ALLAN J. GOTTLIEB

Palindromic Time and a Chemical Pun

Let me use this month's introduction to report and in some cases respond to remarks from several readers. Jim Landau, who sent me a *real* rope problem (i.e. the problem statement included a real rope), writes to report that according to *The Ashley Book of Knots* his rope contained #1164 as well as a variation of #2692. Clearly the impressive part is that there is a book that includes descriptions of thousands of knots. Indeed, judging by the photocopy of a few pages Mr. Landau sent to me, the book can give knot theorists and (especially) practitioners countless hours of activity.

John Drumheller suggests that lovers of the yearly problem published each January enjoy their successes on **Y1987** since, for the years that follow, the success rates will be much lower.

Avi Ornstein posts *Puzzle Corner* in his high school classroom and notes that Patel's solution to **1986 DEC 4** marks the third time in about a decade that one of his students was mentioned. John Boynton asks:"What is the maximum number of puzzle submissions, in linearly increasing difficulty, accompanied by a geometrically increasing submission of associated answers, with a fixed ratio of correct:incorrect solutions, that an otherwise sane college math professor can handle in a year without committing homicide/suicide?" I wonder what he could be referring to?

Mike Green sells disentanglement puzzles. Interested readers should write to Puzzletts, 24843 144th Place SE, Kent WA 98042.

Longtime participant Phelps Meaker writes that a recent cataract operation has left him with somewhat diminished vision. Nonetheless, the same letter contains a new problem, which is published as JUL 2 below.

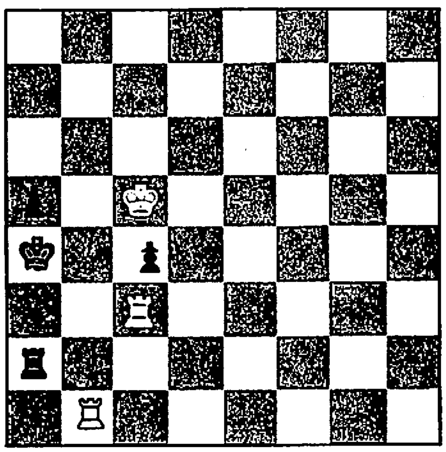
SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

Finally, John Chandler suggests that I publish my electronic address. OK, it is gottlieb@nyu.arpa or in the new (domain) format gottlieb@nyu.nyu.edu.

Problems

JUL 1. We begin with a chess problem from Matthew Ek in which White is to move and mate in three:

BLACK



WHITE

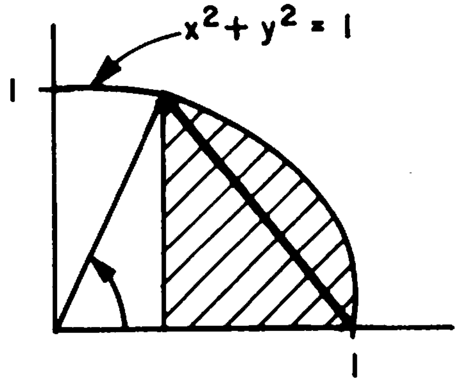
JUL 2. Phelps Meaker wants to know the radius of a sphere circumscribing a regular tetrahedron.

JUL 3. Richard Hess writes that Nob Yoshigahara requested that he submit the following problem:
A (24-hour) digital watch has many times that are palindromic, such as 1:00:01, 23:22:32, :11, 2:44:42, etc. (ignore the colons). Find the two closest such times. Find the two that differ closest to 12 hours. Find the longest time span without a palindromic time.

JUL 4. Dennis Clougherty wants you to find the exact value of (cos 36°) − (cos 72°).

JUL 5. Jerry Grossman sent us a geometry problem from Sherman Stein that appeared in *Focus*:

Let A be the area of the shaded curved region, and B the area of the triangle. Find $\lim_{\theta \to 0} A/B$.



Speed Department

SD 1. Jim Landau writes that in nursery school his son was taught to put his coat on the floor "outside down," with the hood toward him. Why?

SD 2. The Freshman Quiz that appeared in *The Tech* on September 4, 1984, asked for the name of the following compound: BaAuHiJKLMnO.

Solutions

F/M 1. What is the shortest chess game that ends with the move, "Pawn takes pawn, en passant, double check, mate"?

Robert Bart and Lorris Mizrahi found essentially the same solution. Mr. Bart's move order is:

1	P–Q4	P–K4
2	P×P	P–Q4
3	N–QB3	P–Q5
4	K–Q2	B–QB4
5	Q–K1	B–KN5
6	P–K4	P×P ep dbl ch mate

This problem appeared on an electronic bulletin board and the solution sent there by Tim Smith is too good not to reprint:

I can beat this by half a move, if I am allowed to assume that the players are Australians playing blindfolded:
W: "G'day, Bruce. d4, mate"
B: "G'day, Bruce. d6,mate"
W: "Bf4, mate"
B: "Kd7, mate"
W: "B×d6, mate"
B: "Nf6, mate"
W: "Ba3, mate"
B: "Ne8, mate"
W: "d5, mate"
B: "e5, mate"
W: "Pawn takes pawn, en passant, double check, mate"
B: (lost on time)

Also solved by Greg Spradlin, Matthew Fountain, Ronald Raines, Thomas Harriman and the proposer, K. Heuer.

F/M 2. The Cubs lost 13 games in a row during the summer of 1985; what is the *a priori* probability that this is the major league record?

John Chandler found an earthshaking solution. He writes:

Calculating an *a priori* probability of a certain length losing streak is like predicting earthquakes—both require a lot of (probably unwarranted) assumptions. First, assume that the year's worst team in the major leagues has a probability p of winning any game, regardless of opponent and schedules. Next, assume that p is invariant from year to year, even though the worst team won't be the same. Also, assume that "streaks" must start and end in a single season, so that the same probability p applies throughout. Now, the probability of any game being part of a losing streak of length at least 1 is $q = 1 - p$, i.e., the probability of losing that game; the probability of being in a streak of at least 2 is $q(1 - p^2)$, i.e., the probability of losing times the probability of not winning both surrounding games. In general, ignoring end effects, the probability of any game being in a losing streak of length at least N is $q^n (N - [N - 1]q)$. Thus,if the season were $kN - 1$ games long, the probability of *not* having a losing streak of at least N is just$\{1 - q^N(N - [N-1]q)\}^{k-1}$, the probability that none of the set of $k - 1$ evenly spaced games is in such a streak. Now for an approximation: the corresponding probability for L seasons of length M is $\{1 - q^N(N - [N-1] q)\}^{L(M-N-1)/N}$. This amounts to about 1.5×10^{-5} probability that no losing streak of 14 games occurs in 100 seasons of 100 games each if the worst team has a 1/3 probability of winning each game. Assuming that the second-worst team is even noticeably better than the worst, it will have a far higher probability of escaping that fate: p=2/5 gives about 5.5×10^{-2} probability of avoiding a 14-game losing streak. In any case, the probability that 13 is the record is very small.

Also solved by Matthew Fountain, Michael Jung, Peter Silvenberg, Robert Bart, Mark Markish, and the proposer, Thomas Harriman.

F/M 3. A general Fibonacci series is formed by taking $x_1 = A$, $x_2 = B$, $x_{n+2} = x_n + x_{n+1}$ for n = 1,2,3, . . . and its sequent series is formed by taking $x_1 = A$ and $x_2 = B + 1$. If the tenth terms of a series and its sequent series are 127 and 161, respectively, what are the 11th terms?

I guess Judith Longyear put it best in her parenthetical remark, "I can't believe Frank Rubin [a frequent contributor of problems of unusually high quality—ed.] goofed. What was the *real* question? The solution to the printed question is "whatever you like,' R and R+55." A detailed explanation of the last point comes from Steve Feldman:

The series:
x1 = A, x2 = B, x3 = A + B, x4 = A + 2B, x5 = 2A + 3B, x6 = 3A + 5B, x7 = 5A + 8B, x8 = 8A + 13B, x9 = 13A + 21B, x10 = 21A + 34B, x11 = 34A + 55B . . .
The sequent series:
xs1 = A, xs2 = B + 1, xs3 = A + B + 1, xs4 = A + 2B + 2, xs5 = 2A + 3B + 3, xs6 = 3A + 5B + 5, xs7 = 5A + 8B + 8, xs8 = 8A + 13B + 13, xs9 = 13A + 21B + 21, xs10 = 21A + 34B + 34, xs11 = 34A + 55B + 55 . . .
We are told that x10 = 127 (we do not need to be told that xs10 = 161, because xs10 = x10 + 34). We need to solve the diophantane equation 21A + 34B = 127. This has infinitely many solutions if A and B are allowed to take on both positive and negative integer values. One such solution is A = 19, B = −8.
The resulting series is 19, −8, 11, 3, 14, 17, 31, 48, 79, 127, 206 . . .
The resulting sequent series is 19, −7, 12, 5, 17, 22, 39, 61, 100, 161, 261, . . .
To generalize, A = 19 + 34N, B = −8 − 21N, where N is any integer.
The generalized series is 19 + 34N, −8 − 21N, 11 + 13N, 3 − 8N, 14 + 5N, 17 − 3N, 31 + 2N, 48 − N, 79 + N, 127, 206 + N, . . .
The generalized sequent series is 19 + 34N, −7 − 21N, 12 + 13N, 5 − 8N, 17 + 5N, 22 − 3N, 39 + 2N, 61 − N, 100 + N, 161, 261 + N, . . .
Thus, the eleventh terms of the two series are 206 + N and 261 + N, and can therefore take on any integer value.

Also solved by Avi Ornstein, Greg Spradlin, Harry Zaremba, Jorgen Harmse, Jim Rutledge, John Chandler, John Weissberg, Mary Lindenberg, Matthew Fountain, Michael Jung, Raymond Kinsley, Robert Bart, Steven Feldman, Mark Markish, and Thomas Harriman.

F/M 4. Assume that when a radio weather report cites the temperature on both Fahrenheit and Celsius scales, the Fahrenheit reading represents the actual temperature to the nearest degree and the Celsius number is simply obtained from a chart which gives, for each integer F, the whole degree Celsius temperature nearest to F°. Under some reasonable uniformity assumptions, what is the probability that the reported Celsius temperature is wrong, i.e. not the actual temperature to the nearest whole degree Celsius?

The following solution is from Matthew Fountain: The reported temperature has a probability of 0.133 of being wrong. We note that the F or Fahrenheit temperature report may be plus or minus 0.2778 Celsius degrees incorrect. The Celsius reports differ from the F reports by 0.0000, 0.1111, 0.2222, 0.3333, and 0.4444 degrees Celsius, as will be shown later. To cause the Celsius report to be wrong, the error in the F report must increase the error due to the conversion past 0.5000. This cannot happen when the conversion errors are 0.0000, 0.1111, and 0.2222. The F error must lie between 0.1667 and 0.2778 to increase 0.3333 to 0.5. As the F error has an equal chance of being any value from 0.0000 to 0.2778, half the time being additive and half the time being subtractive, the probability of a conversion error of 0.3333 being raised to 0.5000 is (0.2778 − 0.1667)/[(2)(0.2778)] = 0.2. Similarly, the probability of a conversion error of 0.4444 being raised to 0.5 is (0.2778 − 0.0556)/(0.5555) = 0.4. The formula for converting Fahrenheit to Celsius is C = (5/9)(F − 32). When we apply this formula to consecutive integer F values ranging from 32 to 40, we obtain 0.0000, 0.5555, 1.1111, 1.6667, 2.2222, 2.7778, 3.3333, 3.8889, and 4.4444. Rounding off these nine Celsius temperatures to the closest integer produces errors exceeding 0.2778 in only four cases, the errors being 0.4444 twice and 0.1667 twice. We conclude that weather report temperatures of 32, 34, 36, 37, and 39° F always convert to the closest integer Celsius, that 33 and 40°F fail to convert to the closest integer Celsius with a probability of 0.4, and that 35 and 38°F fail to convert with a probability of 0.2. If we repeat this procedure with higher temperatures, we find the same distribution over any nine-Fahrenheit-degree range—that is, five temperatures have no possibility of error, two have a 0.2, and two have a 0.4 possibility of error. Given a reasonable spread in the temperatures reported, we conclude the probability of the Celsius reading to be wrong is

$$[(5)(0.0)+(2)(0.2)+(2)(0.4)]/9 = 0.133.$$

Also solved by Dick Allphin, Dick Mackler, Jim Rutledge, John Chandler, John Weissberg, Ken Rosato, Peter Silvenberg, Raymond Kinsley, Robert Bart, Steve Silberberg, Steven Feldman, Thomas Harriman, Mark Markish, and the proposer, Jerry Grossman.

F/M 5. Given a roller coaster car that just sits on the rails and is not otherwise held down to the track, is it possible for an object in the car, say a Kupie doll sitting on your lap, to be thrown out as the car goes over the hills on the track? Assume a track with no turns to either side and that the car stays in contact with the rails at all times. Ignore air resistance.

Robert Bart notes that the vertical acceleration occurring when the roller coaster crests a circular hill with center O is $\omega^2 r$, where ω is the angular velocity and r is the radius of curvature. For both the car and the doll, r is measured from O to the center of mass of the object. If the doll is higher than the center of mass of the car, then there are values for ω such that

$$\omega^2 r_{car} < g < \omega^2 r_{doll},$$

where g is the acceleration due to gravity. For these speeds the car will stay down but the doll will not.

Also solved by John Chandler, John Weissberg, Matthew Fountain, Raymond Kinsley, Thomas Harriman, Mark Markish, and the proposer, Robert Cherry.

Better Late Than Never

1986 OCT 1. Robert Bart, Jorgen Harmse, and Greg Spradlin noticed that the final move is not mate since the Queen can be captured. Bart and Spradlin supplied the same corrected solution:

1 Q−R1 ch	K−R2
2 N−N5 ch	Q×N
3 Q−KN1 ch	K−R1 or K−R3
4 Q−R7 ch	K×Q
5 P×Q	K−R1
6 P−R6	K−R2 for . . . P×P, see below
7 P−R7	K−R1
8 P−R8=R!	K−R2
9 R−R1	K−R1
10 R−R1	mate
or	
6	P×P
7 P−N6	any
8 P−N7 ch	K−R2
9 P−N8=Q ch	K−R3
10 Q−N6	mate

OCT 5. Robert Bart believes that the possibility of a pentagonal base should not have been ignored since the problem did not require a convex solid.

1987 JAN 3. Peter Lawes has responded.

F/M SD2. Roy Schweiker and Robert Bart note that there is a difference between biweekly and semimonthly.

Proposer's Solutions to Speed Problems

SD 1. If he puts his arms into the sleeves and then raises them over his head, the coat will slide onto him right-side-up. [This is a very valuable technique; in NYC it is called flipping on your coat—ed.]

SD 2. Barium Goldwater (barium gold H-to-O).