ALLAN J. GOTTLIEB

Where Squares and Office Parties Mix

Jim Landau sent me quite an unusual package. I have often received mail marked fragile, but never before one mailed indestructible. Inside was a rope tied into a special knot and an accompanying problem. Although I do not as yet see how to present the problem without having each issue of *Technology Review* include an attached rope, I have had fun with the problem myself. I should also remark that this indestructible parcel seemed to get considerably better treatment en route than many unmarked packages I have received. Perhaps stores should try sending crystal in packages marked indestructible.

On a personal note, I am pleased to report that I have received tenure at NYU.

Problems

N/D 1. We begin with a computer-related problem from Al Weiss, who is given the coordinates of four points and wants to determine if they form the four corners of a square. No assumption can be made about the order in which the points are presented. For example, a square might occur as:
(8,8), (3,5), (2,4), (4,9).
Mr. Weiss seeks not just a solution, but an elegant algorithm.

N/D 2. Oren Cheyette offers us a seasonal problem. For an office party, each person is supposed to bring a gift for someone else. The recipients are assigned to givers by writing each person's name on a slip of paper, putting the slips in a hat, and having everyone draw a slip. Obviously, it's no fun if someone draws his own name. What is the probability that in an office of n people, no one draws his own name?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

N/D 3. Given a square matrix A (not necessarily invertible), satisfying
$AA = AA'$,
where the prime signifies transpose operator, Howard Stern wants you to show that
$A = A'$
using matrix operation only, i.e. without using normed algebras and approximating A by an invertible matrix.

N/D 4. Walter S. Cluett asks: What is the lowest number of current U.S. coins (1 cent through $1.00) for which there is no combination of coins that will equal in value a single coin? How many such quantities are there under 100?

N/D 5. Ronald Raines wants you to find functions f and g satisfying
$f[f(x)] = x$
$g[g(x)] = -x$
for all real values of x.

Speed Department

SD 1. Phelps Meaker has a pan with perpendicular ends and sloping sides. It is two inches deep and measures 8″ × 10″ on the bottom. What is the slant height of the sides if the capacity is 200 cu. in.?

SD 2. Jim Landau wants to buy a solid-state digital clock with a 12-hour LED display and wonders at what time would the largest number of LEDs be on? The smallest number?

Solutions

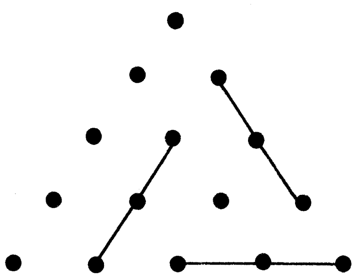
JUL 1. Find the smallest prime number that contains all 10 digits.

Dan Schmoker combined analysis with some computer work to obtain the following solution: The smallest 10 digit number with all digits different is 1,023,456,789. The sum of the digits in this number is 45 which is divisible by 3; hence the number is divisible by 3 and is not prime. In addition, any rearrangement of this 10-digit number would also be divisible by 3 and not prime. It follows therefore, that the smallest prime number that contains all digits must be an 11-digit number. The extra digit cannot be 0, 3, 6, or 9; otherwise the new number and any number formed by rearranging the digits would be divisible by 3, and hence not prime. The smallest number which contains all digits and could smallest on top. The tower is to be restacked on one of two additional sites, moving one disk at a time off the top of one stack either to an empty site or to the top of the stack on one of the other sites, without ever placing a larger disk on a smaller. The original problem was to find the minimum number of moves to transfer the entire stack. The new problem is to calculate the location of each disk after 1,001 moves have been made using the optimum transfer procedure.

Rik Anderson found it easier to take a giant step forward and then 23 small steps back:
Identify the original stack as position A. If disk 1 is initially moved to position C, then disk 2 will initially be moved to position B at move 2, disk 3 will go to C at step 4, disk 4 to B at step 8, etc. Disk n will first move at move $2^{(n-1)}$, to position B if n is even, to C if n is odd. Following this rule, the first time disk 11 moves is at move $2^{10} = 1024$, to position C. At the previous move, number 1023, disks 1-10 would have reached position B. In getting to this position, disk 1 is at any position for two moves, disk 2 moves every 4th move, disk 3 every 8th move, and disk 4 every 16th. Disk 1's moves are in reverse sequence (A,C,B,A,C,B), as are all odd-numbered disks, while even-numbered disks visit the positions in sequence (A,B,C,A,B,C). Working these patterns backwards from step 1023, we would find at step 1001:
Position A - disks 1, 4, and 11 to 64
Position B - disks 2, 3, and 6 to 10
Position C - disk 5.

Also solved by Dennis White, John Prussing, Matthew Fountain, Ned Staples, Richard Hess, Winslow Hartford, and the proposer, Lester Steffens.

JUL 3. Define an n-triangle to be a collection of n(n + 1)/2 points regularly spaced into the shape of an equilateral triangle with n points on a side. Define a 3-line to be a line segment connecting exactly three adjacent points parallel to a side of an n-triangle. (The three adjacent points are said to be "covered" by the 3-line). For what values of n can all points of an n-triangle be covered by non-intersecting 3-lines?



This question appears to be difficult, and I consider the problem to be still open. Harry Zaremba shows that if one drops the requirement that 3-lines are parallel to a side of the triangle, then a solution exists for all
$n = 9m$ and $n = 9m - 1$.
Dennis White, Richard Hess, and Winslow Hartford express the belief that (with the parallel requirement) no solutions are possible. Mr. Hartford notes that to have a multiple of three points, we need
$n = 3m$ or $n = 3m - 1$,
and indicates that an inductive proof should be possible.

JUL 4. A manufacturer makes all possible sizes of brick-shaped blocks such that the lengths of the edges are integral multiples of the unit of length, and that the number of units in the total length of the twelve edges of the block is equal to two-thirds of the number of units of volume in the block. What sizes does he make?

The following solution is from Howard Stern:
AB:CD × E = FG:HI
Since the smallest value that AB can assume is 12, it follows that E cannot be larger than 4. It also cannot be 1 because then the times would be the same. Therefore, E must be 2, 3 or 4. This forces A to be 1. In addition, C, F and H must be 2, 3, 4 or 5 for the times to make sense. Since A = 1, E cannot be 2 because then F would also have to be 2. E cannot be 4 either because then B would have to be 2 or 3, and it would be impossible for C, F and H to be 2, 3 or 5. Therefore, E must be 3. This forces C, F and H to be 2, 4 or 5, and D and I must be 6, 7, 8 or 9. From the multiplication by 3, the only possibilities are D = 6 and I = 8, or D = 9, and I = 7. These restrictions leave only a few feasible times to try and the only one that works is: 18:49 × 3 = 56:27.

Also solved by Avi Ornstein, Dennis White, Frank Carbin, Harry Zaremba, Henry Hirschland, Matthew Fountain, P. Michael Jung, P.V. Heftler, Richard Hess, Steve Feldman, Winslow Hartford, and the proposer, Phelps Meaker.

JUL 5. An ideal pulley system supports a bucket of water on one rope and a monkey on the other. The bucket and monkey are in static equilibrium and at the same vertical level. Suddenly the monkey began climbing up its rope. Describe the motion, if any, of the bucket.

Everyone agrees that the bucket follows the monkey exactly, independent of the mechanical advantage of the pulley system. The argument goes as follows: The monkey's change of state from rest to an upwards velocity implies acceleration which requires force. The reaction force is tension xT in the monkey's rope (where x is the pulley's mechanical advantage with respect to the monkey). The dynamical response of the bucket to the force xT is identical to the monkey's, so that the bucket and the monkey accelerate upwards together. After the monkey's acceleration stops, it continues to climb at some velocity v, and so does the bucket.

Solutions were received from Richard Hess (who attributes the problem to Lewis Carroll), William Moody (who first heard the problem 58 years ago in his 8.01 freshman physics course, taught by Prof. "Hard-Boiled" Lewis Young), Ronald Martin, Winslow Hartford, Harry Zaremba, Dennis White, and the proposer, Bruce Calder.

Better Late Than Never

1980 FEB 3. Warren Himmelberger suggests the solution (33231302212011003)

1983 APR 4. Dick Allphin reports that a similar problem has appeared in *The New York Times*. Furthermore, Mr. Allphin believes that, when travelling in the rain, "for minimum soaking it pays to make haste."

Y1984 Rik Anderson has noticed that $16 = 4^{18/9}$.

1984 APR 2. John Coleman has responded.

1985 F/M 3. John Smith found that x = (sin 60)/(sin 75)a.

F/M 4: Phelps Meaker noticed that this problem was also F/M 4 in 1984. An unintentional yearly problem!

APR 1. Michael Jung has responded.

APR 4. Michael Jung has responded.

APR 5. Frank Carbin has responded.

M/J 4. Jim Landau has responded.

July SD 2. Ben Feinswog wants to play the ♥Q before the ♦K and gives a try for the contract, if West is void in hearts.

Proposers' Solutions to Speed Problems

SD 1. $2(2)^{.5}$.

SD 2. 10:08 (or 10:08:08 if the clock has seconds). 1:11 (or 1:11:11).