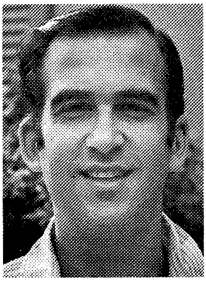
No Way of Knowing the Time of Day?

Puzzle Corner/Allan J. Gottlieb



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Let me begin with thanks to everyone who sent us best wishes for the holidays. David especially asked me to thank Phelps Meaker for the lovely card.

It has been two years since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses arrive during the month, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column, I first weed out erroneous and illegible solutions. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor contributions from correspondents whose solutions have not previously appeared, as well as solutions that are neatly written or typed, since the latter produce fewer typesetting errors.

Problems

APR1 We begin this month with a bridge problem from Doug Van Patter:

```
            ♠ Q 10 3
            ♥ Q 7
            ♦ A Q J 8 6 4
            ♣ J 9
♠ 9 8 7                 ♠ J 6 5 2
♥ 8 3                   ♥ J 10 7 2
♦ 10 9 7 5 2            ♦ –
♣ A 10 6                ♣ Q 8 5 4 2
            ♠ A K 4
            ♥ A K 9 5 4
            ♦ K 3
            ♣ K 7 3
```

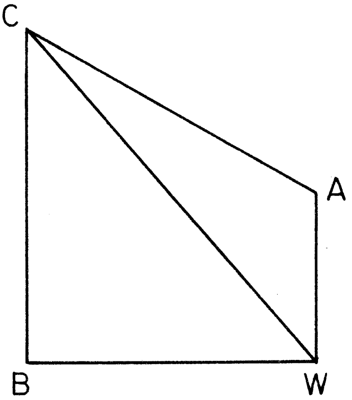
You are West, defending against a six no-trump contract. Suspecting that you may have a diamond stopper, you lead the ♣ A. Do you still expect to set this contract? If so, how.

APR 2 Nob Yoshigahara has sent us an unusual cryptarithmetic puzzle, a multiplication problem involving time:

```
     x  xx
        xx
   ---------
x  xx  xx
```

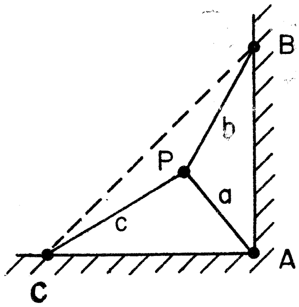
Each of the 10 digits is to be used once. The first number represents minutes and seconds; the third, hours, minutes, and seconds; and the leftmost digit of each number is nonzero.

APR 3 A. Singer wants to know how fast you can commute:



A commuter lives at C and works at W. He normally drives to work via road CW. Occasionally, to break the monotony of the daily commute, he drives to Town A and from there to W. On still other occasions he drives to Town B and from there to W. Towns A and B are each exactly 10 miles from W and roads BW and AW are at right angles. Our commuter always travels at the same speed, regardless of route. His normal trip takes exactly 30 minutes, his trip through Town A takes him 35 minutes, and his trip through Town B takes him 40 minutes. At what rate of speed does he travel?

APR 4 Harry Zaremba's rods are loose; how large a triangle can they determine? In the figure (*next page*), rigid rods of lengths a, b, and c are hinged at point P, and rod a is hinged to the intersection of the vertical and horizontal surfaces. If the free ends of the rods b and c are



always maintained in contact at B and C and permitted to slide along the surfaces, what is the maximum area of the right triangle CAB that can be formed by the rod extremities? (For uniformity and simplicity, let the constant $K^2 = b^2 + c^2 - a^2$.)

APR 5 The M.I.T. undergraduate math club plays with an infinite chessboard but no chess pieces. The squares of an infinite chessboard are numbered by putting zero in the corner and in each other square putting the smallest nonnegative integer that does not appear to its left in the same row or below it in the same column. Find a nonrecursive formula for the number placed in row i column j.

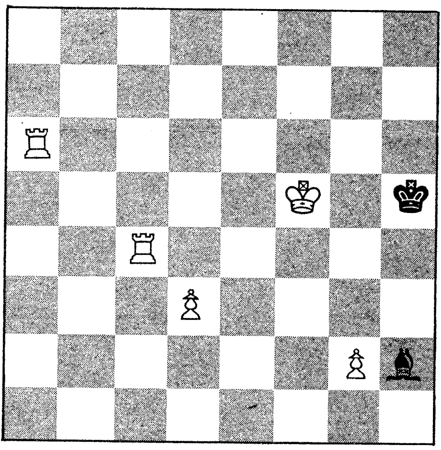
Speed Department

SD1 Phelps Meaker wants to know a simpler way to write the sequence 1, 8.825, 31.544, 77.88, 156.99, 278.38, . . . As a hint he reminds you that the sixth term is greater than 216.

SD2 Joan Baum found the following query in the *New York Times*: If it is 10 a.m. in New York and 5 p.m. in Moscow, what time is it at the South Pole? At the North Pole?

Solutions

N/D1 White is to play and mate in three:



Elliott Roberts solved this for us:

1. R—a7	K—h6
2. R—h4	mate

or

1. R—a7	B—f4
2. R x B	K—h6
3. R—h4	mate

Or

1. R—a7 B—c7
2. R(a7) x B K—h6
3. R—h4 mate

Also solved by Kenneth Bernstein, Avi Ornstein, Robert Bart, Walter Nissen, Matthew Fountain, Winslow Hartford, David Krohn, Norman Wickstrand, Miguel Colina, Ronald Raines, and the proposer, George Farnell.

N/D2 Cancelling d's to give
dy/dx = y/x
is known as "freshman's folly." But similar jokes occasionally work for arithmetic:
64/16 = 4/1
326/163 = 2/1
Can anyone find a four-digit counterpart?

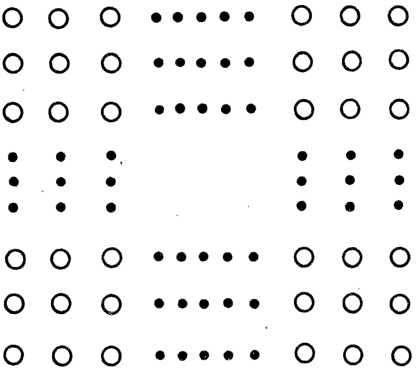
Philip Klenn did so, with acknowledged assistance from Radio Shack:

I was challenged by your "freshman's folly" problem and programmed my Radio Shack home computer to solve it by trial and error. I was surprised to see the large number of possible solutions to the problem, 60 by my count. Naturally, I ignored any four-digit numbers containing a zero, since even a freshman knows that cancelling zeroes is indeterminate. Incidentally, my computer was particularly slow—it took a run of 14 hours to check all possibilities.

3926/1963 = 2/1	9891/2198 = 9/2	8324/6243 = 8/6
6392/3196 = 2/1	2164/1623 = 4/3	8432/6324 = 8/6
6938/3469 = 8/4	2168/1626 = 8/6	8464/6348 = 4/3
9386/4693 = 8/4	3244/2433 = 4/3	8644/6483 = 4/3
9998/4999 = 8/4	3248/2436 = 8/6	8648/6486 = 8/6
8565/2855 = 6/2	3284/2463 = 8/6	8756/6567 = 8/6
6664/1666 = 4/1	4216/3162 = 4/3	8864/6648 = 8/6
6748/1687 = 4/1	4324/3243 = 4/3	9188/6891 = 8/6
7348/1837 = 4/1	4328/3246 = 8/6	3157/1353 = 7/3
7468/1867 = 4/1	4432/3324 = 4/3	9584/3594 = 8/3
7948/1987 = 4/1	4648/3486 = 4/3	2317/1324 = 7/4
9995/1999 = 5/1	4756/3567 = 4/3	4627/2644 = 7/4
9564/1594 = 6/1	4832/3624 = 8/6	6937/3964 = 7/4
8176/1168 = 7/1	4864/3648 = 4/3	7231/4132 = 7/4
9373/1339 = 7/1	6448/4836 = 4/3	7462/4264 = 7/4
9793/1399 = 7/1	6484/4863 = 4/3	7693/4396 = 7/4
1761/1174 = 6/4	6488/4866 = 8/6	5946/4955 = 6/5
6665/2666 = 5/2	7564/5673 = 4/3	6824/4265 = 8/5
9772/2792 = 7/2	7568/5676 = 8/6	2925/2275 = 9/7
8919/1982 = 9/2	8216/6162 = 8/6	9162/7126 = 9/7

Also solved by Dennis Sandow, Harry Zaremba, Winslow Hartford, Matthew Fountain, Walter Nissen, Jr., Robert Bart, and Kenneth Bernstein.

N/D 3 When the conversation grew dull at a party, I discovered that equal numbers of red and black checkers could be arranged in a rectangular tray, with all the black ones on the border, like this.



But when my back was turned, one of the guests ate a few of each color, thinking my checkers to be cookies. I found that I still had an equal number of black and red checkers, and that I could still arrange them in an array with the reds inside and the blacks on the border. How many checkers did I start with, and how many were eaten?

Unfortunately, the diagram printed (unlike the proposer's original) inadvertently suggested that the black border was three checkers wide. Under this interpretation Miguel Colina writes:
The rectangular array that contains the red checkers is of size M × N, and the number of black checkers that surround the red ones is 36 + 6M = 6N. Since there is an equal number of red and black checkers, we have that M × N = 36 + 6M + 6N. Solving for M, we obtain M = (36 + 6N)/(N − 6). The constraints are that both M and N must be integers, and that N must be greater than 6.

To solve this problem, I used my personal computer. A simple program that had a looping variable N was used to find integer values for M. The following data was found:

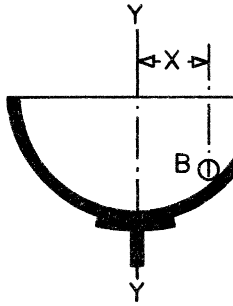
N	*M*	*M x N*
7	78	546
8	42	336
9	30	270
10	24	240
12	18	216
14	15	210

Of the 15 possible combinations of starting and ending numbers of checkers, I chose the one with the smallest difference (starting with 216 and ending with 210 of each color) because of the hint that only a few were eaten. Therefore, we must have started with a total of 432 checkers, and 12 checkers were eaten.

Walter Nissen, Jr. was able to deduce that the indented border is just one checker wide. His solution is 60 at the outset, 12 eaten:
Assuming that the border is only 1 checker wide (I had a hard time deducing this from the diagram which is square and does not distinguish red and black), there was a 5 × 12 array with a 3 × 10 center, and after 12 checkers were eaten there was a 6 × 8 array with a 4 × 6 center. If the border is two or more checkers wide, various gluttonous and even more indigestible solutions emerge. Where the width of the border is x and the dimensions of the red rectangle are h and w, the number of reds must equal the number of blacks in the border, hw = 2wx + 2hx + 4xx. (If Euler could use this notation so can I, especially as superscripts are a pain.) Solving for w,w = (2 hx + 4xx)/(h − 2x). For positive, integral h, find positive, integral w. If x = 1, the only solutions are h = 3,4.

Also solved by Kenneth Bernstein, Avi Ornstein, Matthew Fountain, Winslow Hartford, Harry Zaremba, Norman Wickstrand, Mary Lindenburg, and Gene Henschel.

N/D 4



A hemispherical bowl with a radius of 1 foot 1 inch is mounted on a central vertical shaft YY. A one-pound ball B with a radius of 1 inch is free to roll inside the bowl. On what part of the bowl's surface will the ball tend to ride (i.e., what will be the value of x) if the bowl is spun at 50 R.P.M. about the YY axis? How do you explain this peculiar result? Show that x will increase to about 7 inches at a speed of 60 R.P.M.

Kenneth Bernstein found the ball by looking at the bottom of the bowl!
In the frame of the spinning bowl, the ball experiences a force of 32 lb. vertically downwards and ($w^2 \sin\theta$) horizontally away from the central shaft YY (θ is the angle between YY and the radius of the bowl drawn to the center of the ball). The resultant force must be directed away from the center of the bowl to ensure that, at equilibrium, there is no net force acting to change x. Thus:
$\tan\theta = (w^2 \sin\theta)/32$

One solution is always $\theta = 0$. The other solution, when possible, is

$\cos \theta = 32/w^2$

If $w < \sqrt{32}$, then the second solution is extraneous. In the first part of this problem

$w = 2\pi\, 50/60 \text{ rad/sec} < \sqrt{32}$.

In this case only the solution $\theta = 0$ obtains. The ball remains at the bottom of the bowl and either spins, if there is friction, or remains motionless in the absence of friction. In the second part, $w = 2\pi > \sqrt{32}$ so that the second solution leads to $x = 7.028$ inches. In both parts there is an equilibrium at $x = 0$; however, for the second part this equilibrium is unstable while for the first part it is stable.

Daniel Whitney noted a similar problem that caused great consternation in 8.01 (M.I.T. freshman physics). In this example we spin a cylinder containing water.

Also solved by Thomas Harriman, Bruce Calder, John Prussing, Harry Zaremba, Winslow Hartford, Matthew Fountain, Walter Nissen, Jr., Robert Bart, Norman Wickstrand, and Kenneth Bernstein.

N/D 5 Find two five-digit perfect squares that together contain all ten digits. How many solutions exist?

Sidney Shapiro found eight solutions:

	$N(1)$	$N(2)$	$N(1)^2$	$N(2)^2$
(1)	126	153	15876	23409
(2)	126	198	15876	39204
(3)	144	228	20736	51984
(4)	144	309	20736	96481
(5)	174	228	30276	51384
(6)	174	309	30276	95481
(7)	195	219	38025	47961
(8)	252	267	63504	71289

All the Ns are divisible by 3.

Also solved by David Krohn, Avi Ornstein, Robert Bart, Walter Nissen, Jr., Matthew Fountain, Winslow Hartford, Harry Zaremba, Thomas Harriman, Frank Carbin, Phelps Meaker, and the proposer, John Rule.

Better Late Than Never

1983 M/J 4 Frank Rubin points out that the twin prime conjecture is now a theorem and the argument given for **M/J 4** must be augmented to give special treatment to low values.

M/J 5 Frank Rubin has a simpler solution.

JUL 1 Sidney Williams has offered a simpler solution.

JUL 4 Mark Nagurka and Anthony Standen have responded.

JUL 5 Laurie Fabens reports that the next two number elements are

$3114^2 = 9696996$

$81619^2 = 6661661161$

OCT 2 Naomi Lieberman has responded.

N/D SD1 Michael Strieby found a South holding that contra-indicates the solution given

Proposers' Solutions to Speed Problems

SD1 1X,2X,3X,5X,6X.

SD 2 When it is 10 a.m. in New York it is 3 a.m. the next day at the South Pole. That is because the only people at the South Pole with any interest in time are the personnel at the United States polar station. They use New Zealand time, which is Eastern standard time plus 17 hours, because the polar station is supported from the American base at McMurdo Sound, which in turn is supported from Christchurch, New Zealand, and it is convenient for the time to be the same at all three. The North Pole is uninhabited, so there is no one to care what time it is; of course, strictly speaking, where all the meridians that govern local time converge, as they do at the poles, there is no way of defining the time of day.