in either 06 or 56. Given all but the last two digits of the computed number, Mr. Brown proceeded as follows:
a. He appended two zeros to the all-but-last-two-digits number; call that number P.
b. He let Q = P+6
c. He solved the quadratic equation
$225x^2 + 225x + 56 = Q$
d. For x equal to an integer, x is the value of the selected number and the last two digits of the computed number are 06.
e. If x is not an integer, Mr. Brown let Q = P+56 and repeated step c. The resulting integer value of x is the selected number and the last two digits of the computed number are 56.
For example, if the all-but-last-two-digits are 2678:
a. P = 267800
b. Q = 267806
c. x = 34 and the computed number is 267806.
If the all-but-last-two-digits are 2835, then:
a. P = 283500
b. Q = 283506
c. x is not an integer
d. Setting Q = 283556, x = 35 and the computed number is 283556.

Also solved by Daniel Morgan, Dick Robnett, Frederick Furland, James Landau, John Cushnie, Larry Bell, Mary Lindenberg, Matthew Fountain, and Michael Jung.

Better Late Than Never

JAN 2. Naomi Markovitz has responded.

JAN 3. Naomi Markovitz has responded.

FEB 3. Naomi Markovitz and Kelly Woods have responded.

FEB 4. Kelly Woods has responded.

Proposers' Solutions to Speed Problems

SD 1. Holding onto the rope with your legs and one hand, use the other hand to tie a bowline in the rope. Make sure that the loop of the bowline is about a foot or two across. Now you can sit comfortably and safely in the loop until you are rescued. Legend has it that someone saved himself from the burning Hindenburg by using this trick.

SD 2. Three quarters of one of the flats of the hexagon.