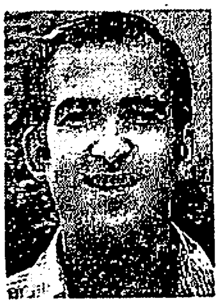
How Norbert Wiener Found His House



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My wife Alice and I are buying a house next week, so things are a little hectic as this is written. It's amazing how many details are involved. I naively thought that selecting the house and obtaining the money were the only time-consuming tasks. Wrong! There's painters, and insurance, and renting a truck, and moving, and . . .

Larry Bell would like recommendations for puzzle books to purchase other than those by Martin Gardner and James Fixx (which he already owns). If readers have particular favorites that you've read (or written!), please let me know.

A quick note: chess and bridge problems are in short supply.

Problems

A/S1. We begin with a chess problem from Roser Powell (and Sam Lloyd): Place the Black King in the center of the board and then place two White Rooks and one White Knight so that the Black King is mated.

A/S2. William Butler offers the following cryptarithmetic problem; he requires only that R = 1:

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  R O O K
      T O
  K I N G
E I G H T
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C H E C K
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A/S 3 Here's one from the M.I.T. Math Club:
It has been said that Norbert Weiner, the great (but absent-minded) mathematician, once lived in a housing development in which all the houses on his street were identical except for their addresses, which were consecutively numbered 1,2,3, . . ., m. In order to remember which house was his, Norbert discovered that his address, n, had the property that the sum of all the addresses less than n was equal to the sum of all the addresses greater than n. For what m and n is this possible?

A/S 4 Please help Irving Hopkins with his dog yard:
Compelled to fence in my dog, I have scrounged some pieces of picket fence from the dump. I have four pieces of lengths 3, 4, 5, and 6 units, and I want to arrange them so that the dog has a maximum area. What is the best configuration; and what is the maximum area?

A/S 5 Jack Parsons wants to know the probability that the World Series (if there is one) will be won by the team that wins the first game. (Two teams contest for the World Series, and the first to win four games is declared the victor; tie games are not possible.)

Speed Department

A/S SD 1 Here's a quickie adapted by Rex Ingraham from a problem posed by L. Boyd; the question is, What did the keeper say?
One time there was, or so it's told, a wise and wealthy Emir who prized his fine Arabian steeds and also his two sons. Before these sons to manhood grew he gave to each a fine Arabian foal

to prize and raise and call his own. When these fine foals had grown to steeds the wealthy Emir, dying, made known his will and named a goal two days distant from his bed; the brothers were to race their steeds, and to the one whose steed were first to reach the goal would go one third of all the Emir's wealth, while to the one whose steed were last would fairly go the rest of all the Emir's wealth. So then the brothers, vying, set out upon the race. But soon they slowed, and slowed and slowed until at nightfall they were nowhere near the goal but near an inn with stables where they then agreed they'd spend the night and also ask the keeper if they might fairly race while vying for the second place. The keeper understood their plight and said, "If you will agree to pay my fee for such advice, then I will say how you can do it." To this the boys agree to pay the fee, and here is what the keeper said: "________."

SD 2 John Woolston needs a fourth flag pole of unknown length: At three corners of a field which measures 96 x 72 yards are flag poles. Those at diagonally opposite corners are 90 and 20 feet in height; the third pole is 60 feet high. It is desired to erect a fourth flag pole on the fourth corner, the *top* of which is the same distance from a point on the ground equidistant from the tops of the others as is that distance. (The solution to the question, "Why?", is left to the readers' fertile imaginations.)

Solutions

APR 1 South to lead and make all remaining six tricks with hearts as trump:

	♠ K 9 ♥ — ♦ 4 ♣ A Q	
♠ Q J 10 ♥ — ♦ 2 ♣ J 6		♠ A 8 ♥ — ♦ — ♣ K 5 4 3
	♠ — ♥ A Q J ♦ 3 ♣ 9 7	

My old Baker House colleague, Peter Sorant, sent us a neat solution: South leads the ♥A followed by the ♥Q, on which dummy discards the ♣Q. Then South leads the ♦3 and ends in dummy with the following position:

♠ K 9
♥ —
♦ —
♣ A

♠ —
♥ J
♦ —
♣ 9 7

East must hold the ♠A and ♠8; otherwise ♠9 will ruff out East's ♠A followed by a club to the ♣A and the good ♠K. West *must* hold the ♠Q and ♠J; otherwise the ♠K will ruff out East's ♠A *and* capture West's last spade, making the ♠9 in dummy good. This leaves *both* East and West with only one club each. North therefore cashes the ♣A and ruffs a spade in the South hand, then cashing the good club.

Mike Bercher sure has a sharp eye. In addition to discovering that the "I" was omitted from "Gottlieb" under my picture, he (as well as Smith Turner and Emmet Duffy) noticed that this problem first appeared in *Technology Review* two years ago. Also solved by Doug Van Patter, Richard Hess, Matthew Fountain, Steven Balbus, Gardner Bent, Ruth Lewort, Joseph Romm, Jerry Grossman, Charlie Maison, Richard Waters, Stuart Schulman, and the proposer, N. Piffenberger.

APR 2 In the game of Red Dog, a player is dealt four cards and bets that he can beat a fifth one by having a higher card in the same suit. Bets won or lost are taken from or added to the pot. What is the probability of winning (before looking at the four cards)? How many of the 48 outstanding cards should a player be able to beat in order to justify betting? (For simplification, assume a two-handed game in which each player, after looking at his/her cards, must either pass (without penalty) or bet the exact amount then in the pot.)

Most respondents believe that the second part is rather easy: if the odds of winning are at least 50 percent, bet. However, the proposer (Smith Turner) and I disagree. To quote Mr. Turner, "But PLAYER's loss is not picked up; it stays on the table, in the pot, and . . . there is a chance that his loss will increase the pot that he has a chance to pick-up on the next deal." For part one, however, there is agreement: for the four-card hand to win, it must hold n $(0<n<5)$ cards in the same suit as the fifth card. The probability of this event is

$C(12,n) * C(39,4-n)/C(51,4)$

in which we use $C(r,s)$ to denote the number of combinations of r things taken s at a time. The probability that such a hand would win is

$n/(n+1)$.

Thus the probability of an unseen hand winning is:

$$1/C(51,4) \sum_{n=1}^{4} \{C(12,n) * C(39,4-n) * n/(n+1)\} = .37723\ldots$$

Also solved by Harry Zaremba, Emmet Duffy, Michael Jung, Jerry Grossman, Richard Hess, and Matthew Fountain.

APR 3 Make a round-robin schedule for 12 teams which bowl on six double alleys, two teams to each double alley per night. Each team is to meet every other team once, with no team bowling either more than twice or twice in a row on any double alley.

Depending on how you interpret the problem, Harry Zaremba's solution is either correct or very close (but not both). Richard Hess scheduled the bowlers but was unable to meet the alley requirements. Mr. Zaremba's response follows:

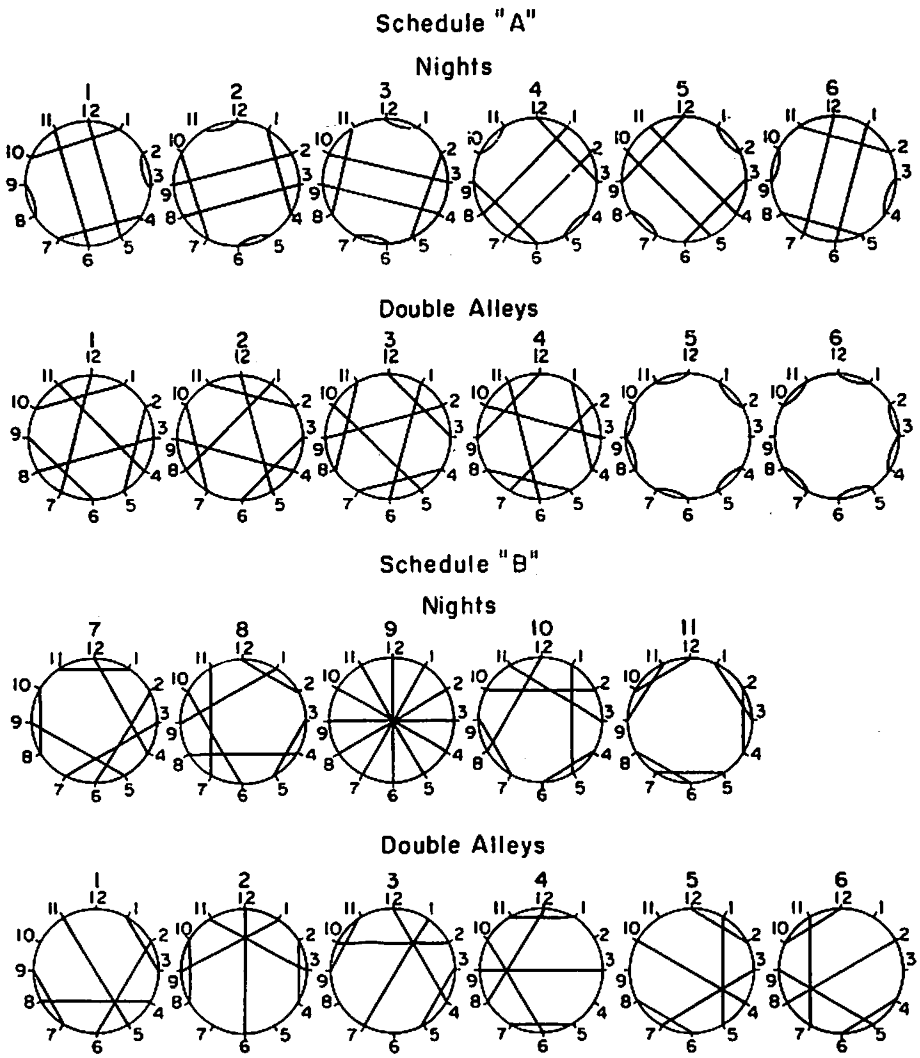
It is presumed here that the "either/or" in the last two conditions means one or the other *but not both*. If the problem intends both conditions, then the solution below falls short by the narrowest of margins. The solution satisfies the following conditions:

- ☐ Each team bowls every other team only once. (Teams are identified by letters A to L.)
- ☐ No team bowls twice in a row on any double alley.
- ☐ All teams bowl at least once and no more than twice on each double alley except team I which bowls three times on double alley # 1.

	1	2	3	4	5	6
1	AG	BL	CK	DJ	EI	FH
2	FI	AH	BG	CL	DK	EJ
3	EK	FJ	AI	BH	CG	DL
4	CH	DG	EL	FK	AJ	BI
5	BJ	CI	DH	EG	FL	AK
6	DI	BK	FG	AL	EH	CJ
7	HK	DF	JL	GI	AC	BE
8	BD	AE	IK	CF	GJ	HL
9	CE	IL	AD	HJ	BF	GK
10	IJ	GH	EF	AB	KL	CD
11	GL	JK	BC	DE	HI	AF

The proposer, Matthew Fountain, submitted the following solution (including the diagram, below) that meets all alley requirements:

My first plan of attack was to see if I could devise a schedule "A" for the first six nights of bowling during which all teams bowled once on each double alley. Next I would try to find a schedule "B" for the last five nights during which each team would bowl once more on five of the six double alleys. Permutations of the nights and alleys in schedules "A" and "B" would then permit meshing the schedules so as to avoid any team's bowling twice in succession on the same double alley. Proper notation was impor-



Diagrams used by Matthew Fountain to construct the bowling schedule required by problem APR 3. Chords in night circles represent teams that meet at night; chords in double alley circles represent teams that meet on that double alley. For example, teams 1 and 10 meet on double alley 1 the first night, as the chord between 1 and 10 appears in the night 1 circle and the alley 1 circle.

tant. Here it was most helpful to represent the teams by their position on a circle like the 12 numbers on a clock. Chords between teams represented meetings on double alleys. After much experimentation I hit upon the following solution which differs from my original conception only in permitting two teams in schedule "A" to bowl a second time on a double alley. The good mesh of schedules "A" and "B" appeared fortuitous, although it can be attributed to the large number of permutations available.

	1	2	3	4	5	6
1	1-10	5-12	4-7	6-11	8-9	2-3
2	3-8	7-10	2-9	1-4	11-12	5-6
3	2-5	4-9	8-11	3-10	6-7	1-12
4	6-9	1-8	3-12	2-7	4-5	10-11
5	4-11	3-6	5-10	9-12	1-2	7-8
6	7-12	2-11	1-6	5-8	9-10	3-4
7	2-6	8-10	4-12	1-11	3-7	5-9
8	4-8	1-9	3-5	6-10	2-12	7-11
9	5-11	6-12	1-7	3-9	4-10	2-8
10	7-9	3-11	2-10	8-12	1-5	4-6
11	1-3	2-4	9-11	5-7	6-8	10-12

Note that in this solution every team bowls at least twice (thrice, with no more than one exception per team) on other double alleys before returning to a double alley.

APR 4 Two well insulated compartments, filled with a "perfect" gas, are maintained at absolute temperatures T_1 and T_2, respectively. If a large tube connects the compartments, the pressures (P_1 and P_2) naturally tend to equalize. But (believe it or not) if the proportions of the tube are suitably reduced, the dynamical theory of gases indicates that a steady state will be reached in which the relationship $P_1/P_2 = (T_1/T_2)^{1/2}$ is approached. Can you verify and explain this formula?

This problem was well understood by Matthew Fountain in 1981 and by Osborn Reynolds in 1879! Mr. Fountain writes:

Connect the two chambers with a minute hole. At equilibrium the rate of molecules entering the hole from chamber (1) equals the rate of molecules entering the hole from chamber (2).

Let R_1 = rate of molecules entering hole from chamber (1)
C_1 = concentration of molecules in chamber (1)
S_1 = average speed of molecules in chamber (1)
P_1 = pressure in chamber (1)
T_1 = temperature in chamber (1)
k, K, and W = appropriate constants.

Then $C_1 = WP_1/T_1$
$P_1 = kC_1S_1^2$
$R_1 = KC_1S_1 = K(kC_1^2S_1^2/k)^{1/2} = K(P_1C_1/k)^{1/2} = K(WP_1^2/kT_1)^{1/2}$

At equilibrium
$R_2 = K(WP_2^2/kT_2)^{1/2} = R_1$ and $P_1/P_2 = (T_1/T_2)^{1/2}$.

Equilibrium will be reached much more quickly if the number of holes is greatly increased by substituting a porous plug for the single hole. Unglazed earthenware is a suitable plug material. According to Sir James Jeans, "Osborne Reynolds investigated this phenomenon in a series of experiments in which the two chambers were kept at temperatures of 8° C and 100° C. When a steady state was attained the pressures were measured, and it was found that, so long as the pressure was sufficiently low, the equation was satisfied with very considerable accuracy. At higher pressures this equation failed, as was to be expected."

Osborne Reynolds' report is in *Philosophical Transactions*, Volume 170 (part II), 1879, p. 727.

APR 5 Substitute digits for letters to make the following addition correct, with some additional restrictions: "ONE" is divisible by 1, "TWO" by 2, "SIX" by 6, and "NINE" by 9; and "NINE" "SIX" "TWO" "ONE."

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  O N E
  T W O
+ S I X
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N I N E
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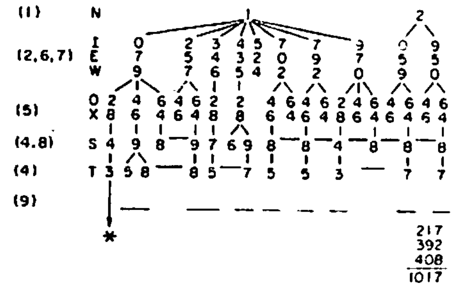
Cryptarithmetic puzzles are often popular and this one was no exception. Everyone used the given conditions to limit the possibilities and then either wrote a computer program or performed an exhaustive manual backtracking search (i.e., trial and error). I have chosen Avi Ornstein's solution since his carefully written diagram makes the (manual) procedure clear:

1. By adding three numbers, N must be 1 or 2.
2. NINE ÷ 9, so: if N=1, I+E = 7 or 16; if N=2, I+E = 5 or 14.
3. O & X are even; O+X = 10.
4. S>T>O.
5. Based on (3) & (4), O=2,4,6 and X=8,6,4, respectively.
6. O+X carries 1 to the next column, so W+I = 9.
7. Based on (5), if N=2, I,E,W ≠ 4 or 6.
8. SIX ÷ 6, so S+I+X ÷ 3.
9. O+T+S+I = NI.



Also solved by Ruth Lewart, Jerry Grossman, Emmet Duffy, W. Smith, Charlie Mason, Stuart Schulman, Dennis Sandow, Robert Mandle, Elliot Roberts, Chester Claff, Harry Hazard, Marlon Weiss, Mike Bercher, Michael Jung, H. Maynard, Harry Zaremba, Steve Feldman, Howard Eglowstein, Frank Schafer, Norman Wickstrand, Frank Carbin, Richard Hess, and the proposer, Abe Schwartz.

Better Late Than Never

1980 M/A 4 Sidney Shapiro has responded
OCT 2 David Lukeus has responded.
1981 JAN 1 Raymond Gaillard has responded.
JAN 4 Rick Decker noticed that this problem is an easy consequence of the invariance of "cross ratios" under "projective transformations."
APR SD 1 Addison Ellis and Stuart Schulman have noticed a musical error. Mr. Ellis writes:
When "Tea for Two" is written in the key of A flat major as in what you call "the strip," the words corresponding to the seven notes shown are, "Me for you and you for me." These notes would correspond to the words, "Tea for two and two for tea" only if the tune were written in the key of G sharp major. And Mr. Schulman adds that another possible correction is to change the G clef to a C clef.

Proposers' Solutions to Speed Problems

SD 1 Tomorrow, boys, each of you must mount his brother's horse.
SD 2 Of course the size of the field is immaterial, but excess data sometimes add spice. The intuitively obvious solution relies on the fact that the locus of all points in a plane (the ground) equidistant from two points outside of the plane is a line perpendicular to the line joining the points in the plane perpendicularly below the points in question—i.e., the side of the field. Considering the intersection of this locus line with the side in question—(use that between 90 and 60 foot poles for example).

If S = length of side
a = distance from the 60' pole
h = height of the 4th flag pole

$(90)^2 + (s-a)^2 = (60)^2 + a^2$
$h^2 + (s-a)^2 = (20)^2 + a^2$ SO
$(90)^2 - (60)^2 = a^2 - (s-a)^2 = h^2 - 20^2$, or
$h^2 = (90)^2 - (60)^2 + (20)^2$; therefore
$h^2 = (70)^2$ or $h = 70$