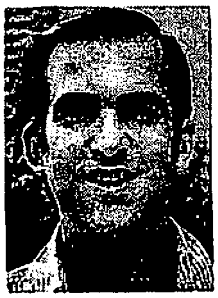
Puzzle Corner
Allan J. Gottlieb

Table for Seven, Please



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Let me once again answer a perennial question: What criteria are used to select solutions for publication?

As responses arrive during the month, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column, I first weed out erroneous and illegible solutions. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor contributions from correspondents whose solutions have not previously appeared, as well as solutions that are neatly written or typed, since the latter produce fewer typesetting errors.

Problems

M/A1 Frank Model wants to know if the following contract can be made against any defense:

West	North	East
	♠7,3,2	
	♥5,4,2	
	♦A,10,9,7,6	
♠K,6,4	♣3,2	♠8
♥K,J,7,6		♥9,8
♦K,J		♦8,5,4,2
♣K,Q,J,5		♣10,9,8,7,6,4
	♠A,Q,J,10,9,5	
	♥A,Q,10,3	
	♦Q,3	
	♣A	

The bidding:

South:	*West:*	*North:*	*East:*
1 spade	double	2 dmds.	—
3 spades	—	4 spades	5 clubs
5 spades	double	—	—
—			

M/A2 Arun Trikha has 39 balls of which 38 are identical in weight, but the 39th one is either heavier or lighter than the others. He needs to devise a method to isolate the "odd" ball with only four weighings using a balancing scale. The weighings should also establish if the odd ball is heavier or lighter than the others.

M/A3 Irving Hopkins asks the following question: Given the lengths of the n sides of an irregular polygon, how should the sides be arranged and what should the angles be in order to maximize the area?

M/A4 Jerry Griggs wonders in how many ways seven people can be seated at a round table so that no person sits next to the same pair (unordered) of people twice?

M/A5 Here is a problem posted on the M.I.T. Mathematics Department bulletin board as a promotion for the Math Club:

We begin with two integers m and n such that $2 \leq m \leq n \leq 99$.

We tell Mr. P. the product mn, and we tell Ms. S the sum m + n. The following conversation then takes place:

Mr. P: "I don't know the two numbers."

Ms. S: "I knew you didn't know. I don't know either."

Mr. P: "Now I know the numbers."

Ms. S: "Now I know, too."

What are m and n?

Speed Department

M/A SD1 Smith Turner has a tennis quickie. In a tournament of 32 entries, seven withdrew and were replaced by byes. When byes replaced two players bracketed to meet in the first round, a bye was advanced to the second round, etc. Assuming that the dropouts were scattered at random throughout the draw, what is the most likely number of matches that must be played to finish the tournament?

M/A SD2 Our last problem is from Phelps Maeker, who has a regular polygon of n sides and wants to know the ratio between the areas of the inscribed and circumscribed circles.

Solutions

N/D 1 What is the shortest possible game of Othello?

Only Matthew Fountain responded; he writes:

Stephen Kimmel reported the following game in the July 1981 issue of *Creative Computing* magazine. It occurred during a tournament involving four computer programs and one human.

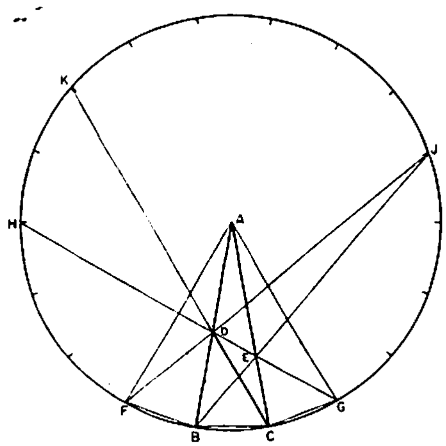
	Flip disc		*Othello-instant*
1	D6		C4
2	F3		F6
3	E6		C6
4	C3		F4
5	F5		F2
6	C5		C2
7	E3		E2
8	D3		D2
Final Score	0	to	20

N/D 2 Using only plane geometry (not trigonometry), find angle EDC in the isosceles triangle ABC in the drawing, given EBC = 50° and BCD = 60°.

The following is from William Schumacher:

I felt intuitively (from appearances and from the difficulty of identifying another angle near that value geometrically) that the unknown angle was 30°, and set about to prove it. Several approaches involved constructing adjacent similar (including congruent) triangles and going through a maze of congruences, but in the end not only were they tedious but they were hardly rigorous enough to serve as proof. Ultimately the attached relatively simple and utterly convincing (I am tempted to say elegant) version emerged. The rationale:

1. Using A as a center and AB = AC as a radius, draw a circle.



2. On the circumference of the circle, lay out points F, G, H, J, and K, respectively 20°, 40°, 80°, 120°, and 140° from B; and alternately clockwise/counterclockwise from it, as shown.
3. Construct chords BJ and GH (120° each) and CK and FJ (140° each).
4. Chords BJ and CK subtend arcs of 120° and 140°, respectively, and define, versus chord BC (subtending a 20° arc), base angles at B and C of 50° and 60° respectively in triangle ABC, corresponding to the problem statement.
5. Chords BJ and GH, subtending arcs of 120° each and symmetrically disposed about radius AC, intersect at point E on AC.
6. Chords FJ and CK, subtending arcs of 140° each and symmetrically disposed about radius AB, intersect at point D on AB.
7. Line segment DE is therefore one side of the unknown angle EDC or GDC, and with its vertical angle HDK the two angles subtend arcs CG (20°) and HK (40°) for a total of 60°. The angles EDC and HDK are therefore each 30°.

Also solved by Matthew Fountain, Harry Zaremba, Winslow Hartford, Ronnie Rybstein, John Woolston, Allan Gifford, Greg Huber, John Rule, F. Farassat, Emmet Duffy, and the proposor, Craig Murphy.

N/D 3 Ask a friend to write down *any* number B. Above B, write another number A, made up of all the digits in B *and any additional digit except O,* arranged in *any* order. Subtract B from A. Ask your friend to tell you the final answer, C. For example,

A	65,835
B	5,653
C	60,182

You can find the unknown added digit (8 in the example) as follows: add together the digits of C, and if this result contains two or more digits, add these together in turn, and so on, until only one digit remains. This will be the extra digit that was added in forming A. Why?

Emmet Duffy solved this easily. He writes:

This problem is nothing but casting out nines. If you add the digits in a number and, if the result is more than one figure, continue to add digits in the sum until only one digit remains, the same result will be obtained by casting out nines except that if the number is a multiple of nine, casting out nines will yield a result that is zero, but adding up the digits as in the puzzle will result in a nine, which is then cast out. Calling the number which remains when nines are cast out the digital number, which will be called a, then if any four-digit number has a digital number a, and if a digit b is included with the four digits to make a five-digit number, then the digital number will be $a + b$ if this is one digit or $a + b - 9$ if the sum of $a + b$ is a two-digit number. Using the cast-out-nines method to check subtraction, we subtract the digital number of the minuend from the digital number of the subtrahend to get the digital number of the remainder. If the digital number of the subtrahend is less than the digital number of the minuend, then add nine to it before subtracting. In either case the digital number of the remainder will be the digit b.

Case 1: $a + b$ is one digit; then $a + b - a = b$, the digital number of remainder.

Case 2: $a + b$ is a two-digit number; then the digital number of subtrahend will be $a + b - 9$. This will be smaller than the digital number of minuend. Then before subtracting a 9 is added making the result: $a + - 9 + 9 - a = b$.

The advantage of adding up the digits instead of casting out nines is that in case the added digit b is 9, the result will be 9, not 0.

Also solved by Ronnie Rybstein, Frank Carbin, Winslow Hartford, Harry Zaremba, Matthew Fountain, and Angel Silva.

N/D 4 Given the one known digit, as shown, fill in all the xs:

```
  xxxx
√xxxxxxxx
 x
 xxx
  xx
  xxxx
  xxxx
   xxxxx
   xxx3x
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Norman Wickstrand presents a lucid explanation:

By inspection, the first digit in the root is 3. The trial divisor for the next digit is then 6x. Since 62 x 2 is a three-digit number, the second digit must be 1. 1xx - 61 is greater than 3900. 3900 ÷ 62x is greater than 6. Hence the third digit is 7, 8, or 9. By inspection the fourth digit cannot be 0 or 1. We have only 24 choices left, all of which we must try. They are 634x • x, and 636x • x, and 639x • x. The only one which has 3 for the next-to-the-last digit is 6384 • 4. Hence the desired root is 3194, whose square is 10,201,636. Incidentally, instead of the digit 3 the digits 1, 5, and 9 also give unique solutions for similar problems.

Also solved by Angel Silva, Harry Zaremba, Matthew Fountain, Winslow Hartford, Ronnie Rybstein, John Woolston, Harry Hazard, Mary McKenockett, Mike Bercher, Carl Sells, Emmet Duffy, Marshall Fritz, W. McGuinness, Richard Celotto, Sandra Zack, and Dennis Sandow.

N/D 5 Given that red columbines produce an average of 100 seeds per pod (with a normal distribution, standard deviation = 10), and yellow columbines produce an average of 99.8 seeds per pod (also with a normal distribution, standard deviation = 10). The number of red plants equals the number of yellow plants, and all plants bear the same number of pods. I might collect only those pods with 110 or more seeds. Alternatively, I might collect all the pods with 110 or more seeds, 90 percent of the pods with 109, 80 percent of the pods with 108, 70 percent of the pods with 107, etc. Whatever I do, the proportionate deficiency of yellow seeds will be approximately equal to the product 0.01•0.2•i, where i is the average number of seeds per collected pod minus 99.9. Show that this last statement is correct.

Harry Zaremba sent us the following solution:

Based on the given data, the standard deviation of seed distribution in the combined seed population must also be equal to 10, and the magnitude of the ordinates in the normal distribution curves for the separate and combined seed populations will be identical. As a consequence, the mean number of seeds in the pods of the total population will be

$m = (100 - 99.8)/2 = 99.9,$

and the number of seeds that the average number of seeds collected per pod (m_c) is in excess of the total population mean is equal to

$i = m_c - m = m_c - 99.9.$

Further, the average number of yellow seeds less than the average number of red seeds per mean number of seeds in the total pod population is given by

$d = (100 - 99.8)/m = 0.2/m.$

Hence the deficiency D of yellow seeds in the collected group of seed pods is

$D = di = (0.2/99.9)\ i = 0.01 \bullet 0.2 \bullet i$, approximately.

Meanwhile, Matthew Fountain comments on the problem itself:

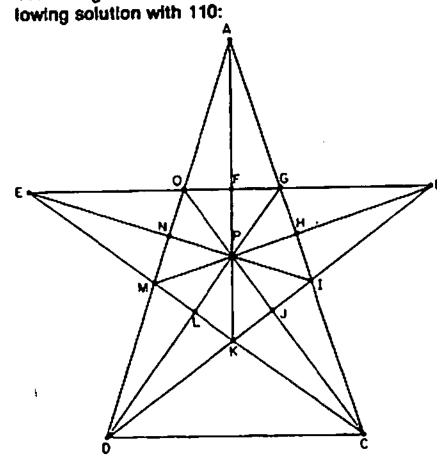
The statement of **N/D 5** glosses over the probability aspect of the problem. The red columbines and the yellow columbines have so closely identical seed distributions that even were one to randomly select 10,000 pods from each, the red pods might have the same or even lower average seed content than the yellow pods. The standard error of the mean for a distribution of means based on a sample size n is the standard deviation divided by the square root of n. With n = 10,000 and σ = 10, the standard error is 0.1. But I do understand the spirit of the problem.

Better Late Than Never

Y1981 Harry (Hap) Hazzard notes that
$71 = 81 - 9 - 1$
$72 = 1 \cdot 81 - 9$
$80 = 81 - 1^9$
giving us a total of 58 numbers.

MAY3 Walter Nissen has responded.
A/S3 Mitchell Serota and Johan Norvik have responded.
A/S4 William Moody has found that the exact answer is $6\sqrt{10}$.
A/S SD1 Jack Page notes that he used this problem in an article he wrote for Technology Review (July/August, 1972).
OCT2 George Flynn has responded.
OCT3 Walter Nissen has responded.
OCT4 Matthew Fountain has responded.
OCT5 Walter Nissen has responded.

N/D SD1 Hank Ferguson and Sandra Zack found 109 triangles and W. McGuinness sent us the following solution with 110:



		CDG			
		CDI			
ACD	BDG	CDJ	DGI	EFK	FGP
ACK	BDM	CDK	DGO	EFP	FOP
ACM	BDO	CDL	DIG	EFI	GHP
ACO	BDP	CDM	DIN	EGL	GIP
ACP	BEI	CDO	DIP	EGP	COP
ADG	BEK	CDP	DJO	EIK	HIP
ADI	BEM	CEG	DJP	EKP	IJP
ADK	BEP	CEI	DKL	ELP	IKP
ADP	BFK	CEO	DKM	EMN	JKP
AFG	DFP	CEP	DKP	EMO	KLP
AFO	BGH	CGL	DLM	EMP	KMP
AGO	BFI	CGO	DMP	ENO	LMP
AGP	BGP	CGP	DNP	EOP	MNP
AHM	BHI	CHM	DOP		MOP
AHP	BIP	CHP			NOP
AIK	BJO	CIJ			
AIN	BJP	CIK			
AIP	BKM	CJK			
AKM	BKP	CKD			
AMP	BMO	CKP			
ANP	BOP	CLP			
AOP		.CMO			
		CMP			

Proposers' Solutions to Speed Problems

SD 1 There are always 24 matches, one player eliminated in each.
SD 2 $\cos^8 (\pi/n)$.