PUZZLE CORNER

ALLAN J. GOTTLIEB

Great Climbing Monkeys!

This past week I was quite ill with a flu-like disease that caused me to miss my first three days of work since arriving at NYU more than five years ago. And guess who strolled into the lab to interview our research group while I was out: the CBS Evening News! I guess it just doesn't pay to get sick so often.

I am sorry to say that, for some unexplained reason, *two* of the October solutions given in the February/March issue were unattributed. Somehow between my original manuscript and the final column, credits to Harry Zaremba and David Griesedieck for **OCT 2** and **OCT 4**, respectively, were omitted. I apologize for the error.

Finally, I would like to acknowledge a touching letter from one of the most active contributors to Puzzle Corner who explained, in a warm and personal way, why his activity would have to decrease. This column is dedicated to John Rule.

Problems

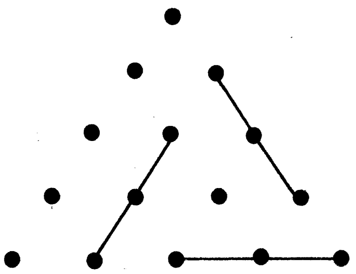
JUL 1. For our computer problem of the month, Matthew Fountain wants you to find the smallest prime number that contains all 10 digits.

JUL 2. Lester Steffens asks us to answer a new question about the widely known Tower of Hanoi problem:

The original tower, first described in 1883, consisted of 64 golden discs, each of a different diameter, stacked according to size, with the smallest on top. The tower is to be restacked on one of two additional sites, moving one disc at a time off the top of one stack either to an empty site or to the top of the stack on one of the other sites, without ever placing a larger disc on a smaller. The original problem was to find the minimum number of moves to transfer the entire stack. The new problem is to calculate the location of each disc after 1,001 moves have been made using the optimum transfer procedure.

JUL 3. Charles Bostick has some points that need to be covered:

Define an n-triangle to be a collection of $n(n + 1)/2$ points regularly spaced into the shape of an equilateral triangle with n points on a side. Define a 3-line to be a line segment connecting exactly three adjacent points parallel to a side of an n-triangle. (The three adjacent points are said to be "covered" by the 3-line). For what values of n can all points of an n-triangle be covered by non-intersecting 3-lines?

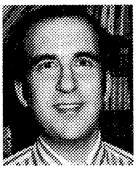


JUL 4. Here is one from a batch John Rule sent me in 1974 and I have been periodically milking ever since:

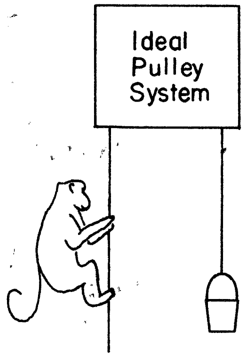
A manufacturer makes all possible sizes of brick-shaped blocks such that the lengths of the edges are integral multiples of the unit of length, and that the number of units in the total length of the twelve edges of the block is equal to two-thirds of the number of units of volume in the block. What sizes does he make?

JUL 5. Here is some monkey business from Bruce Calder:

An ideal pulley system supports a bucket of water on one rope and a monkey on the other. The bucket and monkey are in static equilibrium and at the same vertical level. Suddenly the monkey began climbing up its rope. Describe the motion, if any, of the bucket.



SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, ASSOCIATE RESEARCH PROFESSOR AT THE COURANT INSTITUTE OF MATHEMATICAL SCIENCES, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y., 10012.



Speed Department

SD 1. Phelps Meaker is building a straight sidewalk 54'8" long, using 40 cast-concrete slabs each formed as an equilateral triangle. To square off the ends, he has two extra 30-60-90 half-slabs. What is the width of the walk?

SD 2. A bridge quickie from Doug Van Patter:

Your hand:	*Dummy:*
♠ A K 10	♠ J 5 3
♥ A Q J 10 7 2	♥ 9 6 5
♦ K	♦ A J 7 4
♣ A J 10	♣ Q 6 2

Your contract is six hearts. West leads a low club. You put in the ♣Q, which is covered by the ♣K and ♣A. Can you find a line of play that just about guarantees success?

Solutions

FEB/MAR 1. Management meetings are scheduled on the second Thursday of each month, administrative conferences are the third Friday, and work units have a seminar on the first Monday. Derive an alogrithm which will generate a date given the year, month, day-of-week, and ordinal week within the month. For example, if a meeting were scheduled for the third Friday of August 1984, the algorithm would return "August 17." Note that a meeting on the fifth Tuesday in March would be fine for 1983, 1985, and 1986 but not for 1984 (there are only four Tuesdays in March of 1984). In this case the algorithm could return January 0.

The following solution is from Al Weiss, who also notes that there are only four Tuesdays in March in 1985 and 1986:

Programmers, like all good craftsmen, have their own bags of tools. One tool that has been in my bag for a long time is a routine that will convert a date into the number of days since November 24, −4713 (this is not the same as 4713 B.C., since there was no year zero). The number generated by this routine is called the Julian day. It is similar to a "shop date," which is the number of days since the beginning of the year. This number itself is not too useful, but it can be used to calculate the number of days between two dates. There is also a companion routine which converts the number back into a date. These two routines can be used of find the date 90 days from today. Using these two routines it is possible to solve Alfred Anderson's problem. My procedure is as follows:

1. First we determine the Julian Day of the 1st of the month desired (assuming we are looking for the third Friday in July 1985, this would return the number 2446248).
2. We then determine what day of the week this is (in this case the program returns a 1 telling us that this is a Monday). Next we find the date of the first Friday (the computer tells us that this is the 5th).
3. Then we calculate the date of the 3rd Friday (in this case the 19th).
4. Finally the program validates the answer by calculating the Julian day of this date and then reconverting the Julian day back to a real date. If the month of this new date is the same as the original month, we have a solution. As an example of an impossible date: if we were looking for the 5th Friday in July, the computer would calculate it to be the 33nd of July. When this is converted to a Julian day and then back to a real date it comes back as August 2nd. The computer recognizes that August is not the month requested and sets the date to zero.

Mr. Weiss submitted the program described above, but inadequate space prevents us from reprinting the program and we apologize. A copy can be had by return mail from the *Review*.

Also solved by Frank Carbin, Harry Zaremba, Jim Landau, John Patterson, Matthew Fountain, Robert Slater, Winslow Hartford, and James Abbott (who also included a TI-59 program card containing his solution.)

FEB/MAR 2. Two coins, loosely coupled, are flipped simultaneously such that if either one is heads, the other has probability 7/8 of also being heads, but if either one is tails, the other is equally likely to be either heads or tails. Find the probability of each individual coin turning up heads, and the probability of their both being heads simultaneously (or prove that the problem statement and data are inconsistent).

Michael Tamada found a couple of solution techniques for the loosely coupled coins:
Each coin has probability of .8 of being heads. The probability that both are heads is .7:

		Coin 2: H	T	
Coin 1	H	.7	.1	.8
	T	.1	.1	.2
		.8	.2	

I have two ways of deriving the answer, one using a "contingency table" approach and one using a "conditional probability" approach.

Contingency Table Approach:
We wish to find the unknown probabilities a, b, c, and d:

		Coin 2: H	T
Coin 1	H	a	b
	T	c	d

We know that a + b + c + d = 1. If Coin 1 is tails (i.e., if we're in the second row, which contains c and d), we are told that Coin 2 has equal probabilities of being heads or tails. In other words, c/(c + d) = 1/2. Similarly, if Coin 2 is tails, we are told that b/(b + d) = 1/2. These two equations tell us that b = c = d, so we know that a + 3b = 1. We are told that if Coin 1 is heads, then Coin 2 has a probability of 7/8 of being heads. In other words, a/(a + b) = 7/8. Substituting a = 1 − 3b in the above equation, we get (1 − 3b)/(1 − 3b + b) = 7/8, or b = .1. Since b = c = d, we know c = d = .1 and thus a = .7.

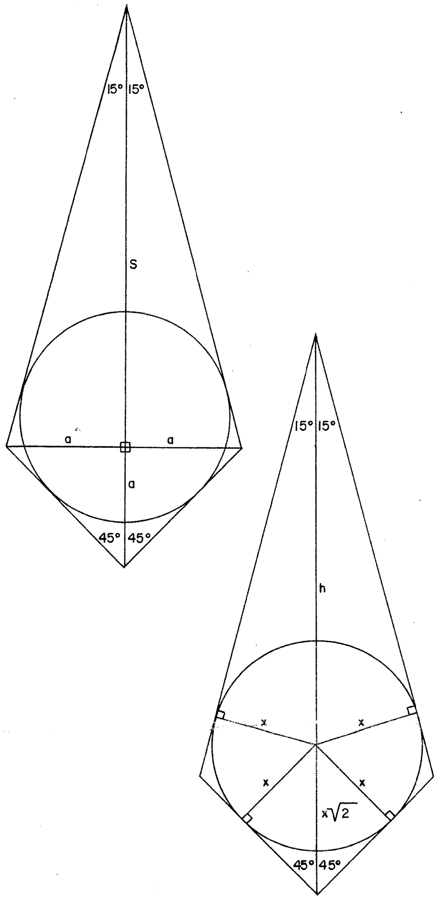
Conditional Probability Approach:
Let "H1" and "T1" stand for the probability that Coin 1 is heads or tails respectively. Obviously H1 = 1 − T1 (and H2 = 1 − T2 for Coin 2). Let p(H1:H2) stand for the probability of event H1 given that H2 occurs. We are told that p(H1:H2) = 7/8 and p(H2:H1) = 7/8. Similarly, we are told that p(H1:T2) = 1/2 and p(H2:T1) = 1/2. The unconditional probability of any event is equal to the sum of its conditional probabilities (weighted by the

probabilities of the condition occuring). I.e.,
p(H1) = p(H1:H2) × p(H2) + p(H1:T2) × p(T2). So
p(H1) = (7/8)p(H2) + (1/2)[1 − p(H2)]
p(H1) = 1/2 + (3/8)p(H2). (1)
Similarly, for Coin 2 we get
p(H2) = p(H2:H1) × p(H1) + p(H2:T1) × p(T1)
p(H2) = (7/8)p(H) + (1/2)[1 − p(H1)]
p(H2) = 1/2 + (3/8)p(H1). (2)
Combining equations (1) and (2),
p(H1) = 1/2 + (3/8)[1/2 + (3/8)p(H1)] = 4/5.
We also find that p(H2) = 4/5. Again using laws of conditional probability, we know that
p(H1 & H2) = p(H1:H2) × p(H2) = 7/8 × 4/5 = .7

Also solved by David DeLeeuw, Leon Tabak, Matthew Fountain, Michael Jung, Richard Hess, Winslow Hartford, and the proposer, William Stein.

FEB/MAR 3. A horizontal line of length 2a forms the common base for two isosceles triangles. The near side the triangle is 45° −45° −90°, and on the opposite side 75° −75° −30°. Determine the radius of the circle tangent to all sides of the composite lanceolate figure, and locate the center.

Avi Ornstein makes it look easy:



Let x be the radius of the inscribed circle, let h be the line segment from the circle's center to the vertex of the 75° −75° −30° triangle, and let s be the length of the bisector of this triangle. From the diagram, we see:
$s = a/\tan 15°$ and $h - x/\sin 15°$
$a + s = x2^{1/2} + h$.
Thus we have
$a + a/\tan 15° = x2^{1/2} + x/\sin 15°$
$x = a(1 + 1/\tan 15°)/(2^{1/2} + 1/\sin 15°)$
$x = 0.896575472a$
$x2^{1/2} = 1.267949192a$

Also solved by David DeLeeuw, Everett Leroy, George Parks, Harry Zaremba, Mary Lindenberg, Matthew Fountain, Mel Garelick, Naomi Markovitz, Richard Hess, Steve Feldman, Winslow Hartford, and the proposor, Phelps Meaker.

FEB/MAR 4. Find a four-digit number whose square is an eight-digit number whose middle four digits are zero.

Most solutions were brute-force computer searches. Pierre Heflter reduced the search vastly by employing some pre-analysis:
The answer is 6,245, whose square is 39,*000*,025. Trivial answers of 4,000, 5,000, 6,000, 7,000, 8,000 and 9,000 should have been excluded in the statement of the problem. Since the square must lie between 10,*000*,001 and 99,*000*,099, the number itself must lie between 3,163 and 10,000. Absent any way to predict the occurrence of zeros in the middle of a square, one could square each number between 3,163 and 10,000 (6,144 numbers in all if endings in zero are omitted) and hope to find a square with four zeros in the middle. A tedious search without a computer, almost two hours on my HP97. For a more efficient search, consider the following: the square root of 10,*000*,099 is 3,162.293; the square root of 10,*000*,000 is 3,162.277. The difference is 0.0156. In the same test for numbers 99,*000*,099 and 99,*000*,000, the difference in square roots is 0.00498. It follows that if X is a number ranging up from 10,*000*,000 and if $\sqrt{X}$ comes out with a fraction which is more than 0.0156, there is no number between X and X − 99 which is a perfect square. So, using 0.0156 as a discriminant, take the square root of 90 numbers of the form ab,*000*,099, where ab ranges from 10 to 99 (easy on any pocket calculator), discard any square root if its fractional part exceeds 0.0156, and round out the rest to the next lowest whole number. Then discard any left that end in zero. Three remain that are worthy of being tested. One of them, 6,245, satisfies the problem. The other two do not because the discriminant did not decrease to 0.00498 (as it should have to be a necessary *and sufficient* test) as ab ranged up to 99. This search took just over two minutes on my HP97.

Also solved by Jerry Cogan, Frank Carbin, Chester Claff, Avi Ornstein, Dennis Sandow, George Byrd, George Parks, John Prussing, Lee Fox, Matthew Fountain, Michael Jung, Michael Tamada, Naomi Markovitz, Nicholas Strauss, Richard Hess, Robert Slater, Robert Turner, Ronald Raines, Thomas Stowe, and Winslow Hartford.

FEB/MAR 5. Consider two dipoles. The lower dipole is fixed, and the upper dipole is constrained to move along a horizontal line. (This is roughly the geometry encountered in magnetic stirring.) Find the conditions for which the upper dipole tends to center (the force is in the opposite direction to the displacement from the center line). When does the motion of the upper dipole approximate simple harmonic motion?

Only Matthew Fountain and Richard Hess tackled this hard problem. Mr. Fountain's impressive solution would have been published except for a confrontation between its length and the available space in this issue. Readers may obtain a copy (together with our apologies, which go also to reader Fountain) by return mail on request to the *Review* office.

Better Late Than Never

JAN 4. Pierre Heftler has responded.

Proposers' Solutions to Speed Problems

SD 1. 27.733

SD 2. First cash the ♦K, then lead the ♥Q. If this holds (it did) and both defenders follow suit, play the ♥J. Presumably, a defender will take the ♥K. Now the ♥9 provides an entry to dummy, and you can pitch the ♠10 on the ♦A. If East shows out on the first trump trick, play a small trump toward the ♥9, with the same result. This hand occurred at the Talleyville Club in Wilmington, Del., which boasts of the highest percentage of life masters in the U.S. West actually held three hearts to the king. Several declarers made 12 tricks.