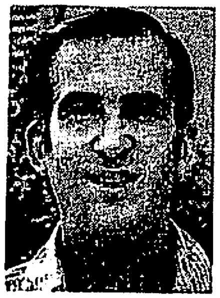
Puzzle Corner
Allan J. Gottlieb, '67

From Pins to Terrapins



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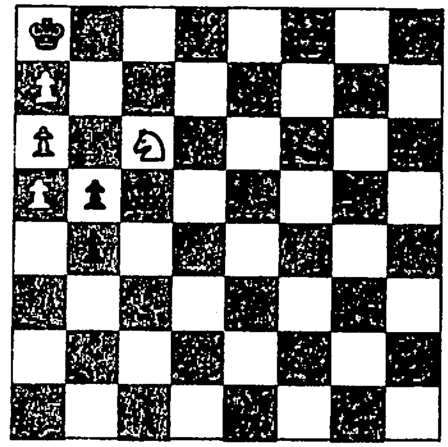
Let me once again answer a perennial question: What criteria are used to select solutions for publication?

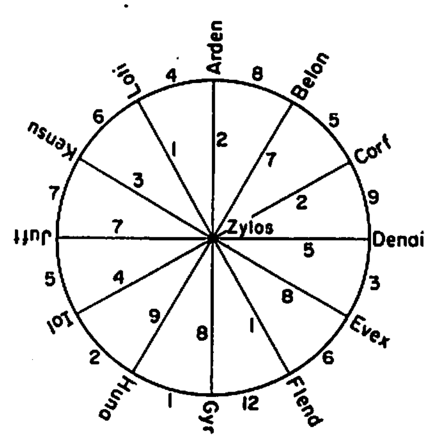
As responses arrive during the month they are simply put together in neat piles, with no record as to their date of postmark or of arrival. When it is time for me to write the column, I first weed out erroneous and illegible solutions. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select for each problem a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor contributions from correspondents whose solutions have not previously appeared as well as solutions that are especially neatly written or typed, since the latter produce fewer typesetting errors.

Problems

MAY 1 We begin with a chess problem from George Farnell:
White to play and mate in two moves:



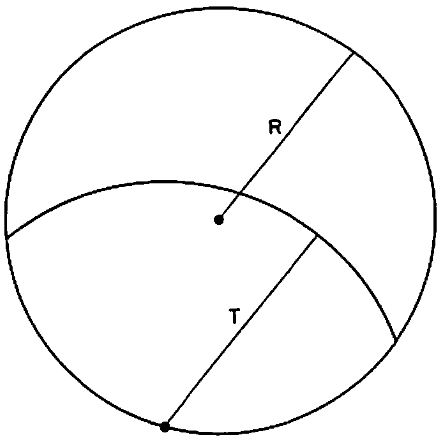
May 2 Frank Rubin has one for all the "trekkies" in the audience:
The Starship Enterprise is conducting negotiations on Arden with the Emperor of the Zylos System. Suddenly there is an alarm: a Klingon warship has entered the sector. The Enterprise must gather all of the leaders of the 12 outer planets and bring them to a conference on Zylos. It must use only the established space routes, whose travel times in zorpets are indicated in the following chart. How soon can the conference be held?



May 3 Akbar Ahmed wants to know the sum of
$1^{-1} + 2^{-2} + 3^{-3} + \ldots$

May 4 Barle Gilbert likes to construct N-by-N matrices of letters such that each row is an N-letter English word (read from left to right) and so is each column (read from top to bottom). Such a matrix is called N-perfect if the 2N words are all distinct. What is the largest N-perfect matrix you can find? Mr. Gilbert doubts that a 6-perfect matrix exists.

May 5 Frank McHargue knows a farmer who rents out one half of a circular, fenced pasture. A cow is to be tethered to a point on the fence so that she is able to graze on exactly one half of the pasture area. What is the length of the tether (T) if the radius (R) of the pasture is 100 feet?



Speed Department

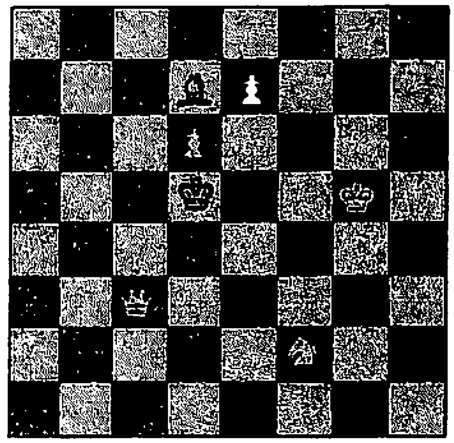
SD 1 A quick one from James Landau: In the November 1979 issue (*page A32*) you made the following incorrect statement: "But there is no time for infinite arguments in a speed problem." The great John von Neumann demonstrated the fallacy in this statement when he was asked to solve the following problem: "A train is traveling at 30 miles per hour. When it is 60 miles from the station, a bird which flies at 50 miles an hour flies from the train to the station, back to the train, back to the station again, etc. When the train reaches the station, how far has the bird flown?" Von Neumann immediately gave the correct answer. "I see you used the shortcut," his questioner said. "I was expecting you to use infinite series." "What shortcut?" asked von Neumann. "I *did* use infinite series." Question: What was the shortcut, and what was the correct answer?

SD 2 Solomon Golumb wants to know the equivalent of each of the following scientific units:

10^{12} microphones	5 holocausts
10^{12} pins	10^6 bicycles
10^{-12} boulevards	10^9 micrometers
10^{21} picolos	10 monologues
10 rations	2×10^3 millinaries
10 millipedes	10^{-5} dollars
1 centipede/second	1 milli-Helen nano-nano
3⅓ tridents	

Solutions

A/S 1 (as modified in January) White to mate in two:



The new diagram published in January omitted the White knight from KB2 (it's included in the diagram above). However, many readers who had worked on the original problem, with the white King on KR_8 instead of KN_8, put the knight back in and solved the problem. The following solution is from Michael Jung:

The key move is 1: N—K4. Now if K×N then B—B6; if P×P then N—B6; if P—K3 then B—B6; and if P—K4 then Q—Q3.

Responses were also received from Matthew Fountain, Joel B. Freilich, Gerald Blum, Abraham Fineman, Peter Steven, and Elliot Roberts.

JAN 1 What is the probability of picking up a bridge hand containing no suit with exactly three cards?

The following solution is from Peter Steven: Simply take a list of all the possible bridge hand suit distributions which do not include three-card suits (there are 25 out of 39 different possibilities); calculate the number of suit permutations for each (that's either 4, 12, or 24 depending on the number of suits of different length); multiply that by the number of combinations of cards that could fill each suit (varying from 1 for a void to 1716 for six- or seven-card suits); sum them up (there are 163.6 billion); and divide by the total number of possible hands (the combinatoric 52 items taken 13 at a time or 635 billion); and the answer is . . . .2577, or a 25.8 percent chance of picking up a hand containing no three-card suits.

Also solved by Matthew Fountain, Emmet Duffy, Harry Zaremba, Winslow Hartford, Michael Jung, and Steve Feldman.

JAN 2 Find, if possible, a set of five distinct positive integers such that the sum of each pair is a perfect square.

The only solution is from Walter Penny: A = 7442; B = 28658; C = 148583; D = 177458; and E = 763442. $A + B = 190^2$; $A + C = 395^2$; $A + D = 430^2$; $A + E = 878^2$; $B + C = 421^2$; $B + D = 454^2$; $B + E = 890^2$; $C + D = 571^2$; $C + E = 955^2$; and $D + E = 970^2$.

JAN 3 Given a balancing scale and 15 billiard balls, of which 14 are known to be identical in weight but the fifteenth is either heavier or lighter, what is the minimum number of balancings needed to isolate the "odd" ball?

Michael Heney argues that at least four weighings are needed: using a balance-type scale, at most three pieces of information can be obtained from each weighing (either the piles are equal, the left is heavier, or the right is heavier). Thus with N weighings, at most 3^N states can be distinguished. So to determine which of the possible states occurred, N must be at least 4.

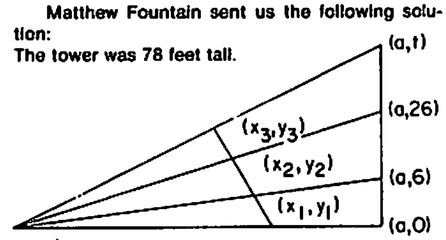
Emmet Duffy sent us a four-weighing method: Place seven balls on each side of the scale. If the scale balances, the odd ball (not on the scale) is heavier or lighter. Test it against a known good ball to find out if it is heavy or light. Total of two balancings. If one side of the scale goes down, mark all balls on that side H for heavy and mark all on the other side L for light. Put three H balls and three L balls on each side of the scale with an H and an L off the scale. If the scale balances, the odd ball is the H or the L off the scale. Test either one against a known good ball. If it doesn't balance, it is the odd one; and if it does balance the odd ball is the other one. This takes three balancings. If the scale does not balance with six balls on each side, then the odd ball is one of three H that went down or one of the three L that went up. Take these six and place an H and an L on each side and an H and an L off scale. If the scale balances the odd ball is the H or L off scale; and if the scale does not balance the odd ball is the H that went down or the L that went up. In either case, make the same test as before to determine which of the pair is odd. This takes four balancings.

Also solved by Joel Freilich, Steve Feldman, Winslow Hartford, Michael Jung, Peter Steven, Matthew Fountain, Marion Weiss, and Richard Kruger.

JAN 4 One good way to estimate the height of an object is to take a known height, sight along a ruler until the known object subtends an easy length to work with, and then take the proportional height subtended by the unknown object. Recently Frank Rubin attempted to measure the height of a bridge in this way. He had a friend who was exactly six feet tall stand next to the bridge tower. He held the ruler so that his friend appeared to be one inch tall; the height of the roadway then appeared to be four inches and that of the supporting tower ten inches. Hence Frank estimated that the roadway was 24 feet above the ground and that the tower was 60 feet high. Later Frank found that the roadway was actually 26 feet above the ground; clearly his error was because he did not hold the ruler precisely vertical. What, then, was the correct height of the tower?

Matthew Fountain sent us the following solution:

The tower was 78 feet tall.



Let the eye be at the origin (0,0) and the top of the bridge tower be at point (a,t). Then (x_1,y_1), (x_2,y_2), and (x_3,y_3) are the 1″, 4″, and 10″ points on the ruler in line with (a,6), (a,26), and (a,t), respectively.

$y_1 = y_2/4$, $y_3 = 10y_2/4$, $x_1 = ay_1/6 = ay_2/24$, $x_2 = ay_2/26$, $x_3 = x_2 + 6(x_2 - x_1)/3 = 3ay_2/26 - 2ay_2/24 = 5ay_2/156$, and $t = ay_3/x_3 = (10ay_2/4)/(5ay_2/156) = 78$.

Also solved by Michael Jung, Emmet Duffy, Winslow Hartford, Harry Zaremba, Richard Kruger, and Peter Steven.

JAN 5 How many people must be present to give a 50-percent probability of having two coincident birthdays in one year? (Most people are surprised to find that only 23 are needed.) But what is the minimum number of people to give at least a 50-percent probability that there would be three coincident birthdays in the year?

The following solution is from Frank Carbin:

The method of attack is to compute $1 - P_1 - P_2$, where

P_1 = Prob (no shared birthday given N people); and

P_2 = Prob (1 or 2 or . . . n/2) dates with exactly two people having that birthdate, given N people.

The enumerations are done via the expansion of

$(x_1 + \ldots + x_{365})^N$

The no-shared-birthday case corresponds to the terms

$$\frac{N!}{(1!)^N} x_{i_1} \ldots x_{i_N},$$

of which there are $\binom{365}{N}$ giving a probability of

$$\frac{N! \binom{365}{N}}{365^N}$$

The one-date-has-two-people case corresponds to the terms

$$\frac{N!}{2!\,(1!)^{N-2}}\, x_{i1}^2 \cdot x_{i1} \ldots x_{i(N-2)},$$

of which there are $\binom{365}{1}\binom{365}{N-2}$.

giving a probability of

$$\frac{\frac{N!}{2!}\binom{365}{1}\binom{364}{N-2}}{365^N}.$$

More generally, the K-dates-have-two-people case corresponds to the terms

$$\frac{N!}{(2!)^K\,(1!)^{N-2K}}\, x_{i1}^2 \ldots x_{iK}^2 \cdot x_{i1} \ldots x_{i(N-2K)},$$

giving a probability of

$$\frac{N!}{2^K}\binom{365}{K}\binom{365-K}{N-2K}.$$

Thus, the probability that at least three people share a birthday given N people is

$$1 - \frac{N!}{365^N}\sum_{L=0}^{N/2} 1/2^L\binom{365}{L}\binom{365-L}{N-2L}.$$

For N=87 (resp. 88), the probability is .49945 (resp. 51107). Thus, 88 people are required for a 50-percent likelihood.

Also solved by Floyd Klavetter, Winslow Hartford, Matthew Fountain, Emmet Duffy, Harry Zaremba, and the proposer, Ernest Steele.

Better Late Than Never

1980 M/A 3 The proposer submitted a solution for eight contestants and 50-50 probability, a copy of which is available from the editor on request.

A/S 2 Frank Rubin notes that, since the problem did not specify *positive* integers, improved answers are possible. For K=2, {0,2,3}; for K=3, {−9,3,9,11,17}; and for K=4, {−9,1,3,7,12}. Mr. Rubin adds that he has formed a puzzle contest company, and contest entries are available from him at 59 DeGarmo Hills Road, Wappingers Falls, N.Y. 12590 (include a self-addressed stamped envelope).

OCT 2 Michael Jung has responded.
N/D 1 Robert Bart has responded.
N/D 2, 3 Robert Bart and Matthew Fountain have responded.
N/D 4 Robert Bart, L. Upton, and Matthew Fountain have responded.
N/D 5 Robert Bart and Matthew Fountain have responded.

Proposers' Solutions to Speed Problems

SD 1 The train takes two hours to reach the station. In those two hours the bird flies 100 miles.

SD 2 The following is the conversion table for scientific units:

10^{12} microphones	= 1 megaphone
10^{12} pins	= 1 terrapin
10^{-12} boulevards	= 1 pico-boulevard
10^{21} picolos	= 1 gigolo
10 rations	= 1 decoration
10 millipedes	= 1 centipede
1 centipede/second	= 1 velocipede
3⅓ tridents	= 1 decadent
5 holocausts	= 1 Pentecost
10^6 bicycles	= 2 megacycles
10^9 micrometers	= 1 kilometer
	= 200 pentameters
10 monologues	= 5 dialogues
	= 1 decalogue
2×10^3 millinaries	= 4 seminaries[1]
	= 1 binary
10^{-5} dollars	= 1 Millicent
1 milli-Helen	= the amount of beauty required to launch 1 ship.
nano-nano	= a prefix designating 10^{-18}

[1]The enlightenment generated by a *seminary* is measured in *luminaries*.