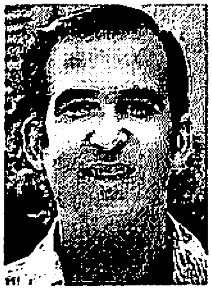
Puzzle Corner
Allan J. Gottlieb

Dropping a Rock in Borneo



Allan J. Gottlieb, '67, is at the Courant Institute of Mathematical Sciences, New York University. He studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y., 10012.

Not very much to report this month. I am very busy helping raise our son David and managing a research group in "parallel processing" (using many cooperating computers to solve large problems much faster than is possible with a single machine). I find both parenthood and computers to be extremely interesting and rewarding; I just wish every day were a little longer so I could spend more time on each.

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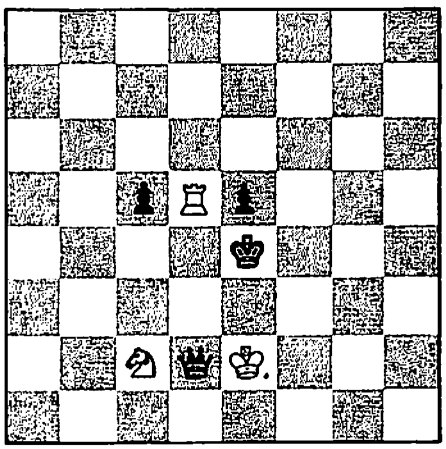

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$$y^6x = 9(x^2 + 1)y^3$$

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North:
♠ K 6
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Your partner (North) opened one club and raised your one-heart bid to four. The opening lead is ♠Q. What play gives you the best chance of making your contract?

Solutions

OCT 1 South is on lead with hearts as trump and is to take all tricks against the best defense:

West	North	East
	♠ A J	
	♥ 5	
	♦ K Q	
	♣ 10 8	
♠ K 2		♠ 4 3
♥ —		♥ 6 3
♦ J 9 8		♦ 7
♣ 9 7		♣ 6 5
	♠ Q	
	♥ 8 4	
	♦ A 10 3 2	
	♣ —	

The following solution is from Matt BenDaniel. There are three considerations:
- ☐ We must pull East's trumps; this can only be done by finessing from the dummy.
- ☐ The bad offensive fit combined with East's diamond singleton limit transportation for the offense.
- ☐ West must discard the protection from his potential winners.

The basic plan, then, is to get to board in spades, finesse East once, and play another round of hearts. This will pull East's trumps and will also cause West to discard. West's discards will determine the declarer's play for the rest of the hand. The following diagram gives the details of play:

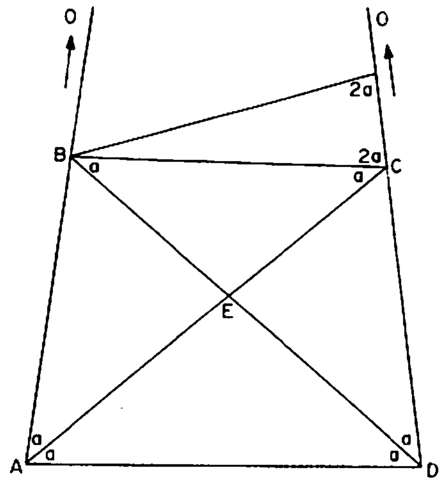
notes that more "in-order" solutions are possible if one permits "negative solutions," e.g. $-63 = 19 - 82$), and David DeWan, who writes, "As a newcomer to this problem, I was astounded by the variety of numbers that can be created with only four digits, five operators, and lots of time. I am once again in awe of the power of permutation. And genes are permutations of just four "digits" also: A, G, T, and C; no wonder there are lady bugs, redwoods, penguins, and porcupines."

A/S 1 Place white bishops on a1 and a3 and place black bishops on e1 and e3. By moving each color alternately, such that a black bishop can never take a white one and conversely, and restricting moves to ranks 1 to 4 and files a to e (i.e., to ten black squares), exchange the position of the bishops so that the black bishops end up on a1 and a3 and the white bishops on e1 and e3.

Michael Jung and Matthew Fountain seem to have convincing evidence that no solution is possible. How about it, Mr. Chen?

A/S 2 An isosceles triangle has a bisector of one of the two equal angles that is 6 inches long. If the base is 5 inches, without using trigonometry, find the length of the two equal sides.

The following solution is from Phelps Meaker:



The diagram represents the base section of a slender isosceles triangle AOD. The base is 5 units, the diagonals AC and BD 6 units; the diagonals bisect the base angles. Fold the triangle ABD about its side BD, so that AD lies along CD, extending 5 units to F, and side AB forms the member BF. Return triangle ABD to its original position, noting that triangles CBF and AOD are similar. Members AB, BC, BF, and CD are equal. Triangles BCD, BEC, and AED are isosceles and similar, providing three ratios as follows:

$$BC/BD = BC/6 \quad (1)$$
$$BE/BC = (BD - ED)/BC = (6 - AE)/BC \quad (2)$$
$$AE/AD = AE/5 \quad (3)$$

Combining (1) and (3):
$BC/6 = AE/5$; $AE = (5 \cdot BC)/6$
Combining (1) and (2):
$BC/6 = (6 - AE)/BC$; $\overline{BC}^2/6BC = (36 - 6AE)/6BC$
Combining the above:
$\overline{BC}^2 = 36 - 6(5 \cdot BC)/6$; $\overline{BC}^2 + 5 \cdot BC - 36 = 0$;
$BC = (-5 \pm \sqrt{25 + 4 \cdot 36})/2 = (-5 + 13)/2 = 4$.
$CD = BC$. Since $FD = 5$ and $CD = 4$, $FC = 1$.
In triangle CBF, the sides are four times as long as the base. Hence the side AO in the similar triangle AOD is four times AD; and the triangle given in the problem has sides of $5 \cdot 4 = 20$.

Also solved by Harry Zaremba, Mary Lindenberg, John Prussing, Everett Leroy, Steven Schmelling, David Evans, Matthew Fountain, Leon Bankorf, and the proposer, Emmet Duffy.

A/S 3 Write the prime numbers and take successive absolute differences:

2 3 5 7 11 13 17 19 23 ...
1 2 2 4 2 4 2 4 ...

Note that the second row starts with a 1. Next repeat the process of taking successive absolute differences, obtaining

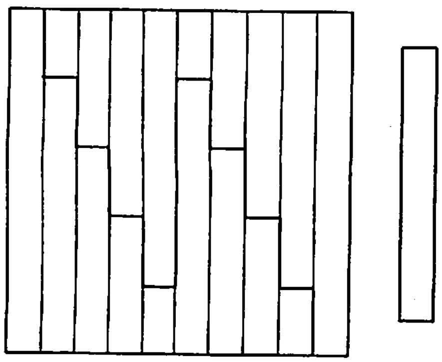
2 3 5 7 11 13 17 19 23 29 ...
1 2 2 4 2 4 2 4 6 ...
1 0 2 2 2 2 2 2 ...
1 2 0 0 0 0 0 ...
1 2 0 0 0 0 ...
1 2 0 0 0 ...

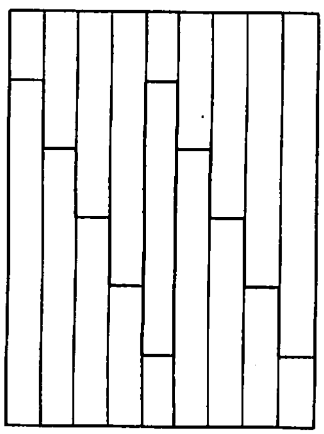
The problem is to prove or disprove that the series of leading 1's continues forever.

Matthew Fountain and Winslow Hartford submitted indications that the sequence of ones continues indefinitely. However, a proof is still wanting. Note that the difference between consecutive primes is not bounded (because the density of primes approaches zero).

A/S 4 The "rug puzzle": you want to put wall-to-wall carpeting into a room that is 9 × 12 feet. You have two pieces of carpet, one 10 × 10 and the other 1 × 8. These do add to the correct square footage, but obviously the 10 × 10 must be cut. The challenge is to devise *one continuous* cut through the 10 × 10 piece such that the two resulting pieces will exactly fit the 9 × 12 area with one gap left over into which the 1 × 8 remnant can fit to complete the job.

The following solution is from Ken Haruta:





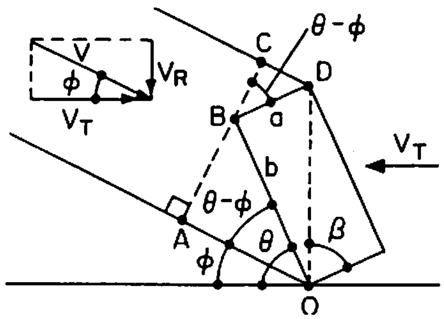
Also solved by Fuhsi Ling, Sidney Williams, David Evans, Matthew Fountain, Avi Ornstein, and the proposer, John Fogarty.

A/S 5 One rainy day Fat Timothy was riding a donkey in the countryside ten miles away from the nearest shelter when he was caught in a strange rain which had a uniform mass density and fell straight down. Worried about being exposed to this strange precipitation, Timothy rode the faithful donkey as fast as he could to the nearest shelter. Assuming that the speed of the donkey was uniform and sufficiently high, find:

1. The amount of rain which fell on Timothy as a function of θ_T, the angle Timothy's body makes with the ground.
2. The minimum and maximum amount of rain which might fall on Timothy.
3. The ideal situation in which Timothy would be wet least.

Approximate Timothy's body as a box.

The following solution is from Harry Zaremba:



In the drawing, assume the dimensions of Timothy's body are a, b, and c where c is perpendicular to the page and a < c < b. Also assume that the donkey's velocity V_t is such that the rain falls on the top and front of Timothy. If V_r is the rain's rate of fall, then the relative velocity of the rain with respect to Timothy is
$V = (V_r^2 + V_t^2)^{\frac{1}{2}}$, and
$\phi = \tan^{-1}(V_r/V_t)$.
The area of rainfall that will be intercepted by Timothy's body is
$A = c(AB + BC) = c[a\cos(\theta - \phi) + b\sin(\theta - \phi)]$,
and the time required to reach the shelter is
$T = D/V_t$,
where D = 10 miles, the distance to the shelter. In time T, the amount of rain that will fall on the rider is
$R = AVT = c[a\cos(\theta - \phi) + b\sin(\theta - \phi)]\sqrt{V_r^2 + V_t^2} \cdot D/V_t$.
Noting that
$\cos\phi = V_t/\sqrt{V_r^2 + V_t^2}$,
the total amount of rain on the rider becomes
$R = [a\cos(\theta - \phi) + b\sin(\theta - \phi)] \cdot Dc/\cos\phi$. (1)
The minimum amount of rain that can fall on Timothy occurs when his topside surface is perpendicular to the apparent path of the rain. In this event, $\theta = \phi$, and the amount of rain from equation (1) becomes
$R_{min} = Dca/\cos\phi$.
For maximum amount of rain, the riding position must be such that the diagonal OD of Timothy's body is perpendicular to the relative velocity V of the rain. Under this condition, the projected area of Timothy's body on a plane perpendicular to the rain's apparent path is
$A = c\sqrt{a^2 + b^2}$.
Thus, the maximum amount of rain is
$R_{max} = \sqrt{a^2 + b^2} \cdot Dc/\cos\phi$.
The ideal situation for getting wet the least is to ride at an angle where $\theta = \phi$ and where the smallest area of the body is exposed to the rain. In the current case, the smallest area equals ac, the topside area of Timothy. When equation (1) is expanded, divided by $\cos\phi$, and simplified, the result is
$R = Dc[a\cos\theta + b\sin\theta + (a\sin\theta - b\cos\theta)\tan\phi]$.
It is noted that the term with factor $\tan\phi$ will be eliminated when the factor $(a\sin\theta - b\cos\theta) = 0$, or $\tan\theta = b/a$. The interesting things about this is that, when the rider's body makes an angle of $\theta = \tan^{-1}b/a$ with the ground, the amount of rain to fall on the rider remains constant, irrespective of the velocity V_t of the donkey. This holds true as long as $V_t \neq 0$. The amount of rain that will fall on the rider is $R = Dc(a\cos\theta + b\sin\theta)$.

Also solved by Matthew Fountain, M. Pope, and Minn Chung, editor of the Physics Department newsletter from which the problem was originally taken.

Better Late Than Never

JAN 2 Henry Fisher has responded.

JUL 3 H. Spacil has sent photos showing that clocks in Asia are displayed at 10:09.

Proposers' Solutions to Speed Problems

SD 1 5. Each term is the number of lit elements when a calculator displays the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, . . .

SD 2 Do not ruff the second spade but discard a club from dummy. Now West cannot force declarer. Take out two rounds of trumps, then knock out the ♦A. The ♥J takes the third trump and provides an entry to dummy's clubs.

Massachusetts Institute of Technology

Report of the President

For the Academic Year 1981-82

The Institute's principal mission is centered on the generation of knowledge, and on its preservation and transmission to new generations of students. During the past year, there has been broad concern about a number of issues which remind us, once again, of the power of knowledge, and of the fact that knowledge is power. The questions of how closely knowledge is held, how freely it is shared, and with whom, have permeated most of the major events and issues of this academic year — linking together several domains which, at first glance, may not appear to have much in common.

These issues include: first, changing patterns in the organization and support of research; second, questions relating to the international transfer of technology; and third, access to higher education. They illustrate the ways in which the generation, dissemination, and control of knowledge influence the intellectual and organizational development of MIT.

The Generation of Knowledge: Changing Patterns of Research Organization and Support

During the past several years, there has been a steady increase in the scale of industrial support of sponsored research in universities. While, on a relative basis, the scale of such support is still small, it is growing rapidly. At MIT, for example, industrial support of such research has grown from $6.7 million in the 1977-78 academic year to $19.7 million in this past year, and now constitutes about a tenth of the sponsored research conducted on the MIT campus.

For some, the growth in industrial research sponsorship raises almost as many questions as opportunities, and has led to debate within the academic community on how best to ensure the transfer of new ideas and technology from the laboratory to the wider society.

The opportunities generated by industry's increasing support of research are manifold. They include the prospect of stable, long-term funding which may complement government support of basic research, which has declined in real terms in recent years. In addition, and importantly, both universities and industry can benefit from closer communication and ties. After all, both rely, for their own evolution and growth, on the talent and new ideas generated by a vigorous system of higher education. Further, new ideas and technologies which are born in an academic setting must be nurtured and developed before they lead to broad practical uses. This development usually occurs in a business setting, and is aided by effective communication between universities and business. Finally, universities can better chart their own future development in certain areas, such as engineering, if they know about the changing directions and needs in the world of business and industry. This does not mean that universities will forsake their traditional independence and reliance on their faculties to pursue shifting frontiers of knowledge. It does mean, however, that our development can, and should, be informed by these perspectives.

At the same time that we recognize the opportunities and mutual advantages of closer ties between academia and industry, we must consider the questions arising from such association.

The sponsorship of research by business and industry has, at times, created considerable anxiety within universities, and has been the subject of considerable public interest and commentary in the media as well. This is not surprising, because such collaboration carries the possibility of conflict between the essential openness and public accountability of the universities, on the one hand, and the private and proprietary interests of industry on the other. In a context in which knowledge becomes not only power but wealth, questions arise about intellectual property rights, about the closing down of communications between research colleagues, and about the effect of varying degrees of openness or secrecy on the progress and integrity of research. For some, industrially sponsored research appears to increase the probability that universities will be compromised in their independence and objectivity, or that their priorities and activities will be distorted by private influence.

These kinds of questions led to a gathering last March of people from five major research universities (the California Institute of Technology, Harvard University, MIT, Stanford University, and the University of California) at Pajaro Dunes, California, to explore the issues generated by commercial sponsorship of research and other forms of interaction between industry and academe. The meeting was

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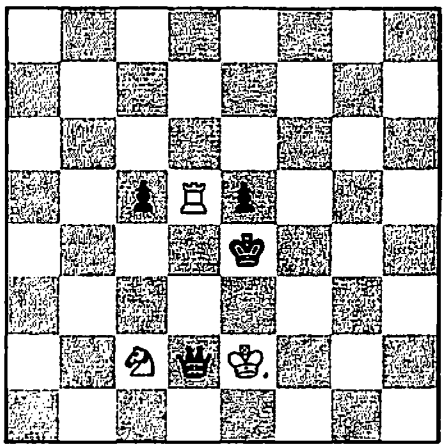

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with implementing M.I.T.'s commitment to research and teaching in health economics and policy. Funds from the Henry J. Kaiser Family Foundation will be available for student support.

Dorothy Eleanor Westney, who arrived at M.I.T. to be assistant professor in international management last fall, is now the first Mitsubishi Career Development Professor. The chair is intended for junior faculty with a strong interest in comparative management who will devote at least a portion of their research and teaching to better understanding Japanese culture and institutions. Dr. Westney, who is fluent in Japanese, holds degrees in sociology from the University of Toronto and Princeton and for four years held a joint appointment in sociology and mangement at Yale.

David L. Bodde, S.M.'73, is currently assistant director, Congressional Budget Office. ... **William M. Nuckols,** S.M.'64, has been named vice-president for business planning and analysis at the Penn Central Corp., New York City. ... **Bernard J. Jourdan,** S.M.'70, writes, "I am back in the states as executive vice-president for the Campagnie Generale des Cieux U.S. Group. I moved to Philadelphia, Penn., from Paris in January 1982. I married in late 1981 and my wife is expecting a baby." ... **Gordon C. Shaw,** DS.M.'60, professor of administrative studies at York University, Canada, has just completed a two-year study to forecast the number of Canadian-registered dry-bulk vessels required to serve Great Lakes cargoes in 1990. The final report has been published by the University of Toronto/York University Joint Program in Transportation.

The following graduates participated in the M.I.T. Alumni Fund upgrading telethon: **Samuel Appleton,** S.M.'57; **Carol Bratley; Peter Condakes,** S.M.'80; **Stephen Hall,** S.M.'62; **Howard Hillman,** S.M.'60; **Walter Lehmann,** S.M.'75; **Howard Miller,** S.M.'63 (coordinator and top caller); **Constance Stubbs,** S.M.'79; and **Thomas Thompson,** S.M.'57.

Sloan Fellows

Donald H. White, S.M.'70, senior vice-president at Hughes Aircraft Co., Culver City, Calif., has been named a director of the company. ... **Hugh E. Witt,** S.M.'57, vice-president, government liaison, United Technologies Corp., has been elected vice-chairman of Aerospace Industries Association of America, Inc., Washington, D.C. ... **Claudia B. Liebesny,** S.M.'80, is currently product manager—fused and sintered specialty products at the Norton Co., Worcester, Mass.

Jere Drummond, S.M.'77, is currently vice-president of Southern Bell Telephone & Telegraph Co., Charoltte, N.C. ... **C. Clement Patton,** '77, is currently vice-president of Southern Bell Telephone & Telegraph Co., Atlanta, Ga.

Senior Executives

Alexander M. Williams, '63, former president—U.S. Division of the Campbell Soup Co., Camden, N.J., is currently president—International Division. ... **Joseph A. Baute,** '64, chairman and chief executive officer of the Markem Corp., has recently been named a director of Houghton Mifflin Co., Boston, Mass.

Julian Hartwell, '67, of Cohasset, Mass., and a long-time executive with New England Telephone Co., passed away on September 25, 1982.

XVI

Aeronautics and Astronautics

What happened to the Navy's fleet defense missile system known as Talos? It was the first ramjet-powered guided missile, first conceived in 1945 and delivered to the Navy in 1955. But it was never used to its full capability, say two analysts—Rear Admiral **Wayne E. Meyer,** '47, and Captain (Ret.) **Richard W. Anderson,** S.M.'60—in the spring of 1982 issue of John Hopkins Physics Laboratory's *Technical Digest.* Talos' electronics became obsolete before their time, maintenance was high, and detection systems were inadequate to locate targets at a distance large enough for Talos to be fully effective, write authors Meyer and Anderson. Among the reasons is that Talos never had an at-sea development site except on operational ships of the fleet; White Sands Missile Range was the only test facility that could handle Talos.

Professor Emeritus **C. Stark Draper,** '26, received the Medal of the City of Paris as guest of honor at the 33rd Congress of the International Astronautics Federation in Paris late last summer. During the Congress "Doc" Draper resigned his post as president of the IAF's Academy of Astronautics after 19 years of service. Preceding his engagements in Paris, "Doc" had attended the UNISPACE '82 Conference in Vienna and the History of Astronautics and Rocketry Conference in Moscow, the latter to celebrate the 25th anniversary of Cosmonaut Yuri Gagarin's first manned spaceflight. "Doc" was joined in Paris by **Janet B. Jones-Oliveria,** a graduate student in the department at M.I.T., who received the IAF's Edmond A. Brun Medal for her paper on the design of an environmental research facility for low earth orbit.

Sir **William R. Hawthorne,** Sc.D.'39, master of Churchill College, Oxford, who is senior lecturer at M.I.T., delivered the Calvin W. Rice Lecture at the 1982 winter meeting of the American Society of Mechanical Engineers and at the same meeting was designated an honorary member of ASME. The Rice Lecture topic: "World Energy versus the Environment—Conflict or Compromise."

James A. Martin, S.M.'69, has recently completed (February 1982) his D.Sc. degree from George Washington University. ... **Leslie M. (Bud) Boring,** S.M.'64, reports, "Not too much in the way of news this past year, although the construction and engineering contractor banking business in France has gone well, the change in government notwithstanding, and our two children (girl 2 and boy 6) continue to grow marvelously. On the avocation front, have made good progress with my painting, which I started at evening classes at the Musuem of Fine Arts while still at M.I.T. (now beginning to be decades ago!). I was accepted for exhibition at the Grand Palais in Paris for both the Salon d'Automore and Salon des Artistes Francais and even won a medal in our hometown exhibition, the Prix de la Ville de Croissy-sur-Seine! Have been contacted by several galleries, although their initial demonstration of enthusiasm drops to zero when they learn I'm a banker and do not paint full time. It seems volume of productoin is what ultimately counts. Anyway, it's a lot of fun and doubly interesting as we live within 20 minutes' walk of several scenes painted by the Impressionists."

James Harrill, S.M.'64; **Gaylord MacCartney,** S.M.'53; and **Robert Stern,** Sc.D.'63, participated in the M.I.T. Alumni Fund fall upgrading telethon.

William B. Abbott III, S.M.'61, reports that he has retired from the U.S. Navy after 30 years of service. For the past 19 years he served in the Strategic Systems Project Office where he was Navy manager of the Polaris, Poseidon, and Trident submarine-launched ballistic missile programs.

XVII

Political Science

A new curriculum leading to a master's degree in political science and public policy has been approved by the faculty, and the first students will enter next fall. The program differs from the regular master's program in political science in its emphasis on policy studies in one of four fields: defense and arms control; science, technology, and public policy; communications; and international development. Mid-career professionals as well as recent bachelor's graduates are welcome as applicants; they will finish the program in one to two years, depending on preparation, and will be qualified for further studies toward the Ph.D. and for positions in government, business, and nonprofit institutions. Professor **Donald L. M. Blackmer** assured the faculty that the department is "committed to the training of policy analysts with a broad outlook. Technical, analytical skills are important," he said, "but we believe that policy analysts must also have a larger understanding of what they are doing—of long- as well as short-term consequences of public intervention, of unintended as well as intended consequences, and of wider as well as narrower effects of policies."

XVIII

Mathematics

Professor **Daniel M. Kan,** a member of the M.I.T. faculty since 1959, has been honored by election to the Royal Netherlands Academy of Arts and Sciences; a native of Holland, Professor Kan's bachelor's and master's degrees are from the University of Amsterdam.

Jerry Grossman, Ph.D.'74, reports that he is currently associate professor of mathematical sciences at Oakland University, Rochester, Mich., and assistant department chairman. He was married to Suzanne Zeitman on August 15,, 1982. ... **Margaret Freeman,** S.M.'34, was a caller in the M.I.T. Alumni Fund fall upgrading telethon.

XIX

Meteorology

George F. Collins, S.M.'48, has joined the Environmental Resource Planning Division of Charles T. Main, Inc., Boston, Mass., as head of air sciences. ... **Ryland Y. Bailey,** '52, is currently senior engineer of the State Corporation Commission of Virginia and a member of the Methodist Church, Richmond.

XXI

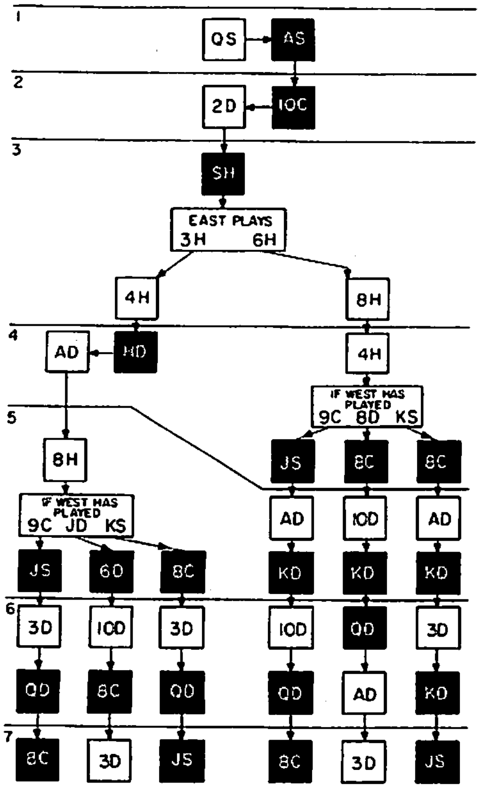
Humanities

Carl Kaysen, David W. Skinner Professor of Political Economy who is director of the Program in Science, Technology, and Society, is a member of a committee of the American Academy of Arts and Sciences to oversee the academy's new association with the International Institute for Applied Systems Ánalysis, near Vienna. American support for IIASA had previously been organized through the National Academy of Sciences in Washington, but political pressure stemming from White House concern about security and possible technology transfer to the East caused NAS to withdraw.

Science, Technology, and Human Values, a quarterly edited by **Marcel La Follette** in the Program in Science, Technology, and Society and cosponsored by Harvard's Kennedy School of Government, is now being published by John Wiley and Sons, Inc., New York; it has previously been in the periodicals group of the M.I.T. Press.

Technology and Policy Program

Barbara Herrmann, S.M.'81, is currently vice-president of Consulting Resources Corp., a management consulting firm which specializes in serving chemical process industries. Her firm has recently been recognized nationally for its work in commodity and specialty chemical markets. ... **Tariq Mahmood,** S.M.'80, has accepted a new position with the Power Rates Section of the Department of Public Service, Albany, N.Y.—Professor Richard de Neufville, '60, Chairman, Room 1-138, M.I.T., Cambridge, MA 02139.



Also solved by Gian Holderness, John Woolston, Matthew Fountain, Doug Van Patter, Matt Daniel, Winslow Hartford, John Boynton, and the proposer, Emmet Duffy.

OCT 2 Solve the set of four cryptarithmetic puzzles, entitled "Seven and Twelve," created by Nobuyuki Yoshigahara.

Only Harry Hazzard was able to solve this one and even he could not find a solution to the third puzzle.

1:

13677	13788	17566	37166
47870	48280	36460	96460
47870	48280	36460	96460
47870	48280	36460	96460
157287	158628	126946	326546

2:

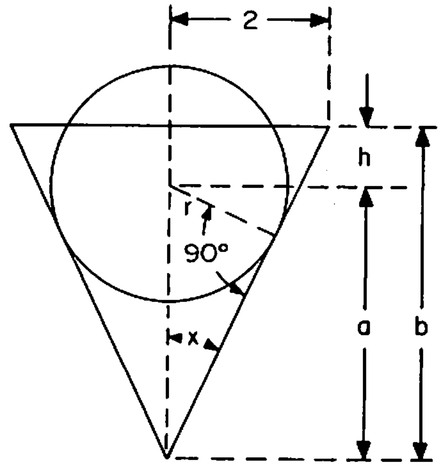
107	107	129	567	567
47570	87570	69892	27476	37476
47570	87570	69892	27476	37476
47570	87570	69892	27476	37476
47570	87570	69892	27476	37476
47570	87570	69892	27476	37476
237957	437957	349589	137947	187947

4:

8162
8162
42623
42623
42623
42623
202623
202623
592062

OCT 3 Given an ice cream cone filled with water, how large a sphere displaces the most water? Let the half angle of the cone be x, and let the radius of the base be 2.

Emmet Duffy sent us a fine solution of his own as well as a reference to a similar problem in the classic 1904 calculus book of Granville et al. Leo Harten used the well known symbolic algebra package MACSYMA and obtained an "imaginary" sphere (see the end of the following solution). Mr. Duffy writes:



Assume the sphere is tangent to the side of the cone. Let b = depth of cone and a = distance from center of sphere to bottom. Then $2/b = \tan x$ and $b = 2/\tan x$, or $b = (2\cos x)/\sin x$; $r/a = \sin x$ and $a = r/\sin x$. Then

$h = b - a = (2\cos x - r)/\sin x.$

The volume, v, of displaced water is given by:

$v = \pi\,(2r^3/3 + r^2h - h^3/3)$, where h is not greater than $+r$ nor less than $-r$.

$v = \pi\,(2r^3/3 + 2r^2\cos x/\sin x) - r^3/\sin x - (2\cos x - r)^3/3\sin^3.$

$dv/dr = \pi\,[2r^2 + 4r\cos x/\sin x - 3r^2/\sin x - (2\cos x - r)^2(-1)/\sin^3 x]$

$dv/dr = \pi\,[2r^2 + 4r\cos x/\sin x - 3r^2/\sin x + (4\cos^2 x - 4r\cos x + r^2)/\sin^3 x]$

Set dv/dr to zero, clear fractions, and divide by π. Then:

$2r^2\sin^3 x + 4r\cos x\sin^2 x - 3r^2\sin^2 x + 4\cos^2 x - 4r\cos x + r^2 = 0$

Collecting terms:

$r^2(2\sin^3 x - 3\sin^2 x + 1) + r(4\cos x\sin^2 x - 4\cos x) + 4\cos^2 x = 0.$

$r^2(2\sin^3 x - 3\sin^2 x + 1) + 4r\cos x(\sin^2 x - 1) + 4(1 - \sin^2 x) = 0.$

Factoring:

$r^2(2\sin x + 1)(\sin x - 1)(\sin x - 1) + 4r\cos x(\sin x + 1)(\sin x - 1) + 4(1 + \sin x)(1 - \sin x) = 0.$

Divide by $(\sin x - 1)$:

$r^2(2\sin x + 1)(\sin x - 1) + 4r\cos x(\sin x + 1) - 4(\sin x + 1) = 0.$

Solving this using the quadratic formula yields the expressions for r shown at the bottom of this column; the desired value of r is

$r = -4\cos x/2(2\sin x + 1)(\sin x - 1) = 2\cos x/(2\sin x + 1)(1 - \sin x)$ which converts to:

$r = 2\cos x/(\sin x + 1 - 2\sin^2 x)$ or

$2\cos x/(\sin x + \cos 2x).$

The other value of r is

$r = (-8\cos x\sin x - 4\cos x)/2(2\sin x + 1)(\sin x - 1).$

This simplifies to $r = 2\cos x/(1 - \sin x)$, the radius of an imaginary sphere tangent to the imaginary extension of the cone and also tangent to the tip of the water level of a full glass, displacing no water.

Also solved by Winslow Hartford, Norman Wickstrand, Matthew Fountain, David Evans, Harry Zaremba, William Moody, and the proposer, Edmund Nadler.

OCT 4 Find the closed form (not infinite series) solution of $dy/dx = x - y^2$.

The following solution is from John Wrench:

The differential equation $dy/dx = x - y^2$ is a special case of Riccati's equation and is transformable to a

$$r = \{-4\cos x(\sin x + 1) \pm [16\cos^2 x(\sin x + 1)^2 + 16(2\sin x + 1)(\sin^2 x - 1)]^{\frac{1}{2}}\}/2(2\sin x + 1)(\sin x - 1)$$
$$r = \{-4\cos x(\sin x + 1) \pm [16\cos^2 x(\sin x + 1)^2 - 16\cos^2 x(2\sin x + 1)]^{\frac{1}{2}}\}/2(2\sin x + 1)(\sin x - 1).$$
$$r = [-4\cos x(\sin x + 1) \pm (16\cos^2 x\sin^2 x)^{\frac{1}{2}}]/2(2\sin x + 1)(\sin x - 1)$$
$$r = [-4\cos x(\sin x + 1) \pm 4\cos x\sin x]/2(2\sin x + 1)(\sin x - 1)$$

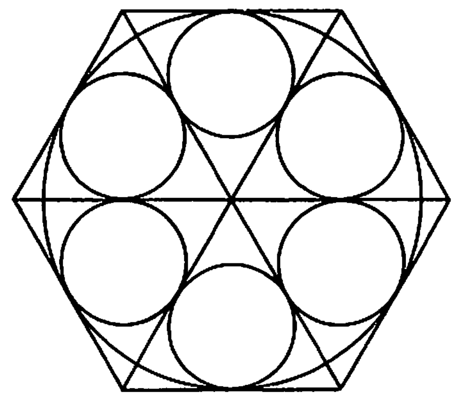
second-order linear homogeneous differential equation (called Airy's equation):
$d^2u/dx^2 - xu = 0$,
by the substitution $y = 1/u\ du/dx$. The general solution of the transformed equation is
$u = c1\, x^{.5}\, I_{1/3}(2/3\, x^{1.5}) + c2\, x^{.5}\, I_{-1/3}(2/3\, x^{1.5})$,
where the Is are modified Bessel functions of the first kind of orders 1/3 and −1/3. If we use the differential formulas
$d/dx\, [u^p I_p(u)] = u^p I_{p-1}(u) du/dx$, and
$d/dx\, [u^{-p} I_p(u)] = u^{-p} I_{p+1}(u) du/dx$,
we obtain
$du/dx = c1\, x\, I_{-2/3}(2/3\, x^{1.5}) + c2\, x\, I_{2/3}(2/3\, x^{1.5})$.
Thus, the general solution of the given differential equation may be written

$$y = \frac{c1\, x^{.5} I_{-2/3}(2/3\, x^{1.5}) + c2\, x^{.5} I_{2/3}(2/3\, x^{1.5})}{c1\, I_{1/3}(2/3\, x^{1.5}) + c2\, I_{-1/3}(2/3\, x^{1.5})}$$

Clearly, that solution involves only one independent constant—that is, c1/c2 or its reciprocal.

Also solved by Leo Harten, Antony Beris, Matthew Fountain, and David Evans.

OCT 5 Given a regular hexagon with an inscribed circle, what is the ratio of the area of the six smaller circles (see drawing) to the area of the inscribed circle?



Roger Milkman had little trouble with this one: The answer is two-thirds, assuming that the ratio desired is that of the total area of the six small circles to the area of the large one. Set the radius of the inscribed large circle at 1. Draw it vertically upward from the center; it is perpendicular to, and bisects, the top side of the hexagon. Each side of each of the equilateral triangles is therefore sec 30° (since the radius so drawn also bisects the lowest angle of its triangle). The radius of (for example) the upper left hand small circle will be perpendicular to, and bisect, any side of its triangle that it reaches. It will also bisect the opposite angle if prolonged. The radius of the small circle is thus [½ sec 30°] tan 30°, or 1/3. The ratio of the areas is 6 times the ratio of the squared radii, or 6/9, or 2/3.

Also solved by John Woolston, Emmet Duffy, David Evans, Norman Wickstrand, Winslow Hartford, Naomi Markowitz, Irl Smith, Mary Lindenberg, Matthew Fountain, Jordan Wouk, Steve Feldman, David Lukens, Phelp Meaker, and Raymond Gaillard.

Better Late Than Never

M/A 4. Robert Kennedy has responded.
JUL 1, JUL 3, and **JUL 5.** Richard Hess has responded.
A/S 5. John Woolston has responded.
Y1982. Harry Hazard has responded.
OCT SD1. The proposer doesn't feel the problem is speedy.
OCT SD2. The proposer believes that calculating the last eight digits is more fun.

Proposers' Solutions to Speed Problems

F/M SD 1 The level drops 1.2 inches.
F/M SD 2 The ♠6. West may continue a second spade instead of shifting to a diamond honor.

Puzzle Corner
Allan J. Gottlieb

Sneaky vs. Freddy

Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y., 10012.

Since I've not reviewed the ground rules of "Puzzle Corner" for more than a year, let me do so now for the benefit of new readers.

In each issue we present five regular problems (the first of which is chess- or bridge-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and one submitted solution for each problem is selected for publication three issues later. We also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the August/September issue. Since *I* must submit that column sometime in May (today is January 15), you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the section entitled "Better Late than Never." For example, comments appear in this issue on previously published solutions to two problems, **JUL 1** and **Y1982.**

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, solutions to the "speed" problems printed in this issue are given at the end of this column. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes we go back into history to republish problems which remained unsolved after their first appearance.

All problems come from readers, and all readers are invited to submit favorites. I'll report on the size of the backlog, and on criteria used in selecting problems for publication, in future issues.

Problems

APR 1 Here is a zany problem from Arthur Polansky, who recounts a conversation with a friend:

"I know you've played duplicate bridge with life masters and once received an invitation to the Brisbane regionals. But have you ever played mixed-pairs quadruplicate?" my friend asked. Reluctantly I confessed that I had not. My friend explained: "Each team of four players is split amongst four tables so that one member plays North, one South, one East, and one West. Each deal of the cards is played four times, once at each table."

"But that means that I always get a partner from an opposing team," I complained.

"Precisely," my friend said. "Cuts down on preplanned conventions, like the 17-19-point left-eye-scratch. Also does wonders for the scoring." I could imagine. "Consider deal 23 from last night. Team 5, the Four Spades, reached four spades at all four tables."

By now I was mildly disgusted. "With such bidding prowess, why do you allow these turkeys to keep playing?"

"They own the club," was my friend's answer.

Hastily bidding adieu (which was immediately doubled for takeout), I raced home. Then I realized that the self-styled tournament director had failed to show me the deal in question. Can you? (That is, find a deal such that four spades, when played from *any* position, will be set at least seven tricks (i.e., the defense will make four spades). Naturally, the set will be perforce: the declarer will do everything in his power to prevent being set and to minimize the damages if being set becomes inevitable.)

APR 2 Frank Rubin wants a defect-free plane of words:

Shown below is a simple word square.

A C E
P A R
E R A

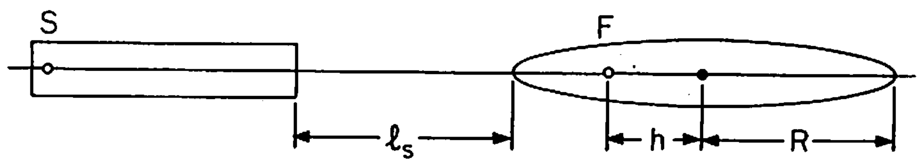
We could fill the plane with such word squares so that every cell of an infinite grid lay at the intersection of two words. However, such an arrangement would have two defects: (1) the pattern would be repetitive, and (2) the words would no all be interconnected. Show how thes defects can be fixed; i.e., fill the plan with an infinite number of words so tha the pattern does not repeat in any row o column, every cell lies at the intersectio of two words, and every word is con nected to every other one by a chain o intersecting words. Do not use any two letter words.

APR 3 Here's one from Ming Chung who first published it (and the diagram a the bottom of this page) in the studen newsletter of the M.I.T. Physics De partment:

One afternoon in a swamp Sneaky th snake, whose mass is M_s and length ℓ_s was resting on the left end of a log o length L and mass M_L when he suddenl remembered that he was hungry. After a few moments of looking around, he founc Freddy the frog sitting on a large disk-shaped leaf of radius R and mass M_0 h cm. away from the center of the disk. Initially the disk was 4/5 ℓ_s cm. away from the right end of the log. Quite naturally Sneaky started moving toward Freddy, and Freddy, who was ignorant of the laws of physics, started jumping for his life. However, when Freddy remembered that Sneaky had his best friend for breakfast in the morning he was overcome by the rage and desire for revenge, and he turned around to charge against Sneaky. Well, normally it would have been a dumb move. But the laws of physics reward the courageous; when Freddy stopped at a point on the disk and realized that his attempt was futile if not suicidal, the disk was just out of the reach of Sneaky, who was on the right end of the log by that time. Sneaky could not swim, so the good guy was saved and the bad guy had a hungry afternoon. It's a dumb story, you say. Well, if you'd like to entertain your intelligence, consider the following question: Can you locate the point where Freddy stopped? Assume all motions occurred on a line, and the water offered no resistance. Both the log and disk have a uniform mass density.

APR 4 Ronald Burde doesn't seem to know enough to come in out of the rain. Instead he asks:

Assuming the sudden onset of steady rainfall, will a person remain dryer by walking or running any given distance to shelter? (Mr. Burde remarks that this problem has reached the level of a friendly "cause celebre" among some



ous yacht builders and owns his own company building traditional cruising sailing yachts in molded fiberglass. . . . **Donald Distant,** S.M.'80, has been appointed chief engineer (electrical and marine) with the Port of Singapore Authority in May 1982, responsible for all electrical engineering and marine engineering activities in P.S.A.

Theotokis S. Milas, S.M.'71, is presently superintendant engineer with Cove Shipping, New York City. . . . **Robert J. Bosnak,** '60, chief, Mechanical Engineering Branch, Division of Engineering of the U.S. Nuclear Regulatory Commission, was a recipient of the 1982 American Society of Mechanical Engineers' Bernard F. Langer Nuclear Codes and Standards Award. The award was for "outstanding professional and technical contributions in the development of ASME nuclear codes, standards, and accreditation programs." . . . **Robert I. Price,** '53, recently retired as the third ranking officer of the U.S. Coast Guard, commanding both the New York District and the Atlantic area, has received the annual Vice Admiral "Jerry" Lane Medal for "outstanding accomplishment in the marine field," by the Society of Naval Architects and Marine Engineers. His naval career has focused on the improvement of national and international standards for maritime safety and environmental protection.

XIV

Economics

Arthur G. Ashbrook, Jr., Ph.D.'47, reports that he has retired from the Central Intelligence Agency after 28 years of service as an economist. For four years he was assigned to the faculty at the National War College, Washington, D.C., and intends to continue teaching and research in the "dismal science." . . . Two graduates of the department have been re-elected to Congress: **Howard E. Wolpe,** Ph.D.'62, won his third consecutive term in Congress from Michigan's third Congressional district, the area encompassing Kalamazop; and **Les Aspin,** Ph.D.'66, won his seventh consecutive term in Congress from Wisconsin's first district.

XV

Management

Project management will be so different by 1990 that "a new breed of manager will be needed to cope with changes," says **Albert J. Kelley,** '48, president of the Arthur D. Little Program System Management Co. The changes: increasing constraints by such external factors as environmental and government regulation, multiple financial arrangements, new risk due to advanced, unproved technology, and inflationary pressures. These issues and others are the subject of *New Dimensions of Project Management* (Lexington Books, 1983), a book of 15 essays by Dr. Kelley and other project management specialists including **John R. White,** S.M.'63, senior vice-president—strategic planning at Arthur D. Little, Inc.; Professor **Mel Horwich** of the Sloan School of Management; **John F. Magee,** president of Arthur D. Little, Inc.; and **Robert C. Seamans, Jr.,** Sc.D.'51, Henry R. Luce Professor of Environment and Public Policy at M.I.T.

Richard A. Michaelson, S.M.'77, reports, "I have recently been made vice-president at MetPath, Inc., responsible for billing and receivables. MetPath, a wholly-owned subsidary of Corning Glass, is in the clinical laboratory business. . . . **Warren H. Hausman,** Ph.D.'66, has been appointed chairman of the Industrial Engineering and Engineering Management Department at Stanford Univeristy, Palo Alto, Calif. His area of expertise is in production/operations management and analysis. . . . **George E. Williams,** S.M.'49, corporate vice-president of United Technologies Corp., has taken an early retirement and has joined Kensington Management Consultants, Stamford, Conn., as a senior vice-president.

Charles C. Holcomb, S.M.'75, is currently the commanding officer of USS *Canopus* and resides in Charleston, S.C. . . . **Aristea Xafa,** S.M.'75, has left her London-based position as vice-president—finance of Saudi REDEC and is now in charge of Boston-based Global Investments Limited. She writes that she would like to hear from alumni in microelectronics and computer manufacturing concerns and those engaged in R&D equity financing. . . . **W. John Swartz,** S.M.'67, has been appointed executive vice-president of Santa Fe Industries, Inc., Chicago, Ill. . . . **William O. Schach,** S.M.'50, is currently senior vice-president of Merrill Lynch, Pierce, Fenner, and Smith, Inc.

Frederic C. Westendorf, S.M.'64, is currently business planning manager for IBM Europe/ Middle East/Africa, White Plains, N.Y. . . . **Debra Greenberg,** S.M.'78, reports that she has founded her own consulting firm in New York City.

Otto H. Poensgen, Ph.D.'64, who was assistant professor at M.I.T.'s Sloan School (upon his graduation until 1967) and later was appointed to a chaired professorship in industrial management at the University of Saarland, Saarbrucken, Germany, passed away on October 29, 1982. He was well known in the U.S. and abroad for his research and teaching in management science, business strategy, and organizational structure. During a sabbatical leave in 1980, he continued his research at M.I.T. . . . **Donald G. Robbins, Jr.,** of Fairfield, Conn., passed away on October 11, 1982; no details are available.

Sloan Fellows

Ormand J. Wade, S.M.'73, president and chief operating officer of Illinois Bell Telephone Co., has been elected to the boards of Harris Bankcorp, Inc., and Harris Trust and Savings Bank. . . . **Leroy E. Day,** S.M.'60, writes, "I retired from NASA and the Space Shuttle Program to start my own consulting business for aerospace and management—business expanding. I was also awarded NASA's Distinguished Service Medal for work on the space shuttle." . . . **Brian J. Kelly,** S.M.'73, vice-president of marketing of Bell of Pennslyvania and Diamond Star Telephone Co., has been elected to the Board of Trustees of Hahnemann University, Philadelphia, Penn.

John C. Davis, S.M.'56, senior vice-president and a director of Santa Fe Railway, Chicago, retired from the firm in December 1982. . . . **Eric W.A. Lange,** S.M.'62, has joined the staff of Failure Analysis Associates, Troy, Mich. . . . **Sam R. Willcoxon,** S.M.'65, formerly vice-president of American Telephone & Telegraph Co., New York City, has become the firm's executive vice-president of marketing (interexchange organization). . . . **William H. Springer,** S.M.'68, executive vice-president of finance for Illinois Bell Telephone Co., has become its senior vice-president and secretary.

Steven J. Miller, S.M.'79, writes, "I was recently married to Janet Patricia Beal on September 11, 1982, and am currently director of marketing—western region for Data Resources, Inc., a leader in economic forecasting and consulting." . . . **William S. Wheeler, Jr.,** S.M.'54, reports, "I took early retirement as a senior vice-president of Englehard Industries earlier this year and with an associate bought a small manufacturing company which manufactures bulk material handling equipment. The company is Buck El, Inc. Thoroughly enjoying the independence of being the C.E.O. of my own company." . . . **Cline W. Frasier,** S.M.'72, a senior technical member of the Manufacturing Automation and Computation Department at the Charles Stark Draper Laboratory, Cambridge, has been appointed head of the department.

Robert E. Workman, S.M.'58, a retiree after 39 years of service at Goodyear Tire and Rubber Co., Akron, Ohio (his latest position was vice-president of general products development), passed away on November 18, 1982. He was a past president of the International Institute of Synthetic Rubber Producers, a member of the American Chemical Society and of the American Institute of Chemical Engineers.

Management and Technology Program

Geoffrey N. Andrews, S.M.'82, is currently with the New Opportunities Department at Pilkington Brothers in England, where he recently headed a corporate-wide project called Quest—ferreting out underutilized skills and technology within the firm. He saw Professor **Edward B. Roberts,** Ph.D.'57, briefly during Ed's trip to Pilkington at the end of November.

XVI

Aeronautics and Astronautics

The spread of nuclear power to developing nations is dangerous because it lowers their threshold for nuclear weapons development, says **John P. Holdren,** '65, professor of energy and resources at the University of California at Berkeley. A nuclear power program assures the presence of the technical skills needed to operate a weapons program and often includes facilities for coverting reactor fuel into weapons-usable material, Dr. Holdren wrote early this year in the *Bulletin of the Atomic Scientists*. A power program also reduces the marginal cost of a weapons program, and it lowers some of the political barriers to nuclear weapons commitments, Dr. Holdren said.

Roger Neeland, S.M.'70, writes, "After teaching at the U.S. Air Force Academy for several years in the Department of Astronautics, and two years heading FAA's Airborne Systems Branch in their Systems Research and Development Service, I am now responsible for analysis to support the Air Force Systems Command long-range planning." . . . **Louis H. Benzing,** S.M.'52, executive vice-president of Jofee Marketing International, has taken on the additional position of president and chief executive officer at the Micro Z Corp., Monrovia, Calif. . . . **Tore Christiansen,** S.M.'82, has recently taken up an engineering position in Det Norske Veritas, an international Norwegian classification society for ships and off-shore structures, and is presently working in Oslo, Norway.

Henry F. Lloyd, S.M.'46, reports that he is still a college administrator at Flagler College and has been in this position since retirement from the U.S. Navy. . . . **Robert Schegerin,** S.M.'74, is presently employed at Talasa Law and Technologies, Woodbury, Conn. . . . **Jack E. Steiner,** S.M.'41, vice-president of corporate product development at the Boeing Co., recently served as a witness on hearings on government policy toward aeronautical research chaired by Congressman Dan Glickman, chairman of the Subcommittee on Transportation, Aviation and Materials.

Technology and Policy Program

Julian Villalba, S.M.'80, has recently completed his doctorate at M.I.T. in international technology transfer. He will teach at the Institute des Estudios Superiores y Administracion, Caracas, and also plans to do consulting work on state-owned enterprises and public administration. . . . **David Rubin,** S.M.'81 is working for the city of San Francisco as a member of a task force studying district heating systems.—Richard de Neufville, Chairman, Technology and Policy Program, M.I.T., Room 1-138, Cambridge, MA 02139.

fellow-alumni. He agrees with them that the solution to the problem is straightforward and that it has been solved many times, but agreement is lacking on the answer.)

APR 5 Peter Groot wants you to find positive integers a, b, and c such that $a^3 + b^4 = c^5$.

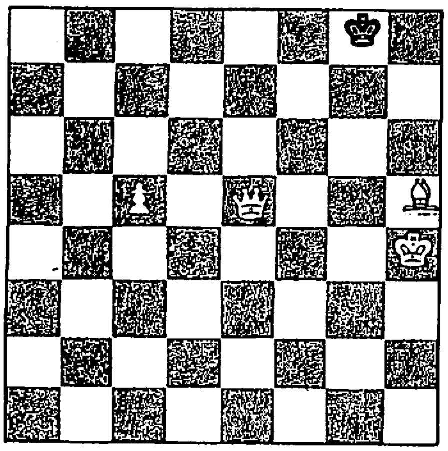
Speed Department

APR SD1 Rufus Briggs will let you have additional information, but not much: Given a glass tube 6 inches in diameter and 60 inches long. The tube is capped and sealed at one end and mounted vertically. The bottom 30 inches of the tube is filled with thick molasses and the top 30 inches with a medium grade of oil. A 1-inch polished steel ball is allowed to fall through the liquids, and it takes $t_0 = 2.73$ seconds to fall half way down the tube. What is the time t_T for the ball to fall the entire length of the tube? To answer this question you may have one and only one additional piece of information. Identify the information you have chosen, and write the equation for solving the problem.

APR SD2 David Evans wonders what common five-letter word is spelled wrong by nearly every M.I.T. graduate.

Solutions

N/D 1 White to mate in three moves:

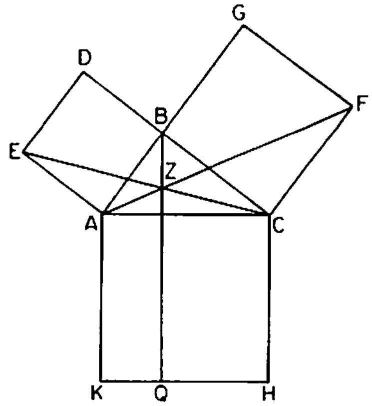


The following solutions are from Darryl Hartman:

Q—B3	K—B1
Q—KN3	K—K2
Q—Q6	
or	
—	K—R2
B—B7	K—R3
Q—R8	

Also solved by Matthew Fountain, George Farnell, Ronald Raines, Robert Holt, Richard Hess, Randy Kimble, Winslow Hartford, and the proposer, Bob Kimble.

N/D 2 Given ABC is a right triangle; B is a right angle; ABDE, BCFG, and ACHK are the squares on the sides; and BQ is parallel to AK. Determine if the three lines EC, BQ, and AF meet at a point Z as they appear to in the drawing.

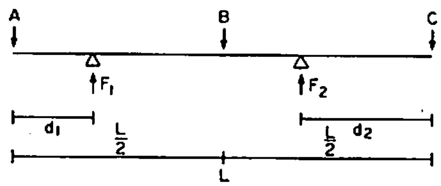


Avi Ornstein found that analytic geometry makes this problem rather easy: Let AC and BQ be the x and y coordinates, respectively. In addition, let A be (−a, 0), let B be (0, b), and let C be (c, 0). Then E is (−a −b, a), F is (b +c, c), and Q is (0, −a −c). The slope of AF is c/(a +b +c) and its y-intercept is ac/(a +b +c). The slope of EC is −a/(a +b +c) and its y-intercept is ac/(a + b +c). AF and EC therefore meet on line BQ at point Z.

Also solved by Winslow Hartford, Richard Hess, Gabrielle and J. Donnay, Paul Sonn, Phelps Meaker, Mary Lindenberg, Steve Feldman, Eugene Boehne, John Woolston, Dave Simen, Harry Zaremba, Emmet Duffy, Matthew Fountain, Robert Holt, and Farrel Powsner.

N/D 3 A massless beam of length L is supported by two stanchions at distances d1 and d2 from the ends. The beam is loaded with point masses A, B, and C at the ends and midpoint. What is the downward force on each stanchion?

Robert Holt sent us a fine solution:



To simplify the notation, say that the point masses have *weights* A, B, and C. (If A is the mass, then the weight is Ag, and A, B, and C must be replaced by Ag, Bg, and Cg, respectively, in what follows.) There are five forces acting on the beam; three are the weights A, B, and C, and the other two are the upward forces due to the stanchions. Call these forces F_1 and F_2. Apparently the beam is supposed to be at rest, so the torque about each stanchion is zero. The magnitude of the torque about the left stanchion is

$Ad_1 - B([L/2] - d_1) - C(L - d_1) + F_2(L - d_1 - d_2)$.

Setting this equal to 0 yields

$Ad_1 - B[L/2] + Bd_1 - CL + Cd_1 + F_2(L - d_1 - d_2) = 0$

$(A + B + C)d_1 - ([B/2] + C)L + F_2(L - d_1 - d_2) = 0$

$F_2 = [(½B + C)L - (A + B + C)d_1]/(L - d_1 - d_2)$.

Similarly, the magnitude of the torque about the right stanchion is

$A(L - d_2) + B([L/2] - d_2) - Cd_2 - F_1(L - d_1 - d_2) = 0$,

and this gives

$F_1 = [(½B + A)L - (A + B + C)d_2]/(L - d_1 - d_2)$.

Of course, by Newton's Third Law of Motion the downward force on the left stanchion is

$[(½B + A)L - (A + B + C)d_2]/(L - d_1 - d_2)$,

and the downward force on the right stanchion is

$[(½B + C)L - (A + B + C)d_1]/(L - d_1 - d_2)$.

Also solved by Richard Hess, John Boynton, Irving Hopkins, Howard Wagner, John Prussing, Haus Meier, George Piotrowski, James Reswick, John Woolston, Matthew Fountain, Robert Slater, and Harry Zaremba.

N/D 4 Will the three hands of a clock (hours, minutes, and seconds) ever divide the clock face into three equal parts? If so, when?

James Reswick sent us an interesting analogy: I'm not much on mathematical proofs, so I tried a model clock in my Radio Shack PC2 computer. The program runs the "clock" very fast but slows down to calculate for each degree of second-hand movement when the angle between the minute and hour hands approaches 120°. At four times, viz: 2:54:32, 5:05:27, 6:54:32 and 9:05:27, it showed the second hand to be only 10° off when the minute and hour hands were exactly 120° apart. Thus there seems to be no solution to the problem but there are four pretty close answers. Reminds me of the old story about a mathematician and an engineer. Each stood at one end of a long room. At the other end stood a lovely young lady. They were told to approach her by walking half the distance, to stop, then to walk half the remaining distance, stop, and to continue until they reached the maid to receive a rewarding kiss. The mathematician declared "But according to Zeno I'll never reach her," and he left the room in disgust. The engineer started walking with alacrity saying, "I learned about Zeno also—but I'll get close enough!"

J. Meier sent us a more conventional treatment: The second hand advances 360° per minute; the minute hand advances 6° per minute; and the hour hand advances 0.5° per minute. The minute hand gains 5.5° per minute [= (11/2)° per minute] on the hour hand. The second hand gains 354° per minute on the minute hand. The first condition is that the minute hand be 120° or 240° ahead of the hour hand. Starting at X:00:00 (where the minute and second hands are at 0° and the hour hand is at X hours, or 30X°), it takes (30X + 120) · 2/11 and (30X + 240) · 2/11, or (60X + 240) /11 and (60X + 480) /11 minutes, respectively, to meet the first condition. This repeats every 12 hours, i.e., X = 0, 1, . . ., 11. The corresponding number of degrees moved by the minute hand are (360 + 1440)°/11 = 360(X + 4)°/11 and 360(X + 8)°/11, respectively. As a check consider X = 7 and X = 3, respectively. The minute hand moves 360° after 7:00:00 and after 3:00:00 to come to 8:00:00 and 4:00:00, respectively, where it is 120° and 240°, respectively, ahead of the hour hand. The second condition is that the second hand be 120° or 240° ahead of the minute hand. Starting at X:Y:00, where the second hand is at 0° and the minute hand at Y minutes, or 6Y° (X does not matter), it takes (6Y + 120)/354 or (6Y + 240)/354 minutes, respectively, to meet the second condition. This repeats every hour, i.e., Y = 0, 1, . . ., 59. The corresponding number of degrees moved by the minute hand are (6Y + 120)°/59 and (6Y + 240)°/59, respectively. As a check consider Y = 39 and Y + 19. In both cases it takes exactly one minute to fulfill the second condition at 40 minutes and 20 minutes, respectively, after the full hour. For both conditions to be fulfilled simultaneously, 360(X + 4)/11 or 360(X + 8)/11 must have a fraction in excess over a full integer equal to the fraction over a full integer of (6Y + 120)/59 or (6Y + 240)/59, respectively. But this is not possible for the stated values of X and Y.

Also solved by Harry Zaremba, Richard Hess, Robert Holt, Winslow Hartford, C. L. Baker, Emmet Duffy, Matthew Fountain, John Woolston, Dave Simen, Norman Wickstrand, Ken Haruta, and the proposer, Alan Davis.

N/D 5 Six different numbers are selected arbitrarily from eight positive consecutive integers. The resulting selection includes the smallest and largest of the eight numbers and can be separated into three pairs of numbers, each of which contains consecutive numbers. The sum of the six integers is three times a number N, and the sum of their cubes equals the cube of N. Find N and each of the integers selected.

The following solution is from John Bucsela: The eight consecutive integers may be written as $n, n+1, \ldots, n+7$ for some integer $n \geq 1$. There are three ways to select six of these integers so that the smallest and largest are included and so that they may be divided into three pairs of consecutive numbers. They are

$n, n+1, n+2, n+3, n+6, n+7$

$n, n+1, n+3, n+4, n+6, n+7$

$n, n+1, n+4, n+5, n+6, n+7$

In the first case, the sum of the six numbers is $6n + 19$. This is not divisible by 3, hence cannot equal 3N. Likewise, in the third case, the sum of the six numbers is $6n + 23$, also not divisible by 3. Thus only the second case can occur. Then, the sum is $6n + 21$, so we conclude that $3N = 6n + 21$, or $N = 2n + 7$. Next,

$n^3 + (n+1)^3 + (n+3)^3 + (n+4)^3 + (n+6)^3 + (n+7)^3 = 6n^3 + 63n^2 + 333n + 651$.

Setting this equal to

$N^3 = (2n+7)^3 = 8n^3 + 84n^2 + 294n + 343$, we obtain $2n^3 + 21n^2 - 39n - 308 = 0$

It is easy to see that the only possible positive integral solution of this equation is 4. Thus N = 15 and the six selected integers are 4, 5, 7, 8, 10, and 11.

Also solved by Richard Hess, Robert Slater, Robert Holt, Howard Wagner, Steve Feldman, Roger Milkman, Samuel Levitin, Howard Sard, Richard Bernicker, Emmet Duffy, Matthew Fountain, David Simen, Frank Carbin, John Woolston, Ronald Raines, Norman Wickstrand, and the proposer, Harry Zaremba.

Better Late Than Never

JUL 1 The answer supplied by the proposer was correct, but I carelessly misread it.

A/S 4 Richard Hess has responded.

OCT 1 Richard Hess has responded.

OCT 3 Richard Hess has responded.

OCT 5 Richard Hess, Michael Jung, and Avi Ornstein have responded.

Y1982 John Fine, Erik Anderson, Thomas Weiss, Phelps Meaker, and Harry Hazard noticed that the published solution was not optimal. A revised solution follows. To clarify the problem let me add that we seek a minimum number of operations (each addition, subtraction, multiplication, division, and exponentiation counts as one operation); and among answers having equal numbers of operations, we prefer those with 1, 9, 8, and 2 used in order.

1	1^{982}	36	$18 + 9 \times 2$
2	$1^{98} \times 2$	37	$1 \times 9 + 28$
3	$1^{98} + 2$	45	$18 \times 2 + 9$
4	$18/9 \times 2$	47	$19 + 28$
5	$9 - 12 + 8$	49	$1 \times 98/2$
6	$9/12 \times 8$	50	$1 + 98/2$
7	$18 - 9 - 2$	54	$(19 + 8) \times 2$
8	$1^{92} \times 8$	57	$1 + (9 - 2) \times 8$
9	$91 - 82$	64	$1^{9} \times 8^{2}$
10	$1^{82} + 9$	70	$81 - 9 - 2$
12	$12^{(9-8)}$	74	$92 - 18$
13	$19 - 8 + 2$	75	$91 - 8 \times 2$
14	$1 + 9 + 8/2$	81	$18 \times 9/2$
15	$19 - 8/2$	82	$1^{9} \times 82$
16	$1^{9} \times 8 \times 2$	84	$1 \times 92 - 8$
18	$18/2 + 9$	85	$1 + 92 - 8$
19	$1 \times 28 - 9$	87	$1 \times 89 - 2$
20	$1 - 9 + 28$	88	$1 + 89 - 2$
21	$1 \times 29 - 8$	89	$1^{2} \times 89$
22	$(19 - 8) \times 2$	90	$1^{2} + 89$
25	$19 + 8 - 2$	91	$1 \times 9 + 82$
27	$18 \times 2 - 9$	92	$1 + 9 + 82$
28	$1^{9} \times 28$	95	$91 + 8/2$
30	$19 \times 2 - 8$	98	$1^{2} \times 98$
33	$(1 + 2) \times 8 + 9$	100	$1 \times 98 + 2$
35	$19 + 8 \times 2$		

Proposers' Solutions to Speed Problems

SD 1 There is one elegant and simple answer to the problem that is quickly seen by those observant people who do not overlook the obvious: 1. They ask for the time t_M required by the ball to fall through the molasses; and 2. They write the equation (called for in the problem). The total time $t_T = t_M + 2.73$.

SD 2 Wrong.

Puzzle Corner
Allan J. Gottlieb

This One Is "Infinitely Hard"

Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10012.

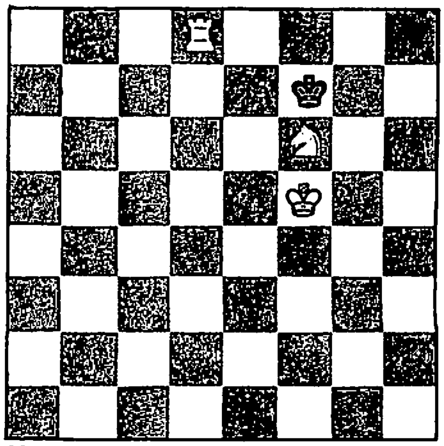
As promised last issue, here's a description of how we choose solutions for publication.

As responses arrive during the month, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column, I first weed out erroneous and illegible solutions. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor contributions from correspondents whose solutions have not previously appeared, as well as solutions that are neatly written or typed, since the latter produce fewer typesetting errors.

Problems

M/J 1. Given the diagram at the top of the next column, John Cronin wants White to move and mate in four.

M/J 2. Frank Rubin sent us an "infinitely hard" problem:
Differential equations crop up frequently, usually first order, but sometimes second order, rarely higher. For *Review* readers, though, we go right to the limit: Find a non-trivial, continuous, real-valued function f(x) possessing continuous derivatives of all orders, and satisfying the *infinite-order* differential equation
$f = f' + 4f'' + 9f''' + 16f'''' + \ldots$



M/J 3. A pair of cryptarithmetic puzzles from Thomas McNelly:

$$\frac{\text{EVER}}{\text{NNNNN}} = .\text{ONANDONANDON}\ldots$$

$$\frac{\text{EVER}}{\text{VVVVV}} = .\text{ONANDONANDON}\ldots = \frac{\text{IRIS}}{\text{IIIII}}$$

M/J 4. Larry Bell asks us about consecutive primes:
Find three consecutive three-digit prime numbers x, y, z, such that $z - y = y - x = 12$. In other words, there are no prime numbers between x and z, other than y. Find four consecutive three-digit prime numbers a, b, c, d, such that $d - c = c - b = b - a = 6$. What is the largest string of consecutive primes $P_1, P_2, \ldots, P_n$ such that:
$P_n - P_n - 1 = P_n - 1 - P_n - 2 = \ldots = P_2 - P_1 = 4$.

M/J 5. Harry Zaremba notes that for any triangle, the ratio of the sum of the squares of the medians to the sum of the squares of the sides equals a constant rational number. What is this number?

Speed Department

M/J SD1. We end with a problem from Phelps Meaker:
To build a straight sidewalk 54′8″ long, there is a pile of 40 cast-concrete slabs, formed as equilateral triangles. To square off the ends, two extra 30°-60° half-slabs are provided. What is the width of the walk?

M/J SD2. A birthday quickie from John Linderman:
On my birthday this year, my age was *not* the product of exactly two primes. However, for a few previous birthdays in succession, it had been. What is the largest number of successive birthdays on which your age can be the product of two primes? Given that I just went past the first such maximal sequence, how old am I? If I live to be 100, how many such maximal sequences will I celebrate?

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Solutions

JAN 1. Unfortunately, the proposer has written that his original analysis was in error and the problem should be withdrawn. Richard Hess and Doug Van Patter noticed that, in fact, only those declarers making the *wrong* play at trick 5 took 11 tricks.

JAN 2. On a map of Massachusetts, each town is connected to its nearest neighbor (assume no ties). Show that no town is connected to more than five others.

Daniel Seidman solved this problem by making two key observations. First, since there were no ties, any three cities form a scaline triangle (i.e., all sides lengths unequal) and thus the longest side would not appear on the map. Second, for a city A connected to six or more others there must exist two neighbors B and C such that angle BAC is less than or equal to 60°. Now it's easy. Triangle BAC has angle sum 180° (or greater if on the globe), so the largest angle cannot be BAC and hence either line AB or AC does not appear.

Also solved by Avi Ornstein, Leo Harten, Naomi Markowitz, and Richard Hess.

JAN 3. Drill a hole through a solid sphere (along a diameter) whose radius is such that the height of the remaining ring is two inches. Find the volume of the ring.

The following pair of solutions is from Avi Ornstein:

Let the radius of the sphere be R. The height of the missing sections of the sphere (top and bottom) is R − 1. The cylindrical hole has a radius which is one side of a right triangle, with 1" as the other side and R as the hypotenuse.

The volume of the sphere is $4\pi R^3/3$.

The volume of the cylinder is πr^2h, or $\pi(R^2 - 1)2$.

Each segment of the sphere is

$\pi h^2(3r - h)/3$, or $\pi(R - 1)^2(2R + 1)/3$.

The volume of the ring is therefore

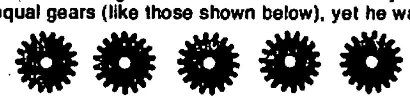
$4\pi R^3/3 - (2\pi R^2 - 2\pi) - 2(2\pi R^3/3 - 2\pi R^2 + \pi/3)$

or $4\pi/3$ in^3.

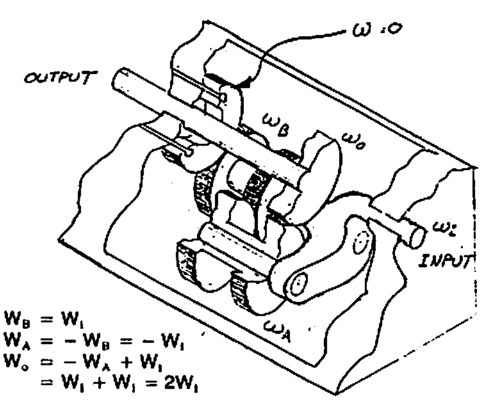
If the original sphere had a diameter of 2", the hole would be nonexistent, and the volume of the sphere would give the same answer, as an alternate way of solving the problem.

Also solved by Phelps Meaker, Ken Haruta, John Prussing, Frank Carbin, Daniel Seidman, Ronald Raines, James Reswick, Fredrick Hutchinson, Richard Hess, Harry Zaremba, Naomi Markowitz, Norman and Ammi Spencer, Roger Milkman, Norman Wickstrand, Richard Marks, Emmett Duffy, Leo Harten, David Evans and the proposer, Dean Edmonds.

JAN 4. In building a model, a man found that he needed a 2:1 gear ratio. He had on hand only six equal gears (like those shown below), yet he was able to obtain the desired ratio, using full tooth conventional meshing. How?



A beautifully drawn solution shown at the bottom of the previous column was submitted by Floyd Kosch.



$W_B = W_I$
$W_A = -W_B = -W_I$
$W_o = -W_A + W_I$
$= W_I + W_I = 2W_I$

Also solved by Richard Marks, Emmett Duffy, Chuck Coltharp, Richard Hess, Luigi Burzio, and James Reswick.

Better Late Than Never

Y1982. Responses received from Robert Sackheim and Harry Garber.

A/S 2. Richard Hess has found a solution if we are not restricted to bishop moves.

OCT 2. Hillary Fisher and R. Morgan believe that, for example, the sum TWELVE $\times$2 indicates that the value for TWELVE should be half the sum. Their solutions are

13744	645
64948	75054
64948	75054
64948	75054
208588	75054
= 104294 × 2	375915
	= 125305 × 3

84502	5394
84502	5394
68387	24947
68387	24947
68387	24947
818387	24947
1192552	404947
= 298138 × 4	404947
	920470
	= 184094 × 5

OCT 3. Nancy Everds believes the cone contains hot fudge and the sphere is ice cream. Thus melting must be considered.

OCT 4. Steve Chilton does not believe that a solution involving Bessel functions (or any other series-defined functions) should be considered closed form.

OCT 5. Jordan Wolk submitted an alternate solution.

N/D 2. Harry Garber and Leo Harten have responded.

N/D 3, N/D 4. Leo Harten has responded.

N/D 5. Harry Garber, Leo Harten, and William Stein have responded.

1983 JAN SD2. Ken Fawcett believes that Van-Patter's line of play is reasonable but not guaranteed to succeed.

1983 F/M SD1. L. Steffens and Jordan Wolk noticed that it is the lock that is 10 by 30 feet.

Proposer's Solutions to Speed Problems

SD 1. 27.733"

SD 2. Three in a row. If two of the years were even, one would be divisible by 4. Since 4 is flanked by 3 and 5, 4 itself cannot participate in the sequence, and neither can 4× anything else. The "middle number" in such a sequence must therefore be 2 × p for some prime p. Browsing around for such numbers flanked by the product of two primes, one first hits paydirt at p = 17, with

33	34	35
3 × 11	2 × 17	5 × 7

So I am (indeed) now 36 (a perfect square, someone insisted on pointing out). Others (I hope I didn't miss any) are:

p = 43 5 × 17 = 85 2 × 43 = 86 3 × 29 = 87
p = 47 3 × 31 = 93 2 × 47 = 94 5 × 19 = 95

So it looks like three by 100.

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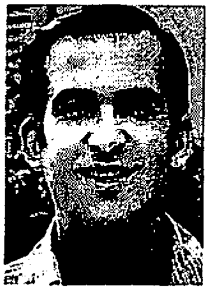
Puzzle Corner
Allan J. Gottlieb

How Many Ways to Play Bridge?



Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y., 10012.

Our son David had his first birthday a few weeks ago (March 14), and he continues to run all around our city apartment and country house. But this activity (as well as lots of others) has been somewhat hindered by the heavy rains this April; I conclude that New York will have a great many flowers this May (and that Mississippi will be awash in blooms).

The published version of **F/M 1** contained a serious misprint, and the (corrected) problem is reopened. Solutions to the revised problem will appear in November/December.

Problems

JUL 1 A bridge problem from Winslow Hartford: Given the hands shown at the top of the next column, South is to make six spades, with the opening lead from West ♥Q.

North:
♠J 9 8 7 2
♥5 4
♦A 5 4
♣A K 2

West:
♠6 5 4
♥Q J 10 9 8 3 2
♦9
♣9 6

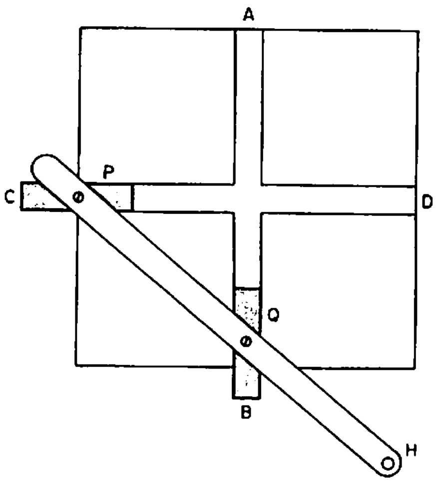
East:
♠—
♥7 6
♦Q J 10 8 7 6
♣Q J 10 8 7

South:
♠A K Q 10 3
♥A K
♦K 3 2
♣5 4 3

JUL 2. Although this problem from Jerry Grossman depends on the rules of bridge, I am classifying it as combinatorial: The number of ways the play of a hand of bridge can proceed obviously depends on the distribution and trump choice. Can some reasonably tight upper or lower bounds be found? How about an average case analysis?

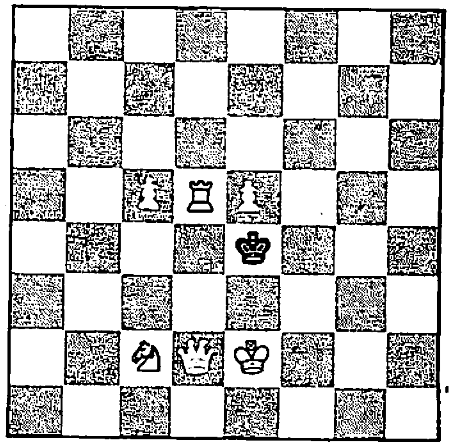
JUL 3. A pair of cryptarithmetic puzzles from Avi Ornstein: Nine is a square, and while NINETEEN isn't actually a prime, at least its smallest factor is greater than 150.
TWO + TWENTY = TWELVE + TEN
(The first three are all divisible by their namesakes.)

JUL 4. Anthony Stanton has a block of wood with slots AB and CD, as shown. In each slot is a wooden piece, P and Q, that can slide in it. Also shown is a bar attached by loose screws to P and Q and extended to a handle H. Is it possible to rotate the handle through a full 360 degrees without having P and Q collide? If this is possible, is the orbit of H an ellipse?



JUL 5. Our Japanese friend Nob Yoshigahara sent us the following list of the first 19 perfect squares containing two distinct digits and not containing a zero. What are the next two elements of the sequence?

$4 \times 4 = 16$
$5 \times 5 = 25$
$6 \times 6 = 36$
$7 \times 7 = 49$
$8 \times 8 = 64$
$9 \times 9 = 81$
$11 \times 11 = 121$
$12 \times 12 = 144$
$15 \times 15 = 225$
$21 \times 21 = 441$
$22 \times 22 = 484$
$26 \times 26 = 676$
$38 \times 38 = 1444$
$88 \times 88 = 7744$
$109 \times 109 = 11881$
$173 \times 173 = 29929$
$212 \times 212 = 44944$
$235 \times 235 = 55225$
$264 \times 264 = 69696$

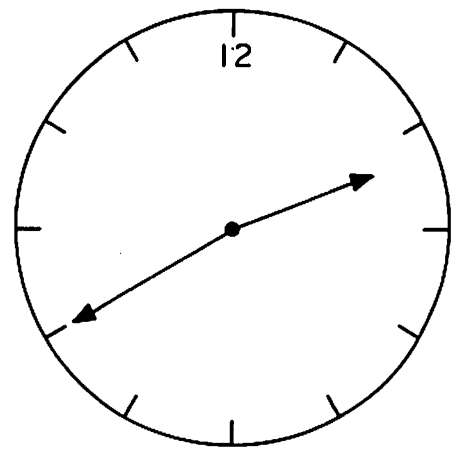


See problem F/M 1, next page

F/M 1. Elliot Roberts and Richard Hess noticed that no solution was possible. Inspecting the proposers' solution, I am able to deduce that the Black queen and pawns should be White. The problem is therefore reopened; White is to mate in three.

Speed Department

JUL SD1. Smith Turner writes: I am told that a clock (which I am not shown) is 1 hour and 20 minutes fast, that the minute and hour hands are interchanged, and that it is placed on the wall behind me. On the wall in front of me is a mirror I can't see, but I see the image on this mirror reflected from a plate glass cover on my desk top. The desk top image is shown below. What is the correct time?

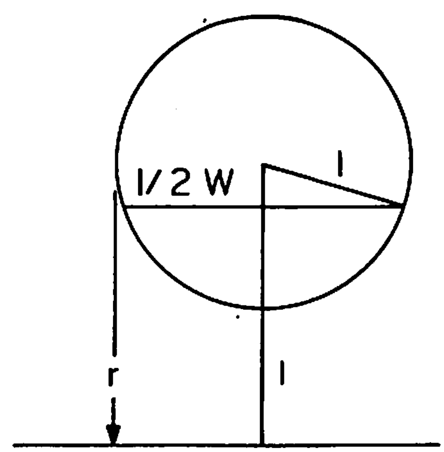


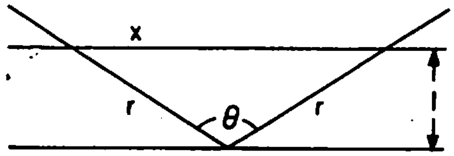
JUL SD2. You are given a three-pint jug and two five-pint jugs, all three empty, and an eight-pint jug full of wine. Divide the wine into two equal portions.

Solutions

F/M 2. Let T be the torus obtained by rotating the circle $(x - 2) + y = 1$ in the x-y plane about the y-axis. Let B be a plane tangent to the torus T at the point (0,0,1). Find the volume and surface area of the small region obtained by slicing T with P.

The following solution is from Matthew Fountain: The smaller part of the torus has a volume approximately 13.03 and a total surface approximately 33,029. The flat portion of the surface is exactly 8.





The volume of a figure of revolution is $\int \theta\, wr\, dr$, where w is the width parallel to the axis of the figure at distance r from the axis, and θ is the angle of revolution of that width. The width of the torus is $w = 2\sqrt{1 - (r-2)^2}$ and $\theta = 2 \sec^{-1} r$, making the smaller part of the torus contain a volume equal to

$$4\int_1^3 r\sqrt{1-(r-2)^2}\,\sec^{-1} r\, dr = 13.03$$

when evaluated by Simpson's rule. The flat surface is

$S_f = \int w\, dx$, where

$x = \sqrt{r^2 - 1}$ and $dx = r/[\sqrt{r^2-1}]\, dr$. Then

$$S_f = 4\int_1^3 r\sqrt{1-(r-2)^2}/[\sqrt{r^2-1}]\, dr$$

$$= 4\int_1^3 r\sqrt{3-r}/\sqrt{r+1}\, dr.$$

Substituting $t^2 = r + 1$ results in

$$S_f = 8\int_{\sqrt{2}}^{2} (t^2 - 1)\sqrt{4-t^2}\, dt = 2$$

$$\left| -t(4-t^2)^{3/2} \right|_{\sqrt{2}}^{2} = 8.$$

The curved surface is

$S_c = \int r\, \theta\, ds$, where

$$(ds)^2 = (dw/2)^2 + (dr)^2$$

$$= [(r-2)^2(dr)^2]/[1-(r-2)^2] + (dr)^2$$

$$= (dr)^2/[1-(r-2)^2]. \text{ Then}$$

$$S_c = 4\int_1^3 [r \sec^{-1} r]/[\sqrt{1-(r-2)^2}]dr.$$

The substitution of $r = 2 + \sin\phi$ converts this to

$$S_c = 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (s + \sin\phi)\sec^{-1}(2+\sin\phi)d\phi =$$

25.029 when evaluated by Simpson's rule.

F/M 3. Solve the simultaneous equations:

$x^6 y = (y^2 + 1)x^3$

$y^6 x = 9(x^2 + 1)y^3$

The following solution is from Charles Sutton: If we discount the trivial solution $x = 0$, $y = 0$, we may divide the equations by x^3 and y^3, respectively, to obtain

$$x^2(xy) = y^2 + 1$$
$$y^2(xy) = 9(x^2 + 1) \qquad (1)$$

Now make the change of variables

$$xy = u$$
$$y/x = v \qquad (2)$$

from which we obtain

$$x^2 = u/v$$
$$y^2 = uv \qquad (3)$$

and equations (2) become $u^2/v = uv + 1$ and $u^2v = 9(u + v)/v$ which, when cleared of fractions, give

$$u^2 = uv^2 + v \qquad (4)$$
$$u^2v^2 = 9u + 9v \qquad (5)$$

Multiplying the first equation by 9 and subtracting the second gives $9u^2 - u^2v^2 = 9uv^2 - 9u$ which, when solved for v^2, gives

$$v^2 = \frac{9u^2 + 9u}{u^2 + 9u} = \frac{9u + 9}{u + 9} \qquad (6)$$

From (4) we have $v = u^2 - uv^2 = u^2 - (9u^2 + 9u)/(u + 9) = (u^3 - 9u)/(u + 9)$ so $v^2 = (u^6 - 18u^4 + 81u^2)/(u + 9)^2$ and substituting this in (6) gives $(u^6 - 18u^4 + 81u^2)/(u + 9)^2 = (9u + 9)/(u + 9)$ which when cleared of fractions gives $u^6 - 18u^4 + 81u^2 = 9u^2 + 90u + 81$, or

$$u^6 - 18u^4 + 72u^2 - 90u - 81 = 0 \qquad (8)$$

We see from (1) and (2) that xy, and hence u, must be positive. Equation (7) is found to have one positive root, located between 3 and 4, and using a calculator, the value of this root is found to be u = 3.888908631. It follows from (3) that v must be positive and from (6) we find v = 1.847647602. Knowing u and v, we can calculate x^2 and y^2 from (3). Finally, we see from (2) that x and y must have the same sign. Thus the nontrivial solutions are

$$\begin{cases} x = 1.45078915 \\ y = 2.68054709 \end{cases} \quad \text{and} \quad \begin{cases} x = -1.45078915 \\ y = -2.68054709 \end{cases}$$

Reino Hakala wondered what MACSYMA (a noted computer software system for algebraic manipulation) would do on this problem. As it happens, Leo Harten did use MACSYMA on his solution, and he reports that after 45 CPU seconds, using its standard solver, MACSYMA found the three solutions given above and ten involving complex numbers. The proposer Norman Wickstrand notes that for a very similar (in appearance) set of equations

$(X^6 + 1)Y = (Y^2 + 1)X^3$

$(Y^6 + 1)X = 9(X^2 + 1)Y^3$

the solution is "solvable by radicals." Readers desiring a copy of Mr. Wickstrand's solution to this should write to the editor.

Also solved by Harry Zaremba, Emmet Duffy, Richard Hess, Matthew Fountain, Irving Hopkins, Frank Carbin, and Jordan Wouk.

F/M 4. A pilot flies south over a spherical earth a distance D, flies due east a distance D, and then flies due north a distance D, arriving back at precisely the starting point. For D equal to the radius of the earth, find all solutions for the starting latitude.

The following countably infinite number of solutions are from Avi Ornstein:

The most obvious solution would be to start at the North Pole. Flying D miles south, then due east, and then north would return to the North Pole. An infinite set of solutions are also possible, however. They would entail starting at a point, flying D miles south, then circling the world an integral number of times in D miles (thereby being at the same point), and then returning D miles north again to the starting point. The formula for the circumference of a circle drawn on the surface of the earth is 2 D sin (r/D), where D is the radius of the earth and r is the radius of the circle. Centering the circle at the South Pole, we can rearrange the equation to be r = D arcsin (1/2 n). The set of solutions would then be D + r, where n is set for the number of times the pilot would circle the world. The fact that the world is not actually a sphere means that slight modification would be required. Likewise, the length of a degree of latitude varies between the equator and the poles, but the error would not be too significant.

The first seven distances north of the South Pole happen to be 1.160D, 1.080D, 1.053D, 1.040D, 1.032D, 1.027D, and 1.023D. The answer is approximately $(1 + 0.160/n)D$, which should be accurate enough.

Also solved by David Evans, Phelps Meaker, Jordan Wouk, John Woolston, Harry Zaremba, Richard Hess, Matthew Fountain, Norman Wickstrand and Emmet Duffy.

F/M 5. A vertical plumb line is hung down the center of a one-mile-deep mine shaft in Borneo. A rock is dropped from next to the top of the plumb line. How far from the bottom of the plumb line will the rock land, if we ignore friction?

Our final solution is from John Prussing: Consider a coordinate system with its origin at the center of the earth. Because Borneo lies on the equator, let the x axis point out the mine shaft, the z axis point out the North Pole, and the y axis complete the right-handed frame. This frame rotates with the earth, and to someone looking down the shaft, the y axis points eastward. The absolute acceleration of the rock is $-ge_x$, where g is the (constant) gravitational acceleration and e_x is a unit vector along the x axis. The acceleration vector of the rock relative to the rotating frame a is given by

$a_r = -ge_x - w \times (w \times r) - 2x v_r$,

where w is the angular velocity vector of the earth, r is the position vector of the rock from the center of the earth, v_r is the velocity vector of the rock relative to the rotating frame, and $\times$ represents the cross product. From this equation the components of a_r are found to be

$d^2x/dt^2 = w^2x + 2w\,dy/dt - g$
$d^2y/dt^2 = w^2y - 2w\,dx/dt$
$d^2z/dt^2 = 0$

where w is the magnitude of the angular velocity of the earth and x, y, z are the components of the position r. The solution to these equations for initial conditions x(0) = earth radius = R, y(0) = z(0) = 0, and all initial components of velocity relative to the rotating frame = 0 is

$x(t) = (R - g/w^2)(\cos a + a \sin a) + g/w^2$
$y(t) = (R - g/w^2)(a \cos a - \sin a)$
$z(t) = 0$

where a = wt. The boundary condition at the final time T is x(T) = R − 1 miles. Using a value of earth radius R = 3960 miles, the value of a which satisfies the boundary condition is 1.320×10^{-3} and y(t) = 4.647 feet. Thus, the rock lands this distance east of the plumb line.

Also solved by Sidney Shapiro, David Evans, Ronald Raines, Bob Pease, Harry Zaremba, Emmet Duffy, Norman Wickstrand, Matthew Fountain, Richard Hess, John Woolston and the proposer, Bruce Calder.

Better Late Than Never

1982 OCT5. David Dreyfuss found a simpler solution.

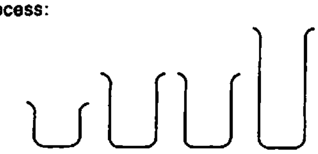
N/D 3. Sidney Shapiro has responded.

N/D 5. Sidney Shapiro has responded.

1983 JAN3. Rick Decker has responded.

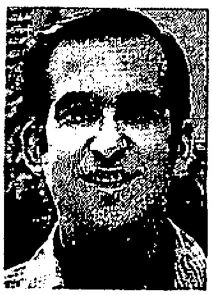
Proposer's Solution to Speed Problem

JUL SD2. The following diagram demonstrates the process:



	3	5	5 .	8
Initial	0	0	0	8
8-5	0	0	5	3
5-3	3	0	2	3
3-5	0	3	2	3
8-5	0	5	2	1
5-3	3	2	2	1
5-5	3	0	4	1
3-8	0	0	4	4

Now Math Is for Money



Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y., 10012.

A recent speed problem has provoked Albert Mullin to offer cash prizes for solutions to two related problems. Such individually funded prizes are not unprecedented; indeed, the famous mathematician Paul Erdos has been doing it for years. Mr. Mullin writes:

"John Linderman's speed problem M/J SD2 (*May/June, 1983, p. A14*) has other interesting aspects that may amuse the readers of 'Puzzle Corner.' Not only are products of two distinct primes connected in nontrivial ways with M.I.T.'s version of public-key cryptosystems (so-called RSA public-key cryptosystems), but the present year and the past two years in the ordinary calendar (yet another *three* consecutive numbers) are products of two distinct primes, too. Thus:

1981 = 7 × 283
1982 = 2 × 991
1983 = 3 × 661

Another such triple of years occurred during World War II:

1941 = 3 × 647
1942 = 2 × 971
1943 = 29 × 67

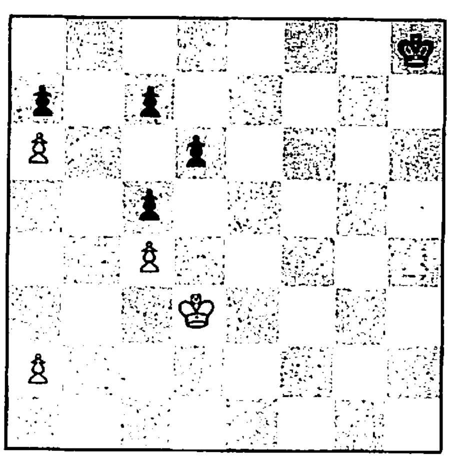
Surely such triples of consecutive numbers have a role in a deep study of the Cabala!

"I will donate $50 to the M.I.T. Alumni Fund for a proof that there exist infinitely many triples of three consecutive positive integers which are products of two distinct primes.

"Further, it can be shown that if a positive integer n is a product of two primes that differ by $d \geq 1$, then $\varphi(n)\sigma(n) = (n - d - 1)(n + d - 1)$, where φ is Euler's totient and σ is the sum-of-divisors function. I will donate $100 to the M.I.T. Alumni Fund for a proof that the *converse* holds for infinitely many $d \geq 1$."

Problems

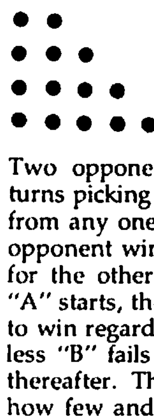
A/S 1 We begin with a chess problem from Bob Kimble:



White is to play and draw. At first it looks too easy: with material balanced and White possessing the only passed pawn, perhaps he should win—not just draw. But watch out for the Black central pawn mass.

A/S 2 Donald Richardson asks us a variation on a well known mathematical game, the "2-3-4-5 coin game." He writes:

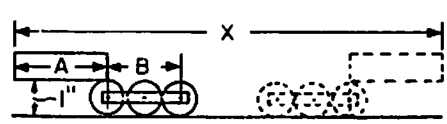
To play the game, 14 coins are arranged in four horizontal rows with 2, 3, 4, and 5 coins, respectively:



Two opponents, "A" and "B," take turns picking up any one or more coins from any one horizontal row until one opponent wins by leaving the last coin for the other opponent to pick up. If "A" starts, there will be no way for "A" to win regardless of his first move, unless "B" fails to make the right moves thereafter. The problem is to identify how few and what configurations "B" can leave for "A" on "B"'s first move (after any starting move by "A") so that "B" can win, regardless of any subsequent move by "A."

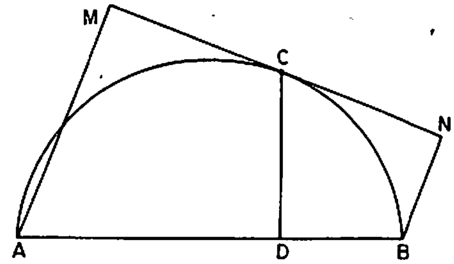
A/S 3 The following puzzle made its *Technology Review* debut in 1942 as part of an advertisement for Caldron Products, Inc.:

The diagram indicates schematically a little problem encountered in our shop a short time ago.



A member A moves on rollers, without slipping, from the solid-line position to that shown in dotted lines. What is the value of X in terms of the lengths A and B?

A/S 4 A geometry problem from Mary Lindenberg:



In the drawing, C is a point on the semicircle with AB as diameter. MN is tangent to the semicircle at C. AM and BN are perpendicular to MN, and CD is perpendicular to AB. Show that CD = CM = CN and that $CD^2 = (AM)(BN)$.

A/S 5 A numeric problem from Susan Henrichs:
Which integers X have the property that 9X is the same as X with the digits in reverse order? There is one other integer multiplier (besides 9 and trivially 1) that reverses digits for an infinite number of integers. What is this multiplier and what are the multiplicands?

Speed Department

A/S SD1 Art Delagrange proposes the following variation on **FM SD1**:
If a 10-by-30-by-1-foot flat-bottomed rectangular barge is in a lock and a 100-cubic-foot chest with a specific gravity of 4 is thrown overboard, what is the change in water level?

A/S SD2 We close with a bridge quickie from Doug Van Patter:

With the hands shown, you and your partner have reached a six-no-trump contract:

♠ A K 10
♥ A 7 3 2
♦ A Q J 9 8
♣ K

♠ Q J 5
♥ K 5 4
♦ 10 6 2
♣ A 9 6 4

The bidding: one diamond, two clubs (you), four no-trump, five diamonds (one ace), six no-trump. The ♠8 (opening lead) is taken with your ♠Q. You play the ♦10—East shows out! You take a second finesse and cash the ♣K—West shows out! Back to your hand with the ♥K. You cash the ♣A, discarding a heart from dummy. Now you finesse diamonds for the third time. What is the best shot at making this contract?

Solutions

APR 1 Find a bridge deal such that four spades, when played from *any* position, will be set at least seven tricks (i.e., the defense will make four spades).

L. Steffens liked this one because it was a problem that he could handle but the expert among his bridge buddies couldn't. The following solution is from Lawrence C. Kells:

I was fascinated to see that this is a retrograde bridge problem—that is, instead of presenting a hand and asking how to reach a particular result, you give the result and ask how to reconstruct the hand. I have doodled some on this kind of problem so I immediately tackled this one. I guessed that the key was that one player had the top spades and high cards in other suits to cash after drawing trumps if he were on lead, but that he would lose his high cards to a crossruff if he weren't. The solution then fell into place:

	♠5 4 3 2 ♥2 ♦5 4 3 2 ♣5 4 3 2	
♠J 10 9 ♥— ♦J 10 9 8 7 6 ♣A K Q J		♠8 7 6 ♥J 10 9 8 7 6 5 4 3 ♦— ♣10
	♠A K Q ♥A K Q ♦A K Q ♣9 8 7 6	

If North or South is declarer, a club lead allows West to take four tricks, after which the defense cross-ruffs six tricks in diamonds and hearts, while South follows and North cannot overruff. South gets only his three top spades. If East or West is declarer, a spade is led and South draws trump in three rounds. Then South can cash his six side-suit winners. North's remaining trump is the tenth trick for the defense.

Thus four spades from any position is defeated seven tricks.

L. Steffens and Stephen Canter noticed that slightly rearranging the diamonds (give North the ♦Q and ♦J, West the ♦3 and South the ♦2) enables North-South to set the contract one more trick.

Also solved by Matthew Fountain, Phillip Dangel, Doug Van Patter, and the proposer, Arthur Polansky.

APR 2 Shown below is a simple word square:

A C E
P A R
E R A

We could fill the plane with such word squares so that every cell of an infinite grid lay at the intersection of two words. However, such an arrangement would have two defects: (1) the pattern would be repetitive, and (2) the words would not all be interconnected. Show how these defects can be fixed; i.e., fill the plane with an infinite number of words so that the pattern does not repeat in any row or column, every cell lies at the intersection of two words; and every word is connected to every other one by a chain of intersecting words. Do not use any two-letter words.

Matthew Fountain gave a solution involving the group-theoretic definition, and Richard Hess submitted the following:

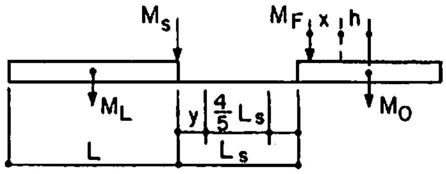
```
     n           k
___________ ___________
ATE...ATEASEATE...ATEASE
TE...ATE
E...ATE
ATE
TE
A
```

The above pattern repeated indefinitely with n and k varied randomly (between 1 and 10, say) will satisfy the conditions of the problem:

T: Always at the crossing of ATE and ATE.
A: At the crossing of EAT and EAT or TEASE and TEASE.
S: At the crossing of TEASE and TEASE.
E: At the crossing of TEA and TEA or TEASE and TEASE.

APR 3 One afternoon in a swamp Sneaky the snake, whose mass is M_s and length ℓ_s, was resting on the left end of a log of length L and mass M_L when he suddenly remembered that he was hungry. After a few moments, he found Freddy the frog sitting on a large disk-shaped leaf of radius R and mass M_0 h cm. away from the center of the disk. Initially the disk was 4/5 ℓ_s cm. away from the right end of the log. Sneaky started moving toward Freddy, and Freddy (who was ignorant of the laws of physics) started jumping for his life. However, when Freddy remembered that Sneaky had his best friend for breakfast that morning he was overcome by rage and desire for revenge, and he turned around to charge against Sneaky. Normally it would have been a dumb move, but the laws of physics reward the courageous: when Freddy stopped at a point on the disk and realized that his attempt was futile if not suicidal, the disk was just out of the reach of Sneaky, who was on the right end of the log by that time. Sneaky could not swim, so the good guy was saved and the bad guy had a hungry afternoon. Can you locate the spot where Freddy stopped? Assume all motions occurred on a line and the water offered no resistance. Both the log and disk have a uniform mass density.

The following solution is from Harry Zaremba:



In the drawing, assume M_F is the mass of the frog and let x, y, and z be the distances moved by the frog, log, and leaf, respectively, from their original positions. If the velocity of the snake along the log is $V_s = L/t$, the impulse forces between the snake and log will impart a velocity of $V_L = y/t$ to the combined mass of the snake and log. The distance y is in the opposite direction to the snake's motion. By conservation of momentum,

$M_sV_s = (M_s + M_L)V_L$

Substituting the velocities,

$M_sL = (M_s + M_L)y$, or

$y = M_sL/(M_s + M_L)$.

Applying the same principle to the frog-leaf system,

$M_Fx = (M_F + M_0)z$,

in which z is the distance moved by the combined mass of the frog and leaf in a direction opposite to x. Thus,

$z = M_Fx/(M_F + M_0)$.

From the figure,

$y + z + 4L_s/5 = L_s$.

Substituting y and z, the solution for x becomes

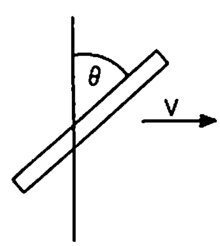
$x = (M_F + M_0)/M_F \cdot [L_s/5 - M_sL/(M_s + M_L)]$.

Also solved by Leo Harten, Harry Zaremba, Michael Jung, and the proposer, Ming Chung.

APR 4 Assuming the sudden onset of steady rainfall, will a person remain dryer by walking or running any given distance to shelter?

Richard Hess believes in running in the rain. He writes:

If you can orient your body at will it is best to run as fast as possible.



Model yourself as a long rectangular solid with end cross area A. If you can move at velocity v you should orient yourself at

$$\theta = \tan^{-1}\left(\frac{v}{v_{rain}}\right)$$

so as to only get the small end wet. The total water received is that in a volume

$V = Atv/\sin\theta$,

where t is the time spent in the rain. But

$t = d/v$

where d is the traveled distance; and

$V = Ad/\sin\theta$.

This becomes less as θ gets larger, and thus
$v = v_{rain} \tan \theta$
should be made as large as possible.

Matthew Fountain notes that with an umbrella it is best to go slow and notes that when he is stuck in the rain it's the inside surface of his glasses that gets wet. Although Fountain says his friends dispute this, I have the same experience. Judith Longyear observes that "*no* animal moves slowly through rain!"

Also solved by Harry Zaremba.

APR 5 Find positive integers a, b, and c such that $a^3 + b^4 = c^5$.

Although no one was able to prove that all solutions had been found, David and Nancy Leblanq (among others) found infinitely many. They write what they call "a computer scientist's solution": Assume a, b, and c are powers of 2. This transforms the problem into
$2^{3x} + 2^{4y} = 2^{5z}$.
(This limits the solution space.) 2^{3x} must equal 2^{4y} in order to sum to a power of 2 (in this case 2^{5z}). Thus $3x = 4y$.
$2^{3x} + 2^{4y} = 2^{3x} + 2^{3x} = 2^{3x+1} = 2^{5z}$
Thus $3x + 1 = 5z$. $3x = 4y$ for all q such that $x = 4q$ (and then $y = 3q$). $3x + 1 = 12q + 1 = 5z$, which has integer solutions for $q = 2, 7, 12, 17, \ldots$ This provides a set of solutions for $a^3 + b^4 = c^5$.

For $q = 2$, $x = 4q = 8$, $y = 3q = 6$, $z = \frac{12q + 1}{5} = 5$

$a = 2^x = 256$, $b = 2^y = 64$, $c = 2^z = 32$.

Also solved by Ronald Raines, Judith Longyear, Richard Hess, Harry Zaremba, Leo Harten, Irving Hopkins, Ruben Cohen, Anthony Beris, Sidney Shapiro, Dick Boyd, Charles Sutton, David Evans, and Robert Slater.

Better Late Than Never

NS 10 B. Laporte has some further partial results.

1981 OCT 4 Matthew Fountain found an exact solution:

$$\frac{\sqrt{2}}{4} - \frac{4}{3}\pi - \frac{5}{2}\cos^{-1}\left(\frac{\sqrt{6}}{3}\right) + 4\cos^{-1}\left(\frac{7}{9}\right)$$

This agrees to seven digits with Harry Zaremba's approximate solution. Copies of Mr. Fountain's derivation can be obtained from the editor.

1982 OCT 2. Matthew Fountain and an anonymous computer found the missing solution:

```
   E I G H T        8 4 5 0 2
   E I G H T        8 4 5 0 2
   S E V E N        6 8 3 8 7
   S E V E N        6 8 3 8 7
   S E V E N        6 8 3 8 7
 E L E V E N      8 1 8 3 8 7
 T W E L V E × 4  2 9 8 1 3 8 × 4
```

Proposers' Solutions to Speed Problems

SD1 None, as the barge had been overloaded by 33 percent and would already be on the bottom.

SD2 Since East has eight clubs, there's a good chance of only two hearts in the East hand. Cash the ♠A and ♠K; if East follows he can have only two hearts originally. Cash the ♥A and throw West in with the last heart, for an end-play in diamonds. If East has only two spades, hope that West has the high heart honor after the ♥A. This was hand 28 from a tournament at Cherry Hill, N.J., on August 21, 1982. The end position was:

```
          Dummy:
          ♠ A K
          ♥ A 7
          ♦ A Q
West:               East:
♠ 7 4               ♠ 9 6
♥ Q 10              ♥ J
♦ K 7               ♦ Q J 10
```

After the ♠A and ♠K and ♥A, West is end-played in hearts. He has to lead a diamond back at trick 12. Some declarers found this line of play; unfortunately, my partner wasn't one of them!

Puzzle Corner/Allan J. Gottlieb

Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10012.

Fannie Dooley Will Gefozzle You

Since this is the first issue of a new academic year, we once again review the ground rules for "Puzzle Corner":

In each issue we present five regular problems (the first of which is chess- or bridge-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later one submitted solution is printed for each problem; we also list other readers whose solutions were successful. In particular, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November (today is July 20), you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, solutions to this issue's "speed" problems are given below. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes we go back into history to republish problems which remained unsolved after their first appearance.

All problems come from readers, and all readers are invited to submit their favorites. I'll report on the size of the backlog, and on the criteria used in selecting ~~problems~~ solutions for publication, in a future issue.

Problems

OCT 1. We begin with a seven-card bridge problem from Emmet Duffy. South is on lead and is to take all seven tricks against the best defense:

North: ♠ K; ♥ 2; ♦ A 4 3 2; ♣ K

West: ♠ J 6 5; ♥ 3; ♦ 10; ♣ 8 3

East: ♠ 9 8 2; ♥ —; ♦ K Q; ♣ Q J

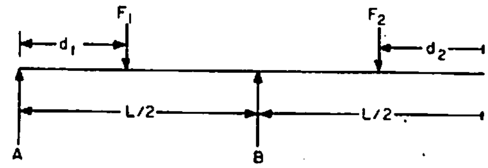
South: ♠ A 10 7; ♥ K; ♦ 5; ♣ A 10

OCT 2. The next (presumably serious) offering is from Robert Pease:

But of course, Fannie Dooley loves to Baffle people. And Gefozzle them, too. You see, Fannie loves vanilla, but not chocolate. She loves Allan J. Gottlieb but not Robert Pease. She likes doors but not windows. She likes Mississippi, but not Alabama, California, or New York. . . . Also, she loves Arrowroot cookies . . . but doesn't care for tapioca. I told a couple of my friends, Lenna Mooree and William Llewellyn, and *they* figured it out. Fannie also loves the Massachusetts Institute of Technology and Harvard College, but not Princeton University . . . she likes cherry-wood furniture but not mahogany. Now do you see why? It's a fun game for people of 5th-grade mentality . . . Of course, you have figured this out, but if you can't figure out why Fannie Dooley prefers all this hoop-la, to plain ordinary BLAH, ask a VOODOO doctor.

OCT 3. Benjamin Madero proposes a variation on **1982 N/D 3**; by writing "I think a more interesting problem would result if the loads and reactions on the beam were inverted, that is, two concentrated loads and three reactions." Then the problem would be:

A massless beam of length L is supported by three stanchions A, B, and C at the ends and midpoint. The beam is loaded with point loads F_1 and F_2 at distances d_1 and d_2 from the ends. What is the downward force on each stanchion?



OCT 4. Greg Schaffer wants you to show that for all $N \geq 3$

$$\sum_{k=0}^{N-1} \cos\left(\frac{4\pi k}{N}\right) = 0$$

OCT 5. John Woolson proposes six root extraction problems (the last of which he claims is not for the faint of heart). In all the problems each x is to be replaced with a digit (duplicates permitted), the numbers are base 10, no leading zeros are allowed, and no zeros are allowed in the roots themselves.

```
   xx                      xxx
√xx xx               ∛xxx xxx xxx
 xx                   xxx
   xx                   x xxx
   xx                   x xxx
                          xxx xxx
                          xxx xxx

    x xx                       x xxx
√xx xx xx            ∛xxx xxx xxx xxx
 xx                   xxx
   xx                   x xxx
   xx                   x xxx
    x xx                  xxx xxx
    x xx                  xxx xxx
                            xx xxx xxx
                            xx xxx xxx

      xx xx                          xxxx
√xx xx xx xx         ∜xxxx xxxx xxxx xxxx
 xx                   xxxx
   xx                   xx xxxx
   xx                   xx xxxx
    x xx                xx xxxx xxxx
    x xx                 x xxxx xxxx
      xx xx                xxxx xxxx xxxx
      xx xx                xxxx xxxx xxxx
```

Speed Department

OCT SD1. Smith D. Turner (also known as $\int dt$) asks what the following three numbers have in common:

1. 371 288 574 2 . . .
237. 581 208 759 3 . . .
3550. 260 181 586 5 . . .

OCT SD2. C. Baker has 5-, 11-, and 13-pint jugs, all empty, and a full 24-pint jug. How does he divide the liquid into three equal portions?

Alan O. Ramo
Michael J. Schaffer
Joel E. Schindall
Charles C. Schumacher
Peter T. Van Aken
Grant M. Wilson

1964
F. Michael Armstrong
Wayne F. B'Rells
Leslie M. Boring, Jr.
Richard A. Carpenter
Leonard Chess
Ernest M. Cohen
Ronald H. Cordover
John P. Downie
Michael B. Godfrey
John N. Hanson
Lester L. Hendrickson
Roger L. Hybels
Mark Joseph
Leon M. Kaatz
Brian R. Kashiwagi
Joseph F. Kasper, Jr.
Joseph L. Kirk
Glenn A. Larson
Donald S. Levy
Stephen B. Miller
Alton B. Otis, Jr.
Peter J. Sherwood
Jay M. Tenenbaum
Thomas C. Vicary

1965
Arnold R. Abrams
William R. Brody
Edward A. Bucher
Arthur A. Bushkin
W. David Carrier, III
L. Scot Duncan
Howard M. Ellis
Peter G. Gerstberger
John J. Golden, Jr.
George L. Hadley
Dawn Friedell Jacobs
William N. Kavesh
Louis A. Kleiman
Peter A. Klock
Alan C. Leslie
Steven B. Lipner
John A. Ottesen
Robert B. Reichelt
John D. C. Roach
Emile Sabga
Gregory L. Schaffer
Donald L. Shulman
Robert L. Silverstein
Douglas C. Spreng
Richard L. St. Peters
G. Wayne Thurman
Carol E. Van Aken
Michael G. Weiss
Ronald Wilensky
Stephen L. Williams
David L. Yuille

1966
Michael R. Adler
Arthur N. Boyars
Richard R. Brady
Paul A. Branstad
William L. Bunce
Richard Y. Chung
Richard T. Cockerill
Peter M. Cukor
Ralph M. Davison
Steven H. Disman
Logan L. Donnel
Bert E. Forbes
Victor K. Fung
Philip M. Jacobs
Michael D. Kinkead
Robert D. Large
Gerald F. Madea
Harry C. Moser
Henry H. Perritt, Jr.
Ralph G. Schmitt
Joseph I. Smullin
Joseph W. Sullivan, Jr.
Frank E. Surma, Jr.
William R. Tippett, II
John Torode

1967
John Acevedo
Donald A. Belfer
James W. Carter
William L. Caton, III
W. Thomas Compton
John M. Davis
David A. Dilling
Carl B. Doughty
Alan B. Hayes
Lutz P. A. Henckels
Edson C. Hendricks
Arthur C. Kwok
William E. Murray, Jr.
John S. Podolsky
John C. H. Reykjalin
Chet Sandberg
John M. Shufelt
Joel M. Steinberg
Arthur S. Warshaw

1968
Harvey Allen
Platte T. Amstutz, III
Paul F. Bente, III
William E. Carlson
Samuel A. Cohen
Claude L. Gerstle
Paul A. Gluck
Daniel M. Green
Peter Groot
John P. Linderman
Scott P. Marks, Jr.
Juan M. Meyer
Charles B. Miller, Jr.
Kenneth P. Morse
William M. Parks
Joel P. Robinson
Leonard H. Schrank
Jonathan D. Shane
Michael Weinreich

1969
Dariush Ashrafi
Melvyn P. Basan
William P. Bengen
Robert A. Berntson
Denis A. Bovin
Marc Davis
Bruce R. Donath
Paul D. Evans
Matthew M. Franckiewicz
Anthony George
Andrew C. Goldstein
Bruce L. Heflinger
Kenneth R. Horner
Joseph A. Horton
Bernard E. Klein
Aaron Kleiner
Alan K. Kudler
Carl W. Kuhnen, Jr.
Michael W. Laird
Samuel E. Polanco
R. Frank Quick, Jr.
Franklin P. Rogers
Christopher R. Ryan
Michael Sporer
Michael P. Timko
James P. Truitt, Jr.
Hal R. Varian
Hing Y. Watt

1970
Gregory K. Arenson
Karen Arenson
David S. Bann
Wendell C. Brase
Gerald L. Brodsky
James C. Bronfenbrenner
James L. Caldwell
Daniel R. Cherry
Eric K. Clemons
Stephen F. Cooper
Harry D. Feldman
Linda L. Furrow
Robert F. Gonsett
John C. Head III
C. Gordon Hunter
Richard W. Ihrie
William C. Michels
William B Parsons
David T. Patten
Anthony C. Picardi
Christopher L. Reedy
James B. Rothnie, Jr.
Stephen R. Takeuchi
Walter Yorsz

1971
Richard A. Aparo
R. A. Castro Alpizar
Thomas C. Kelly
Robert D. Marshall, Jr.
Robert N. Schulte
Laurence Storch
Charles W. Werner
Philip R. Widing

1972
Bradley C. Billetdeaux
John M. Bissell
Jack E. Cater
Leonidas P. Colakis
David A. Davis
Marshall B. Goldman
G. Paul Hendrickson, Jr.
Donald H. Layton
Hampton Pitts
James W. Roxlo
Gary J. Schuitema
Hikaru P. Shimura
John W. Taylor
Robert E. Zahler

1973
John R. Bertschy
John R. Gersh
Debra Judelson
Samer S. Khanachet
Patrick A. Marcotte
Stephen P. Miller
C. Timothy Ryan
Philip M. Sadler
J. Alexander Stevens
Cynthia Day Stratton
John W. Vander Meer, III

1974
James Richard Andrew
Charles Shadell Bruno
Ian Fisher
Roderick John Holland, Jr.
Niels Eric Mortensen
John Emery Plum
Gary David Raymond
Frank M. Sauk
Jay W. Van Dwingelen

1975
Harold M. Cook
Charles Fendrock
Henry G. Heck
David K. Hudson
Paul D. Husby
Mark A. Lysne
Robert W. Mann, Jr.
Richard J. McCarthy

1976
Janis Bestul Ossmann
Jeslie R. Chermak
Garret A. Davies
Jeffrey J. Held
Kelly P. McClellan
Juzer S. Mogri
George W. Todd, IV
James P. Wajda
Robert J. Winkler

1977
Paul J. Ackman
Charles B. Baltimore
Danny Bigio W.
Earl H. Bunker
Walter H. Goodwin
Brian G. R. Hughes
Charles G. Mogged, Jr.
Timothy F. Morison
Michael W. Sonnenfeldt
G. Scott Thompson

1978
Peter C. Coffee
Susan L. Kayton
Tapio L. Kuusinen
Barbara K. Ostrom

1979
Anne M. Michon

1980
Timothy M. Folster
Peter J. Francis

1981
Stuart L. Anderson
Thomas P. Garigan
William I. Ogilvie

1982
Robert D. Powell

Aeronautics and Astronautics
Carl Alexoff '56
Daniel H. Daley '46
Carlo N. De Gennaro '53
David W. Dove '71
James W. Harrill '64
Richard D. Linnell '48
John A. Long '33
Howard A. Magrath '38
James S. McDonnell, III '59
James S. Miller '61
Theodore H. H. Pian '48
John G. Ryan '60
George S. Schairer '35
Leroy P. Smith '49
A. Tobey Yu '46

Architecture
Frederick R. Bentel '50
Bill C. Booziotis '60
Thomas D. Cabot, III '82
C. Rosalie C. Carson '47
Robert P. Cooke '62
Yusing Y. S. Jung '62
Toufic E. Kadri '82
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Edwin Malloy, Jr. '45

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Parr A. Tate '53
Glenn R. Young '77

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Yoshito S. Yamaguchi '82

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Robert B. Anderson '66
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Robert H. Campbell '78
Steve Cenko '64
Franco Chiesa '82
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William R. De Long '60
Donald A. Dick '68
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Carmen Ferraioli '70
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Reinhard Frank '74
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Takashi Iwamura '78
Allan W. Johnstone '64
Joe C. Jones '57
Howard H. Kehrl '60
George Konkol '54
Robert L. Kuhn '80
Allan H. La Plante '78
Carroll M. Martenson '54
William C. Mercer '56
Douglas A. Milbury '73
L. William Miles '70
George W. Morgenthaler '70
Marlin P. Nelson '57
Rita A. O'Brien '77
Ronald W. O'Connor '71
P. Kim Packard '71
Shirley M. Picardi '81
John F. Prendiville, Jr. '62
R. Dewey Rinehart '56
Barry Rosenberg '67
J. Phillip Samper '73
Robert E. Sciffes '50
Robert C. Sharp '81
Nicholas E. Shaw '78
W. Howard Sidner '80
Mary Oakes Smith '77
James I. Spiegel '64
John H. Thacher, Jr. '42
Peter E. Viemeister '69
Sam R. Willcoxon '65
Hugh E. Witt '57
Willis S. Zeigler, Jr. '66

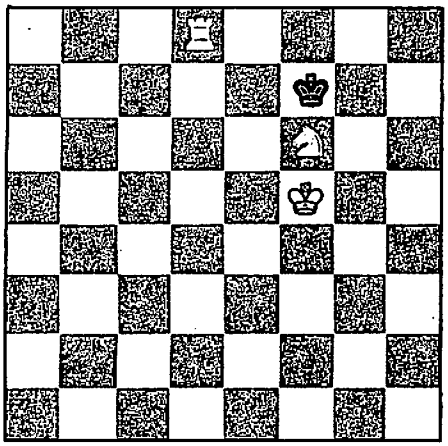
Undesignated
Philip A. Le Bar, Jr. '69
Richard S. McCurdy '70
Elizabeth J. Yeates '74

Urban Studies and Planning
Theodore S. Bacon, Jr. '56
Samuel M. Ellsworth '55
Allen G. Gerstenberger '74
Glynton J. Le Roux '80
Frederick H. Reuter '50
Barbara Z. Sedlin '60
Abbott L. Stillman '73

Honorary
Roy Lamson
James N. Phinney

Solutions

M/J 1. White to move and mate in four.



Elliot Roberts sent us the following solution:

1. N-g8 . . . K-g7	
2. N-h6 . . . if K-h7	2. . . . if KxN
3. K-f6 . . . KxN	3. R-d7 . . . K-h5
4. R-h8 mate	4. R-h8 mate

Also solved by Edward Gaillard, Richard Hess, Peter Hagelstein, Everett Leroy, Ronald Raines, David Evans, Matthew Fountain, and the proposer, John Cronin.

M/J 2. Find a nontrivial, continuous, real-valued function f(x) possessing continuous derivatives of all orders and satisfying the infinite-order differential equation:

$f = f' + 4f'' + 9f''' + 16f'''' + \ldots$

Paul Schweitzer writes:

A solution is

$$f(x) = Ae^{ax} \qquad (1)$$

where A is real and arbitrary, and where a is a real zero of the polynomial $g(x) = 1 - 4x + 2x^2 - x^3$, lying between -1 and 1. To see this, substitute (1) into the differential equation

$$0 = -f + Df + 4D^2f + 9D^3f + 16D^4f + \ldots \qquad (2)$$

to get

$$0 = Ae^{ax}\left[1 - \sum_{n=1}^{\infty} n^2 a^n\right]$$

(sum convergent for $|a| < 1$)

$$= Ae^{ax}\left[1 - \frac{a(1+a)}{(1-a)^3}\right]$$

$$= \frac{Ae^{ax}}{(1-a)^3}\, g(a).$$

Consistent if $g(a) = 0$. Since $g(.25) = 7/64 > 0$ and $g(.3) = -.047 < 0$, a lies between .25 and .3. Also, since $dg(x)/dx = 4(x-1) - 3x^2$ is ≤ 0 for $x \leq 1$, this is the only real choice of a: inside $[-1, 1]$ with $g(a) = 0$.

Also solved by Henry Lieberman, Leo Harten, Charles Sutton, John Prussing, Jerry Grossman, Stephen Persek, Richard Askey, Matthew Fountain, Peter Hagelstein, Richard Hess, and the proposer, Frank Rubin.

M/J 3. Two cryptarithmetic puzzles:

$$\frac{\text{EVER}}{\text{NNNNN}} = .\text{ONANDONANDON}\ldots$$

$$\frac{\text{EVER}}{\text{VVVVV}} = .\text{ONANDONANDON}\ldots = \frac{\text{IRIS}}{\text{IIIII}}$$

Robert Way found 7678/33333 as a solution to the first part and proved the second has no solution. The proposer offers 3638/66666 but does not com-

I	II	III
$m_a^2 = h_a^2 + \left(\frac{a}{2} - x\right)^2$	$x^2 = b^2 - h_b^2$	$(a - x)^2 = c^2 - h_a^2$
$m_b^2 = h_b^2 + \left(\frac{b}{2} - y\right)^2$	$y^2 = c^2 - h_b^2$	$(b - y)^2 = a^2 - h_b^2$
$m_c^2 = h_c^2 + \left(\frac{c}{2} - z\right)^2$	$z^2 = a^2 - h_c^2$	$(c - z)^2 = b^2 - h_c^2$

ment on IRIS/IIIII. Perhaps I have misinterpreted the problem.

Also solved by Richard Hess, Naomi Markovitz, and Matthew Fountain.

M/J 4. Find three consecutive three-digit prime numbers x, y, z, such that $z - y = y - x = 12$. In other words, there are no prime numbers between x and z, other than y. Find four consecutive three-digit prime numbers a, b, c, d, such that $d - c = c - b = b - a = 6$. What is the largest string of consecutive primes $P_1, P_2, \ldots, P_n$ such that: $P_n - P_{n-1} = P_{n-1} - P_{n-2} = \ldots = P_2 - P_1 = 4$.

Charles Sutton writes:

I found this problem particularly interesting, since the basic idea involved, consecutive primes in arithmetic progression, can be considered a generalization of the well-known twin prime problem. One would expect the distribution of pairs of primes differing by 4 to be similar to the distribution of the twin pairs, of which there is conjectured to be an infinite number. Are there also infinitely many sets of three or more consecutive primes in arithmetic progression with a given common difference? An intriguing question!

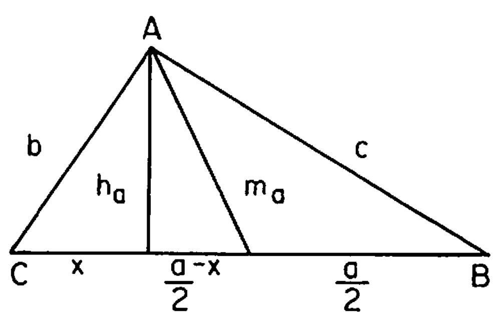
We are asked to find a set of three consecutive three-digit prime numbers in arithmetic progression with common difference 12. One such set is found to be 199, 211, 223. Again, we are asked to find a set of four consecutive three-digit prime numbers in arithmetic progression with common difference 6, and the set 251, 257, 263, 269 is found.

The last question asked is what is the longest string of consecutive prime numbers in arithmetic progression with common difference 4. The surprise answer is that the longest possible string consists of only two primes. Consider the first member of such a string. Since it is prime, its remainder when divided by 3 must be either 1 or 2. If the remainder is 1, then the next prime in the string (just add 4 to the first) will leave a remainder of 2 when divided by 3, but the third will be exactly divisible by 3, so it can't be prime. Thus the length of the string is two. And if the remainder when the first member of the string is divided by 3 is 2, then the string will have length one, since the next number (add 4) will be divisible by 3, so it can't be prime.

Also solved by Phelps Meaker, Frank Carbin, R. Way, Sidney Feldman, David Evans, Peter Hagelstein, Leo Harten, Richard Hess, Matthew Fountain, and the proposer, Larry Bell.

M/J 5. For any triangle, the sum of the squares of the medians to the sum of the squares of the sides equals a constant rational number. What is this number?

Here is what Farrel Powsner describes as "the most publishable solution":



In the diagram, we have triangle ABC with altitudes h_a, h_b, and h_c and medians m_a, m_b, and m_c to sides a, b, and c, respectively. (In order to simplify the diagram, m_a and h_a are the only segments interior to the triangle that are drawn.)

m_b and h_b separate $\overline{CA}$ into segments whose lengths are

$\frac{b}{2}$, $\frac{b}{2} - y$, and y.

m_c and h_c separate $\overline{AB}$ into segments whose lengths are

$\frac{c}{2}$, $\frac{c}{2} - z$, and z.

Using the Pythagorean Theorem gives the results shown in the box above.

Summing II and III and setting the results equal:

$x^2 + y^2 + z^2 = (a - x)^2 + (b - y)^2 + (c - z)^2$

$x^2 + y^2 + z^2 = (a^2 + b^2 + c^2) + (x^2 + y^2 + z^2) - 2(ax + by + cz)$

$2(ax + by + cz) = a^2 + b^2 + c^2$

$$ax + by + cz = \tfrac{1}{2}(a^2 + b^2 + c^2). \qquad (1)$$

Summing I and expanding:

$$m_a^2 + m_b^2 + m_c^2 = (h_a^2 + h_b^2 + h_c^2) + \left(\frac{a^2}{4} + \frac{b^2}{4} + \frac{c^2}{4}\right) + (x^2 + y^2 + z^2) - (ax + by + cz). \qquad (2)$$

Now, summing II:

$$x^2 + y^2 + z^2 = (a^2 + b^2 + c^2) - (h_a^2 + h_b^2 + h_c^2). \qquad (3)$$

Substituting (1) and (3) into (2):

$m_a^2 + m_b^2 + m_c^2 = a^2 + b^2 + c^2 + \tfrac{1}{4}(a^2 + b^2 + c^2) - \tfrac{1}{2}(a^2 + b^2 + c^2)$.

Therefore,

$m_a^2 + m_b^2 + m_c^2 = \tfrac{3}{4}(a^2 + b^2 + c^2)$. And

$$\frac{m_a^2 + m_b^2 + m_c^2}{a^2 + b^2 + c^2} = \tfrac{3}{4}.$$

Therefore, the ratio of the sum of the squares of the medians of any triangle to the sum of the squares of the sides is ¾.

Also solved by Mary Lindenberg, Jack Hiatt, John Prussing, Raymond Gaillard, Norman Spencer, Ken Haruta, Steve Shapiro, Frederick Furland, R. Way, Naomi Markovitz, Phelps Meaker, Sidney Feldman, Peter Hagelstein, David Evans, Henry Lieberman, Ronald Raines, Richard Hess, Matthew Fountain, and the proposer, Harry Zaremba.

Better Late Than Never

F/M 4, F/M 5 Raymond Gaillard has responded.

Proposers' Solution to Speed Problems

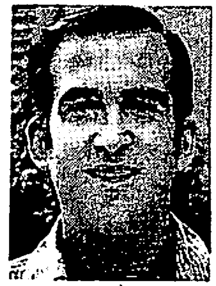
OCT SD1. All equal their logs except for position of decimal point [I didn't get it either—Ed.].

OCT SD2.

Initial	0	0	0	24
24-13	0	0	13	11
13-5	5	0	8	11
13-11	5	8	0	11
5-13	0	8	5	11
24-13	0	8	13	3
13-5	5	8	8	3
5-24	0	8	8	8

Edible Checkers and a Freshman's Folly

Puzzle Corner/Allan J. Gottlieb

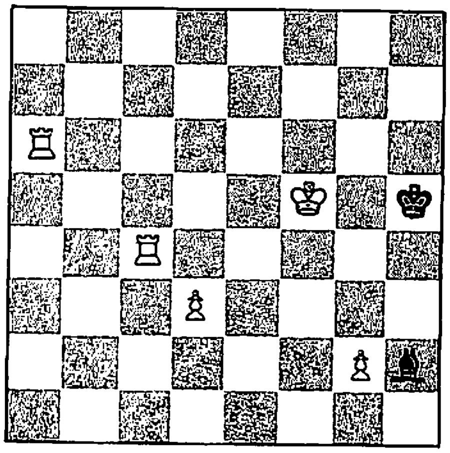


Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10012.

A lovely letter from Mary Lindenberg reports that the M.I.T. and Hunter College alumni associations each met last June 10 at M.I.T.—and she was part of both events. Ms. Lindenberg has an unusual double career; she is a teacher of mathematics and also of painting. She has recently sent us a reprint of her article on originality and creativity—topics germane to both disciplines.

Problems

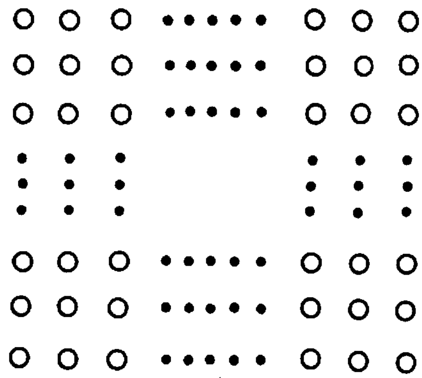
N/D 1 We begin with a chess problem from George Farnell:



White is to play and mate in three.

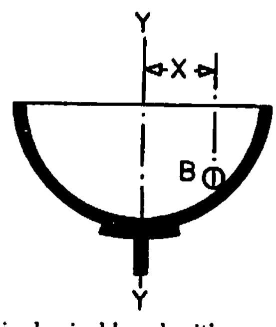
N/D 2 Mitchell Serota recalls that cancelling d's to give
dy/dx = y/x
is known as "freshman's folly." But similar jokes occasionally work for arithmetic:
64/16 = 4/1
326/163 = 2/1
Can anyone find a four-digit counterpart?

N/D3 Niles Ritter and Andrew Bernoff first sent this problem to M^3, the M.I.T. math majors' magazine (why not call it M^4?):
At the Mathematics Department tea, I had a collection of equal numbers of red and black checkers, and when the conversation grew dull I discovered that my checkers could be arranged in a rectangular array, with all the black ones on the border, like this:



But when my back was turned, one of the professors ate a few of each color, thinking my checkers to be cookies. I found that I still had an equal number of black and red checkers, and that I could still arrange them in an array with the reds inside and the blacks on the border. How many checkers did I start with, and how many were eaten?

N/D 4 This problem, described as "a curiosity from the field of speed indication," first appeared in *Technology Review* in 1938 as part of an advertisment for Calibron Products, Inc."



A hemispherical bowl with a radius of 1 foot 1 inch is mounted on a central vertical shaft YY. A one-pound ball B with a radius of 1 inch is free to roll inside the bowl. On what part of the bowl's surface will the ball tend to ride (i.e., what will be the value of x) if the bowl is spun at 50 R.P.M. about the YY axis? How do taught physics at Phillips Exeter Academy, and later he was a member of the faculty at Wayne University. His work with PSSC led to a number of books by members of the project and to several films, including "An Introduction to Optics" which is still used as a teaching aid.

Deceased

John Hall, '04; July 13, 1983; 3838 Halifax Rd., Wilmington, N.C.
Maurice H. Pease, '07; September 10, 1983; 5093 Starfish Ave., Naples, Fla.
John S. Barnes, '08; 1980; 18 Woodland Rd., Lawrenceville, N.J.
Mrs. Earl W. Pilling, '10; 1983; 767 Washington St., Norwood, Mass.
Lloyd C. Cooley, '11; October 2, 1982; Plymouth Harbor Apt. 2205, 700 John Ringling Blvd., Sarasota, Fla.
Clarence R. Woodward, '12; February 24, 1983; 905 Country Club Dr., Greensburg, Penn.
Louis C. Rosenberg, '13; June 9, 1983; c/o Mt. View Convalescent Circle, 1400 Division St., Oregon City, Ore.
Raymond A. Meader, '17; February 8, 1980; 16522 Burr Hill, c/o D F Malder, San Antonio, Tex.
James M. Todd, '18; March 23, 1983; 1489 Clairmont Pl., Nashville, Tenn.
Dean K. Webster, Jr., '19; July 29, 1983; 27 Royal Crest Dr., Lawrence, Mass.
Mendum B. Littlefield, '20; July 4, 1983; Littleton House, Littleton, Mass.
Weston Hadden, '21; June 14, 1983; 22 Monument Ave., Bennington, Vt.
Harry M. Ramsay, '21; May 12, 1983; 11 E Orange Grove Rd. No. 116, Tucson, Ariz.
Edwin L. Rose, '21; July 7, 1983; PO Box 116, Sierra Madre, Calif.
Haywood P. Cavlarly, Jr., '22; June 25, 1983; 1615 North Oleander Ave., Datona Beach, Fla.
William J. Edmonds, '22; July 1, 1983; 2701 Gulf Shore Blvd. N., Naples, Fla.
Morris H. Gens, '22; April 29, 1983; 75 Lee St., Brookline, Mass.
Charles T. McGrady, '22; May 15, 1983.
John J. Breen, '23; June 6, 1974; 174 Summit Ave., Summit, N.J.
Mrs. Kenneth G. Crompton, '23; 1983; 407 Prospect St., Lawrence, Mass.
Kenneth C. Kingsley, '23; June 9, 1983; 649 Via Lido Soud, Newport Beach, Calif.
Theodore M. Burkholder, '24; August 4, 1983; 60 Summit St., Newton, Mass.
Charles E. Herrstrom, '24; September 1, 1983; 700 John Ringling Blvd. No. 905, Sarasota, Fla.
Jacob A. Manian, '24; April 10, 1983; 39 Acorn Circle Apt. 202, Towson, Md.
Samuel Glaser, '25; August 7, 1983; 381 Dudley Rd., New Centre, Mass.
Samuel J. Cole, '26; May 10, 1983; 11 Wilde Ave. Apt. 2, Drexel Hill, Penn.
George J. Leness, '26; August 17, 1983; 31 E 79th St., New York, N.Y.
Lucas E. Bannon, '27; March 29, 1983; 19 No. Columbus St., Beverly Hill, Fla.
Francis T. Cahill, '27; June 29, 1983; McKoy Rd., North Eastham, Mass.
Richard Cutts, Jr., '27; March 3, 1983; 305 Greenwich Ave. Apt. 130B, Warwick, R.I.
Frank G. Kear, '27; July 22, 1983; 501 Portola Rd. No. 8085, Menlo Park, Calif.
Mrs. Thorwald Larson, '28; October 2, 1982; 1535 Pine Valley Blvd. No. 110, Ann Arbor, Mich.
Robert S. Woodbury, '28; September 18, 1983; 12 Meadowbrook Rd., Dover, Mass.
Russell B. Wright, '28; October 14, 1982; c/o Mrs. Henry Giugni, 1518 Adams St., Saint Helena, Calif.
J. Gordon Carr, '29; August 10, 1983; 46 Beechcroft Rd., Greenwich, Conn.
Daniel J. O'Connell, '29; July 14, 1983; 50 Elliot St., Holyoke, Mass.
H. Dayton Wilde, '29; July 20, 1983; 3013 Avalon Pl., Houston, Tex.
Denis R. Agar, '30; August 2, 1983; 2625 Regina St. No. 1706, Ottawa, Ont., Canada.
William Harold Bethel, '30; August 28, 1982; Hallmark Nursing Home, 49 Marvin Ave., Troy, N.Y.
Edward H. Clouser, '31; September 1956; 308 Druid Rd., Clearwater, Fla.
Eliot S. Graham, '31; August 1, 1983; 2331A Avenida Sevilla, Laguna Hills, Calif.
Michael Kundrath, '31; August 6, 1983; 144 Red Oak Rd., Fairfield, Conn.
Edwyn A. Eddy, '32; September 11, 1983; RFD 1, Winsted, Conn.
John W. Leslie, '32; June 29, 1983; 42 Whitney St., Medford, Mass.
Dominic A. Perry, '32; July 7, 1983; 533 Paddock Ave., Meriden, Conn.
Joseph L. Richmond, '32; 1972.
Robert M. Trimble, '33; March 1983.
George R. Forsburg, '35; August 7, 1983; 78 Clisby Ave., Dedham, Mass.
John A. Kleinhans, '36; September 13, 1983; 3064 Kent Rd. Apt. 411A, Cuyahoga Falls, Ohio.
Leo F. McKenney, '36; August 19, 1983; Dogford Rd., Etna, N.H.
Morril B. Spaulding, Jr., '36; 1980; 4341 Montgomery Ave., Bethesda, Md.
Harrison S. Woodman, '36; June 28, 1983; 53 Buena Vista Ave., Rumson, N.J.
Willard R. Beye, '38; September 3, 1983; 4414 Marseilles St., San Diego, Calif.
Francis S. Stein, '38; June 17, 1983; 5 Brooke Ave., Annapolis, Md.
Don Cornish, '39; 1981; 17846 Ballinger Way NE, Seattle, Wash.
Akim S. Zaburunov, '39; June 10, 1983; PO Box 1703, Fort Collins, Col.
Oliver K. Smith, '40; August 11, 1983; 1025 Las Pulgas Rd., Pacific Palisades, Calif.
Merlen C. Bullock, '42; September 17, 1983; 2007 North Medina Line Rd., Akron, Ohio.
Louis V. Sutton, Jr., '42; July 15, 1983; PO Box 3085, Rock Hill, S.C.
Robert A. Bamford, '43; July 2, 1983; 20 Rene Rd., Brockton, Mass.
Frank W. Bailey, '46; 1982; 60 Parkway Dr. E., East Orange, N.J.
Stuart D. Grandfield, '46; August 23, 1983; 328 Barranca Ave. No. 2, Santa Barbara, Calif.
William W. Caudill, '47; June 25, 1983; 10923 Kirwick, Houston, Tex.
Boynton H. Tucker, '49; March 1983; RT 25H Box 34A, Malakoff, Tex.
Arthur H. Schein, '51; September 14, 1983; 22 Puritan Rd. Newton Highlands, Mass.
Walter J. K. Tannenberg, '52; July 12, 1983; 1 Longfellow Pl., Boston, Mass.
Barton Roessler, '55; June 16, 1983; 4 Indigo Rd., Barrington, R.I.
Philip T. Andrews, '57; March 25, 1983; 20 Gunderson Rd., Wilmington, Mass.
William H. Coghill, '57; December 24, 1982; 1111 5th Ave., Sebring, Fla.
Miguel A. Barasorda, '59; December 30, 1981; Orquidea 12, Sta Maria, Rio Piedras, Puerto Rico.
Sylvia C. Bluhm, '59; December 16, 1981; 1253 Cambridge Ave., Morgantown, W.V.
Robert R. Thompson, '59; November 1978.
C. Morgan Harris, '62; August 1977.
Robert K. Bofah, '71; 1983; PO Box 10, Goaso, Ghana.
Harry Boothman, '73; 1977; City of Calgary, Parks and Recreation Dept., Calgary, Alt., Canada.
Robert U. Sautter, '74; May 7, 1982; 54736 Merrifield Dr., Mishawaka, Ind.
Christopher Lawlor, '75; August 25, 1982; 9285 Root Rd., North Ridgeville, Ohio.
William G. McCabe, '77; May 16, 1983; 20 Hillside Ave., Haverstraw, N.Y.
Stephen D. Holland, '79; September 11, 1983; Baker Bridge Rd., Lincoln, Mass.
Stephen M. Paneitz, '80; September 1, 1983; 8 Goodman Rd., Cambridge, Mass.
Sudhir K. Sarin, '83; August 7, 1983; 80 Ranleigh Ave., Toronto, Ont., Canada.

you explain this peculiar result? Show that x will increase to about 7 inches at a speed of 60 R.P.M.

N/D 5 John Rule wants you to find two five-digit perfect squares that together contain all ten digits. How many solutions exist?

Speed Department

N/D SD 1 A bridge quickie from Doug Van Patter:
You (playing East) are defending against a six-heart contract in a high-stakes game:

North (dummy):
♠ 9 6 2
♥ K 4 3
♦ A 8
♣ A 9 7 6 2

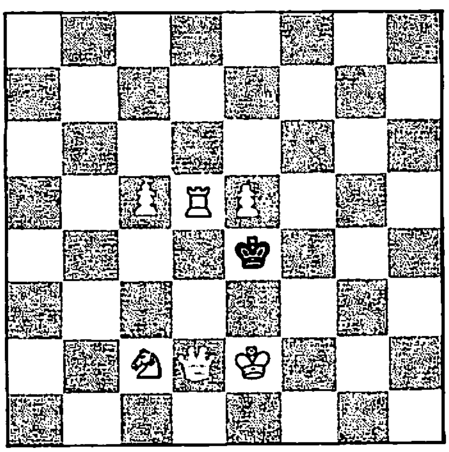
East:
♠ K Q J 8 3
♥ J
♦ K 10 5
♣ K 8 4 3

Your partner leads the ♣5, taken by the ♣A. Declarer pulls three rounds of trump, then leads the ♣Q to your ♣K. Your partner shows out. Can you find a way to save a lot of money?

N/D SD 2 David Evans has drawn a diagonal on each of two faces of a cube so that the diagonals share a vertex. How large is the angle between the diagonals?

Solutions

F/M 1 White to mate in three:



There are several solutions to this problem, which was first published in February/March 1983 and corrected in July 1983. Elliot Robert found the following:

1. R d 6 K x P
2. K f 3 K f S
3. Q f 4

1. K f S
2. K f 3 K x P
3. Q f 4

Also solved by E. Stout, David Evans, Robert Way, Matthew Fountain, Charles Rivers, Randy Kimble, and the proposers, Bob Kimble and Jacques Labelle.

JUL 1 South is to make six spades, with the opening lead from West ♥Q.

North:
♠ J 9 8 7 2
♥ 5 4
♦ A 5 4
♣ A K 2

West:
♠ 6 5 4
♥ Q J 10 9 8 3 2
♦ 9
♣ 9 6

East:
♠ —
♥ 7 6
♦ Q J 10 8 7 6
♣ Q J 10 8 7

South:
♠ A K Q 10 3
♥ A K
♦ K 3 2
♣ 5 4

We received a fine analysis from Robert Way: The experienced bridge player will be disappointed that the spade trumps are divided five-and-five in the declarer's hand and dummy, providing no extra ruffing values. On the other hand, he will note that an "automatic" squeeze is available in clubs and diamonds against East once West has been stripped of these suits and a trick has been conceded to "rectify the count." The latter can be accomplished by losing a trump trick to West after stripping clubs and diamonds. The trick comes back via a "ruff and discard" return and sets up the ultimate "automatic squeeze" against East. In more mundane terms, the simplest play follows. Although the opening lead is specified as the ♥Q, the hand may be made against any opening lead, which declarer wins by following suit with the ♥A, ♠A, ♦K, or ♣A. South must first win these tricks plus the ♠K, ♥K, and ♣K. For these first seven tricks, both the order of play and the cards from the other three hands are not critical, as long as they follow suit. On the eighth lead, South plays the ♠3, following with the ♠2 from North, forcing West to win with a higher spade. West, having been stripped of all other suits, must return a heart which North trumps, and South discards a diamond so as to retain a club as the potential squeeze card. (Note that West, having to return a heart on the ninth trick, has promoted a spade winner in the North hand which otherwise would have fallen under a spade winner in the South hand due to the five-five even distribution, thus regaining the trick lost at trick eight and also delaying the tempo of the game so that East is subsequently squeezed.) North now leads his last spade, which is taken by South's ♠Q; followed by the ♥Q on which North discards a club. The position after 11 tricks have been played: West holds no diamonds or clubs, North holds the ♦A and a small diamond, South holds a small diamond and a small club (!), and East can have only two cards, too. If both are diamonds, South plays his good club and to North's ♦A; if East retains a club, he can hold only one diamond, and after the play of North's ♦A the small diamond is good. Note the problem if the Declarer plays the ♦A (instead of the ♦K) among the first seven tricks: the squeeze position is then transferred to North who must win the last trump trick. This requires Declarer to save the ♠J in the dummy. On the forced heart return at the ninth trick North discards a diamond and South trumps with the ♠Q in order to "unblock" the spade suit. The ♠10 is then led to the North's ♠J and the last spade cashed, discarding a club from South. After 11 tricks, North holds a small diamond and small club, while South retains the ♦K and a small diamond. Likewise, East, reduced to two cards, cannot protect both diamonds and clubs. This line of play is forced if the North-South clubs are interchanged and the East-West clubs are swapped for diamonds. Then declarer must play both the ♦A and ♦K plus a high club to strip West, requiring the final squeeze to be initiated by North.

Also solved by Garabed Zaratarian, Robert Lax, Matthew Fountain, Kenneth Bernstein, Peter De Florez, Conrad Carlson, Doug Van Patter, Tom Harriman, Ben Feinswog, Eric Liban, Charles Rivers, and the proposer, Winslow Hartford.

JUL 2 The number of ways the play of a hand of bridge can proceed obviously depends on the distribution and the choice of trump. Can some reasonably tight upper or lower bounds be found? How about an average case analysis?

Matthew Fountain points out an interesting ambiguity. Consider a hand in which each hand consists of 13 cards of the same suit. For any contract, each player may legally play his 13 cards in any order. Thus we have $13!^4$ possibilities, clearly an upper bound. However, in some sense all these ways of playing the hand are equivalent, so perhaps this deal should be considered an obvious *lower* bound with only *one* possibility. However, since this is a combinatorial and not a bridge problem, we will not try to define equivalent hands. Thus we have an upper bound.

Winslow Hartford has an idea for a lower bound. The lower limit is for a 4-3-3-3 hand, all around, in which one player holds say ♠AKQJ ♥AKQ ♦AKQ ♣AKQ. This will give the fewest choices of play as the winning hand is randomly played out. Thus there are only 432 ways of playing the first trick, as opposed to 28,561 for the case above. This is instictive, but it should be about correct.

Also solved by Robert Way, Tom Harriman, and the proposer, Jerry Grossman.

JUL 3 Two cryptarithmetic puzzles: Nine is a square, and while NINETEEN isn't actually a prime, at least its smallest factor is greater than 150. TWO + TWENTY = TWELVE + TEN. (The first three are all divisible by their namesakes.)

The following solution is from Charles Rivers: NINE is a square and NINETEEN has no prime factor smaller than 150. Therefore N is clearly an odd number. Of the four-digit squares (32^2 through 99^2), only four have equal first and third digits, and of these, only one ($56^2 = 3136$) has equal first and third *odd* digits. Therefore:
NINE = 3136
NINETEEN = 3136X663, where X = 0,2,4,5,7,8,9
Substitution of each possibility in turn and factoring gives the following results:

X =	0	2	4	5	7	8	9
Smallest factor of NINETEEN =	89	3	11	3	617	3	13

Therefore, NINETEEN = 31367663 with factors of 617, 50839.

The second part of the problem is somewhat more difficult:
1. Since TWO is divisible by 2, O is an even number.
2. Since TWENTY is divisible by 20, Y=0 and T is an even number.
3. Since TWELVE is divisible by 12 (and all its factors including 4), E is an even number with the further restriction that if E=0,4,8, V is even, and if E=2,6, V is odd.
4. O + Y = O, an even number with no carryover. Therefore E + N equals an even number, and since E is even, N is also even.
5. All even numbers are accounted for (Y=0,T,O,E,N) so L,V,W are odd and E=2 or 6 (see 3 above).
6. From 4 above, T+W is odd (no carryover). Therefore E + N must be less than 10 (no carryover) so that E + V + carryover (if any) is odd. Therefore: E + N = O and O = 6 or 8; E = 2 or 6:

Y	O	E	N	T
0	6	2	4	8
0	8	2	6	4
0	8	6	2	4

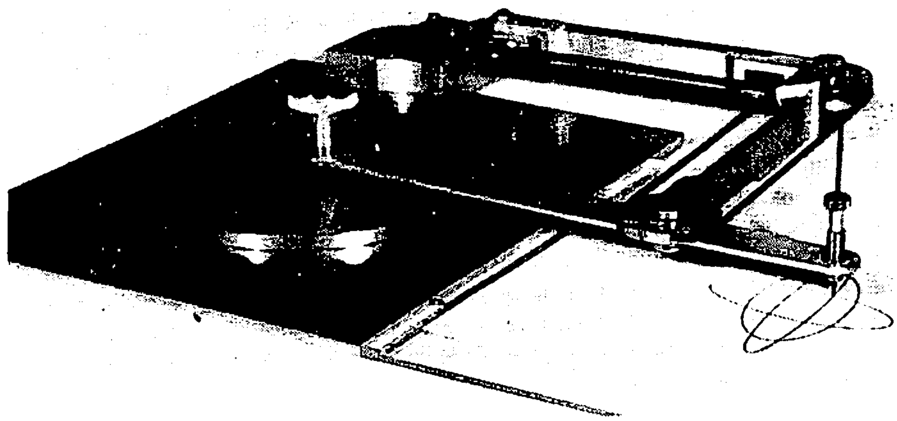
7. From all this, TW + NT = LV + TE and L = N + 1.
8. Rearranging and solving for W - V:
10T + W + 10N + T = 10L + V + 10T + E
W - V = 10(L-N) + E - T
9. Therefore

Y	O	E	N	T	L	W-V	W	V	TWELVE
0	6	2	4	8	5	4	7	3	872532 − 12 = 72711
0	8	2	6	4	7	8	9	1	492712 − 12 = 41059.333
0	8	6	2	4	1	−8	1	9	W ≠ L

10. The solution is:

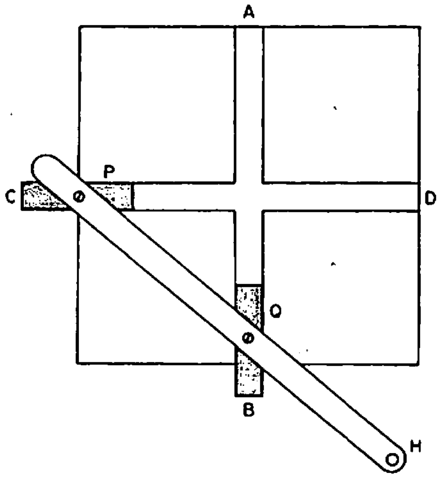
$$\begin{array}{r} 876 \\ 872480 \\ \hline 873356 \end{array} = \begin{array}{r} 872532 \\ 824 \\ \hline 873356 \end{array}$$

Also solved by Matthew Fountain, David Evans, Kenneth Bernstein, Fred Furland, Carlyn Iuzzolino, Winslow Hartford, Robert Way, Sidney Feldman, John Spalding, W. Woods, Harry Zaremba, Frank Davis, John Rule, and the proposer, Avi Ornstein.

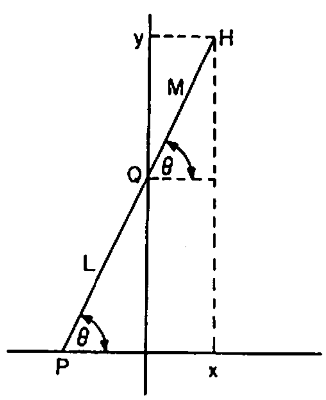


When Anthony Stanton asked "Puzzle Corner" readers (JUL 4) about the geometry of a slotted block with travelers in the slots, Martin Buerger, emeritus professor of mineralogy and crystallography at M.I.T., realized that Stanton's device was essentially equivalent to this machine—Buerger's invention—for drawing ellipses. Buerger's description of the machine is available on request from the editors.

JUL 4 A block of wood has slots AB and CD, as shown. In each slot is a wood piece, P and Q, that can slide in it. Also shown is a bar attached by loose screws to P and Q and extended to a handle H. Is it possible to rotate the handle through a full 360° without having P and Q collide? If this is possible, is the orbit of H an elipse?



The blocks cannot collide since they must remain a distance PQ apart. Kelly Woods shows us that the orbit of H is indeed an ellipse:



Let the distance PQ be designated by L and the distance QH by M. Then, as shown in the diagram,

$y = (M + L) \sin \theta$
$x = M \cos \theta$
The equation of an ellipse is:
$(x/a)^2 + (y/b)^2 = 1$
In this case the minor semi-axis is $a = M$ and the major semi-axis is $b = (M + L)$, from which:
$\cos^2 \theta + \sin^2 \theta = 1.$

Martin Buerger, professor emeritus at M.I.T., constructed such a device in the 1940s, a photograph of which appears above. Professor Buerger has supplied a written description of the machine, which can be obtained from the editor upon request. And Norman Wickstrand notes that Keuffel and Esser once sold such an instrument.

Also solved by Kenneth Bernstein, Matthew Fountain, Tom Harriman, Fred Furland, Eric Liban, Winslow Hartford, Waller Moore, Gary Heiligman, John Prussing, Phelps Meaker, Harry Zaremba, Frank Davis, and Robert Way.

JUL 5 The following list is the first 19 perfect squares containing two distinct digits and not containing a zero. What are the next two elements of the sequence:

$4 \times 4 = 16$	$22 \times 22 = 484$
$5 \times 5 = 25$	$26 \times 26 = 676$
$6 \times 6 = 36$	$38 \times 38 = 1444$
$7 \times 7 = 49$	$88 \times 88 = 7744$
$8 \times 8 = 64$	$109 \times 109 = 11881$
$9 \times 9 = 81$	$173 \times 173 = 29929$
$11 \times 11 = 121$	$212 \times 212 = 44944$
$12 \times 12 = 144$	$235 \times 235 = 55225$
$15 \times 15 = 225$	$264 \times 264 = 69696$
$21 \times 21 = 441$	

Carlyn Iuzzolino found:
$3114^2 = 9{,}696{,}996$
$81619^2 = 6{,}661{,}661{,}161$

Also solved by Matthew Fountain, David Evans, Kenneth Bernstein, Robert Way, E. Stout, Winslow Hartford, Italo Servi, Frank Carbin, Tom Harriman, and the proposer, Nob Yoshigahara.

Better Late Than Never

M/J2 Garabed Zartarian has responded.
M/J5 Arvi Ornstein has responded.

Proposers' Solutions to Speed Problems

N/D SD1 Put the ♦K on the table. This may lose a diamond trick, but it guarantees that the Declarer won't be able to pitch losers on Dummy's last two clubs. (Declarer's ♣J is still blocking the suit.)

N/D SD2 60°. A third diagonal can be drawn forming an equilateral triangle.