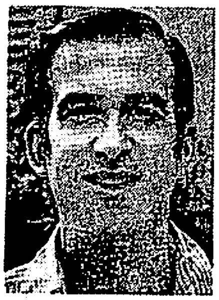
Puzzle Corner
Allan J. Gottlieb

Red or Yellow Columbines?



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Let me begin with responses from several readers.

As you may recall, earlier this year we had a plea from Larry Bell for suggestions of books containing mathematical puzzles. Smith Turner has recommended three: Steinhouse, *Mathematical Snapshots* (Oxford University Press, New York, 1960); Phillips, Shovelton, and Marshall, *Calibaos Problem Book* (T. de la Rue and Co., London, 1933); and Schuh (translation by Gobel), *The Masterbook of Mathematical Recreation* (Dover, New York).

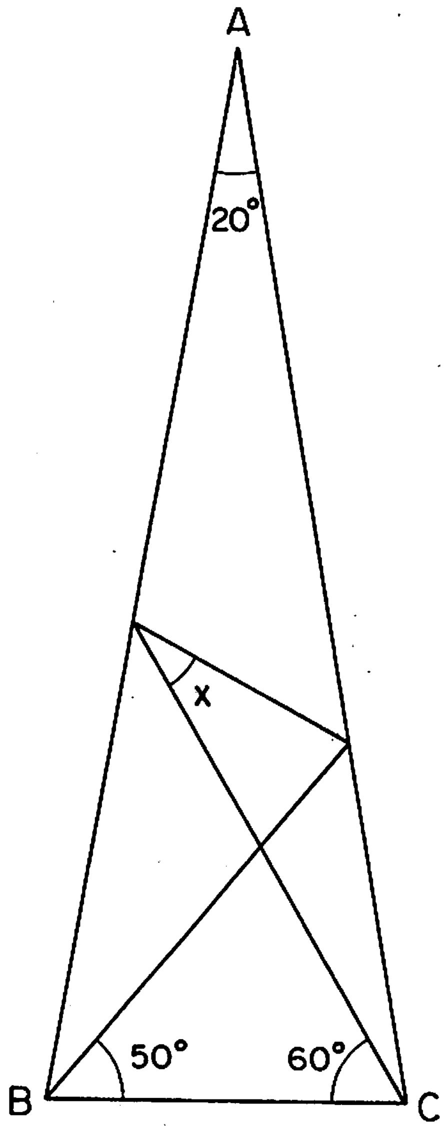
Robert Cutler writes that he has studied the function C obtained by factoring a positive integer into primes, summing the primes, and iterating the procedure until a prime results. By definition C(p) = p when p is a prime and C(1) = 1. [C(4) is undefined.] For example, the calculation that C(98) = 5 proceeds as follows:
98 = 2·7·7; 2 + 7 + 7 = 16; 16 = 2·2·2·2; 2 + 2 + 2 + 2 = 8; 8 = 2·2·2; 2 + 2 + 2 = 6; 6 = 2·3; 2 + 3 = 5.
Mr. Cutler wonders whether C has been studied before. Can anyone help him?

Finally, to answer a question from John Ruttersford: I very rarely play bridge (perhaps 15 hands a year)—and not especially well (I finesse but don't squeeze).

Problems

N/D 1 For variety we begin this month with an Othello problem from Scott Byron: What is the shortest possible game of Othello?

N/D 2 Craig Murphy asks you to find angle x in the isosceles triangle ABC shown below (AB = AC). He demands that the solution use only plane geometry; in particular, he forbids trigonometry. However, he adds, "If you become sufficiently frustrated, you may use trigonometry to find x, since knowing the result may help point the direction in which to look for a geometrical solution."



N/D3 The following problem first appeared in a Calibron Products advertisement in *Technology Review* for June, 1939:
Ask a friend to write down *any* number B. Above B, write another number A, made up of all the digits in B *and any additional digit except O,* arranged in

any order. Subtract B from A. Ask your friend to tell you the final answer, C. For example,

```
A   65,835
B    5,653
C   60,182
```

You can find the unknown added digit (8 in the example) as follows: add together the digits of C, and if this result contains two or more digits, add these together in turn, and so on, until only one digit remains. This will be the extra digit that was added in forming A. Why? (In the example, 6 + 0 + 1 + 8 + 2 = 17; 1 + 7 = 8; and 8 was the added digit.

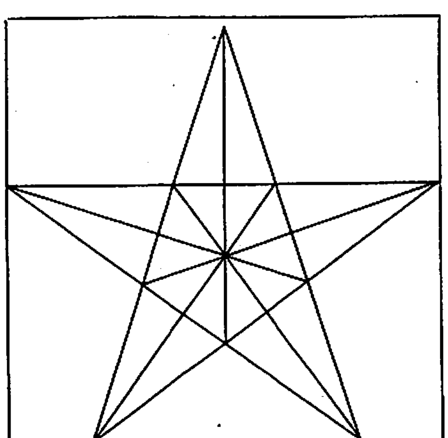
N/D 4 John Trifiletti has a square root problem with only one known digit:

```
        x x x x
  √x x x x x x x x
   x
   x x x
    x x
    x x x x
    x x x x
      x x x x x
      x x x 3 x
```

The problem is to fill in all the x's.

N/D 5 Roger Milkman asks what I believe is our first horticultural problem. He writes:

In my garden are red columbines, which produce an average of 100 seeds per pod (with a normal distribution, standard deviation = 10), and yellow columbines, which produce an average of 99.8 seeds per pod (also with a normal distribution, standard deviation = 10). The number of red plants equals the number of yellow plants, and all plants bear the same number of pods. I might collect only those pods with 110 or more seeds. Alternatively, I might collect all the pods with 110 or more seeds, 90 percent of the pods with 109, 80 percent of the pods with 108, 70 percent of the pods with 107, etc. Whatever I do, the proportionate deficiency of yellow seeds will be approximately equal to the product $0.01 \cdot 0.2 \cdot i$, where i is the average number of seeds per collected pod minus 99.9. Show that this last statement is correct.



Speed Department

N/D SD 1 Adam Becker wants to know how many triangles can be found in the figure at the bottom of column 1.

N/D SD 2 R. Steffens sent us the following counting problem.

1	2	3
4	5	6
7	8	9

A straight line drawn across the figure will traverse a subset of the nine squares, for example 1,2,3 or 1,4,7. How many of the possible subsets so determined include square 1?

Solutions

JUL 1 South to lead and make all seven remaining tricks:

```
              ♠ J 8 6
              ♥ A J 4
              ♦ A
♠ K           ♣ —              ♠ 10 7 2
♥ K 8                          ♥ 9 6
♦ 9           ♠ A 9            ♦ 4
♣ K 8 6       ♥ Q 7            ♣ 9
              ♦ —
              ♣ J 10 3
```

Unfortunately, we failed to specify the trump suit. But several readers, including W. Cuttler, interpreted that omission as part of the problem and deduced the answer. Mr. Cuttler writes:

The last time I submitted a solution to a bridge problem, back when you were a student at M.I.T., I was chided by one of your readers on the grounds that I had peeked at the East-West hands. To avoid that this time, let me say that I refrained from peeking until the problem was solved. However, in solving the problem, I assumed that the key cards in the opponents' hands were so located that the hand was makeable; otherwise the problem would not have been submitted. Although not mentioned, I also assumed that hearts were trumps. I could find no possible way for South to lead and take the seven remaining tricks with any other suit as trumps or in no-trumps. With the above as a preface, here is the solution:

1. South leads ♣ J. West must cover with ♣ K (obvious). North trumps with ♥ 4.
2. North leads ♦ A. South trumps with ♥ Q.
3. South leads ♥ 7 to North's ♥ A over West's ♥ K.
4. North draws remaining trumps. South discards ♣ 3.
5. North leads ♠ 6 to South's ♠ A.
6. South plays ♣ 10. North discards ♠ 8.
7. South leads ♠ 9 to North's good ♠ J.

Also solved by George Holderness, Mike Bercher, Doug Van Patter, Noland Poffenberger, Matthew Fountain, Richard Hess, Robert Garrels, Peter McCall, John Ruttersford, and the proposer, Emmet Duffy.

J' 2 What is the smallest multiple of nine that has n. odd digit, calculated in base 10? What is the answer if other bases are used? And how many ways are there of doing it?

The key to this puzzle is to recall the "casting out nines" rule. For example, Avi Ornstein writes: The digits in any multiple of 9 must add up to produce a number which is also a multiple of 9 (in base 10, that is). [This is "casting out nines."—Ed.] To have no odd digits, the sum must be even, so the digits must add up to a multiple of 18. The smallest number that meets these specifications is 288. If the problem were not limited to base 10, the smallest possible solution would be 22, in various bases. In base 8 this is equal to 18. If you choose to state that 9 does not exist in base 8, then you can use 22 in base 17, which is equal to 4 x 9, or 36, in base 10.

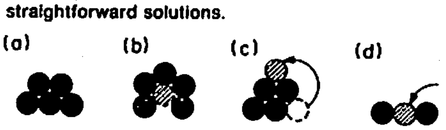
Also solved by Emmet Duffy, Edwin McMillan, Frank Carbin, Marlon Weiss, Richard Hess, Matthew Fountain, Winthrop Leeds, Richard Kruger, Avi Ornstein, Howard Wagner, and Phillip Burnstein.

JUL 3 A falling person reaches a terminal velocity of about 55 m/sec. Entering feet first, one can survive falls into water at speeds of up to 34 m/sec. Starting from rest, what is the maximum distance one can fall before entering water, and survive? Assume that drag is proportional to velocity and that gravitational acceleration is 9.8 m/sec.

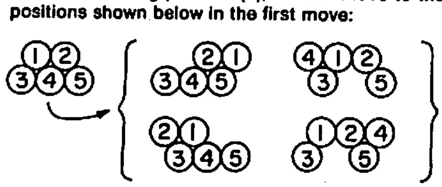
Michael Jung sent us the following solution:
Let the drag acceleration be $d = kv$. Since $55k = 9.8$, $k = 9.8/55$. Starting with $v' = 9.8 - kv$, we obtain $v' + kv = 9.8$
$v'\exp(kt) + kv\exp(kt) = 9.8\exp(kt)$
$[v\exp(kt)]' = 9.8\exp(kt)$
$v\exp(kt) = 9.8/k\exp(kt) + C = 55\exp(kt) + C$
$v = 55 + C\exp(-kt)$.
Since $v(0) = 0$, $C = -55$. Thus
$d = \int v(t)dt = 55t - 55/k\exp(-kt) + C$.
Applying the boundary condition $d(0) = 0$, we obtain
$d = 55(t - [1 - \exp(-kt)]/k)$.
Since the maximum velocity is 34, we see by plugging into the formula for v that the maximum velocity occurs at
$t = \ln(55/21)k$.
Finally, inserting this last value into the formula for d gives
$d = 55/9.8[55\ln(55/21) - 34] = 106.38$ m.

Also solved by Richard Hess, Matthew Fountain, Marlon Weiss, John Prussing, Frank Carbin, Steve Feldman, Winthrop Leeds, Robert Slater, Richard Kruger, William Schoenfeld and the proposer, Eric Piehl.

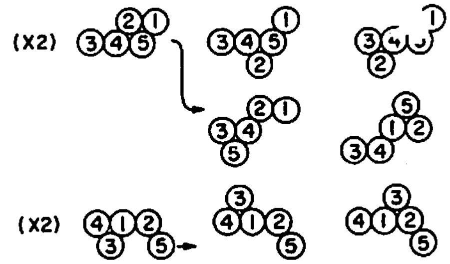
JUL 4 Five coins, arranged as in (a) below, are to be shifted into arrangement (b), using *only four* accurate sliding moves [such as the move shown in (c)]. There is no restriction on the position of arrangement (b) relative to (a), but the new location of any coin moved must be fixed by *definite* contact with two other coins: *estimated* contacts [to form straight lines, as in (d)] are not allowed. Move only one coin at a time, without lifting. Our analysis of this old puzzle shows that there are no less than 24 straightforward solutions.



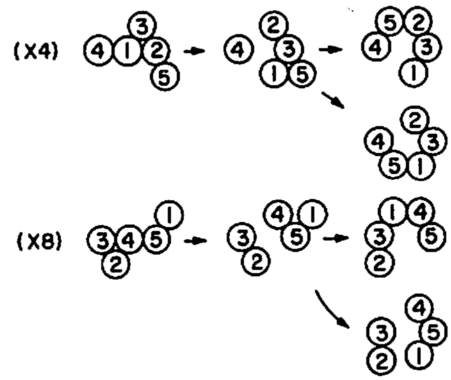
Richard Hess was able to find and describe all 24 constructions:
From the starting position (a), we can move to the positions shown below in the first move:



From the two basic positions created above (each with a multiplicity of two), we can move to the positions shown on the next page in the second move:



From the two basic positions created above (one with multiplicity 4 and one with multiplicity 8), we can move to the positions below in the third and fourth moves:



The two ways in each case to move from the third to the fourth position times the multiplicities of 4 and 8 give the 24 ways to reach the final position.

Also solved by Marlon Weiss and Matthew Fountain.

JUL 5 What is the smallest square matrix in which can be written the names of 50 U.S. states, horizontally or vertically in crossword-puzzle fashion?

Matthew Fountain sent us the following 27 x 27 solution:

```
       WYOMING  SOUTHDAKOTA
         H       R        L
  MISSOURI       E  FLORIDA
  I      O     W G        B
  N         N  E O NEBRASKA
  NEWMEXICO E  S N     R  M
  E  O   A  W  T  LOUISIANA
  S  N   L  Y  V       Z   
  O  T  WISCONSIN  VERMONT 
A TEXAS  F  R  R       N   
L A  N   O  K  C MICHIGAN  
A    A   R     I      E    
S   H PENNSYLVANIA COLORADO
KANSAS   I     I      R    
A   W M  ARKANSAS  VIRGINIA
 NEVADA N       O   L I O  
C   I S E M N N U   L A R R
O W I S W I E O T O I  UTAH
N A   A U S W R H K N   H O
N S D C A S J T C L O K C D
E H E H MAINE H A A I E A E
C I L U P S R D R H S N R I
T N A S S S S A O O   T O S
I G W E H I E K L M I U L L
C T A T I P Y O IDAHO C I A
U O R T R P   T H   W K N N
TENNESSEE INDIANA  MARYLAND
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Also solved by Peter McCall, Richard Hess, and the proposer, Richard Fisher.

Better Late Than Never

JUL SD 1 Alan Faller notes that we did not require a straight hole, so one with side an equiangular (logarithmic) spiral satisfies the problem. However, Mr. Faller shows that such a hole would have infinite size and thus is not constructable.

JUL SD 2 Dan Grayson does not consider solutions involving infinite series "speedy" and proposes an alternate method. Alan Faller comments that most people include the mother and father when giving the size of a family.

Proposers' Solutions to Speed Problems

SD 1 79.
SD 2 27. Specifically, the following subsets: 1, 12, 123, 1236, 1256, 12569, 1258, 12589, 1259, 126, 14, 1456, 14569, 1458, 14589, 1459, 147, 1478, 148, 156, 1569, 158, 1589, 159, 214, 2147, 3214.