Puzzle Corner

Allan J. Gottlieb

Once More in the Never-Never Land of Zymurgy

Allan Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Assistant Professor of Mathematics and Coordinator of Computer Mathematics at York College of the City University of New York. Send problems, solutions, and comments to him at the Department of Mathematics, York College, Jamaica, N.Y., 11451.

John Rule has a question for those of you who have worked on **Y1979** (or any other yearly problem). Are there any four digit years for which all the numbers from 1 to 100 are possible? I suspect that there are not but that the only proof anyone will submit is a blank computer output sheet followed by a large bill.

Several readers have asked about computer attempts to play games other than chess. There are efforts to computerize checkers, go, and bridge — among others. Anyone interested is referred to *Personal Computing* where there are regular articles on computerized games.

Problems

J/J 1 We begin with a bridge problem from Emmet Duffy. South, who is on lead with hearts as trump, is to take all six remaining tricks:

North:
♠K 9
♥K
♦4
♣A Q

West:
♠Q J 10
♥—
♦2
♣J 6

East:
♠A 8
♥—
♦—
♣K 5 4 3

South:
♠J 9 3
♥—
♦J
♣A K 3

J/J 2 For the second consecutive issue our second new problem is similar to **1977 Jan 4.** This month's offering, from Norman Wickstrand, does not require knowledge of the old problem:

A dog swims directly towards his master at two miles per hour. The master is directly across the stream at the start. When the dog is two-thirds of the way across the stream his upstream velocity component equals the velocity of the river. The dog swims five minutes longer than if the water had been still. How wide is the river and what is the velocity of the water in the river?

J/J 3 We now turn our attention to a random geometry problem from William Butler:

Two concentric circles are drawn. The inner circle has a radius of one inch while the outer circle has a radius of two inches. A random chord is drawn within the outer circle by the following method. A point (p_1) is located a random distance (0 to 2 inches) at some random direction for the center. A second point (p_2) is determined by the same method. The chord is drawn through the two points. What is the probability that it will intersect the inner circle?

J/J 4 Edwin Nordstrom sends us a problem that brings back memories of the (cheap energy) days when I drove to and from California. Mr. Nordstrom writes:

I recently took a cross-country trip via car. Part of my efforts to stay awake while driving centered on the odometer. Every once in a while the six digits would show mirror symmetry; i.e., they would be in the form xyz-zyx. The question is: How often does this happen during the complete 100,000-mile sequence through the odometer? Also, is there a particular 1,000-mile trip during which it would occur more often than on any other?

J/J 5 Finally, Kenneth Wise wonders what is the longest English word (no proper nouns or chemical compounds, please) in which no letter occurs more than once — that is, all the letters in the word are distinct.

Speed Department

J/J SD1 Frank Rubin has two circular gears joined by a figure-eight belt. The intersection of the belt is n times as far from the center of gear A as from the center of gear B. What can be said about the sizes of the gears?

J/J SD2 We close with a two-part problem from P. Heftler:

A. In the class of positive integers, expressed in the ordinary decimal system, what proportion is either divisible by 3 or includes the digit 3?

B. Same as question A except that the numbers are written in base 3.

Solutions

FEB 1 Find a knight's tour; that is, with one knig... placed on an empty chess board, make 63 moves that result in the knight having been on each square once.

No one attempted an uncrossed knight's tour, but several responders found closed tours — that is, the knight ends on a square from which it can move to the starting square. The following solution is from Jerome Taylor:

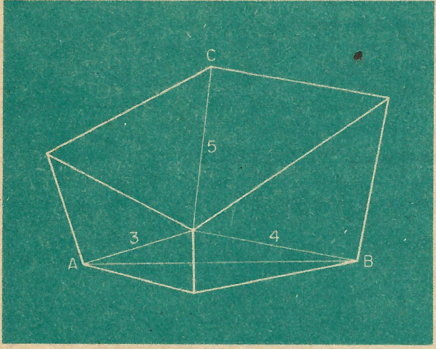
53	46	35	40	51	30	19	26
34	41	52	45	36	27	22	29
47	54	39	50	31	20	25	18
42	33	48	37	44	23	28	21
55	38	43	32	49	12	17	24
64	61	58	5	14	3	8	11
59	56	63	2	9	6	13	16
62	1	60	57	4	15	10	7

Also solved by Harry Zaremba, Ken Zeger, Richard Hess, Emmet Duffy, and Frank Rubin.

FEB 2 Given an equilateral triangle ABC with an interior point P located 3, 4, and 5 inches from A, B, and C, respectively, what is the length of a side of the triangle?

Several readers found solutions involving analytic geometry using cartesian coordinates. The following geometrical solution is from Benn Ross:

The key to solution is to note that the area of the triangle is $x^2\sqrt{3}/4$, where x is the unknown length. Direct use of area formulas leads to intractable-looking equations; however, try the following construction:

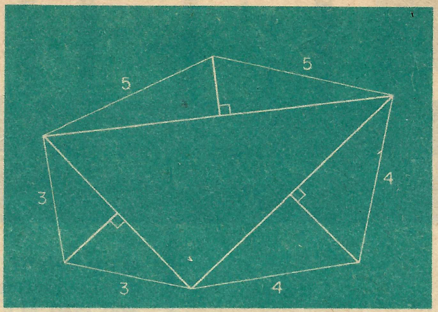


Drop perpendiculars from P to the three sides. Continue the perpendiculars an equal distance beyond the sides. Connect the points so reached with the adjacent vertices of the triangle to form an irregular hexagon (above). Since each small triangle exterior to triangle ABC is congruent to the adjacent small triangle inside ABC, it is easily seen that the hexagon has the following properties:

☐ Its area is twice that of triangle ABC.

☐ The angles at points A, B, and C are all 120°.

☐ The pairs of sides adjacent to A, B, and C are each of length 3, 4, and 5, respectively. We now erase the lines within the hexagon and connect the three other vertices. We also drop perpendiculars from A, B, and C to the three new interior lines:



We now have six 30°-60°-90° right triangles and an inner triangle whose sides can be seen to be $3\sqrt{3}$, $4\sqrt{3}$, and $5\sqrt{3}$. The total area of the hexagon is

$$\tfrac{1}{2} \cdot \sqrt{3}(3^2 + 4^2 + 5^2)/2 + 18 = 25\sqrt{3}/2 + 18.$$

Since this is twice the area of the original triangle $2(\sqrt{3}\, x^2/4)$, we obtain

$$x = \sqrt{25 + 12\sqrt{3}}.$$

Also solved by John Jarvis, Ralphael Justewicz, Monroe Kaufman, Peter McMeriamin, Avi Ornstein, William Katz, Norman Wickstrand, Henry Paynter, Harvey Kaufman, Winslow Hartford, Harry Zaremba, Melvin Garelick, Mary Lindenberg, Oliver Shih, Gerald Blum, Richard Beth, Michael Tersoff, Sidney Shapiro, Robert Kimble, Naomi Markovitz, Winthrop Leeds, Leon Bankoff, Irving Hopkins, Ken Zeger, Bruce Golden, Smith Turner, Richard Hess, Emmet Duffy, Jerome Shipman, Paul Mailhot, Robert Granetz, Frank Rubin, John Wrench, and Roger Powell.

FEB 3 There are nine suspects in a certain crime. When questioned, each answers as follows:

John: "Elvis is guilty."
George: "It was not Elvis."
Ringo: "I did it."
Paul: "It was either Ringo or Tommy."
Elvis: "George isn't telling the truth."
Fabian: "Ringo is guilty."
Chubby: "It was not Ringo."
Tommy: "It was neither Ringo nor I."
Ricky: "Tommy is telling the truth, and it wasn't Elvis either."

Only three of these nine are telling the truth. That being so, who committed the crime?

I have appointed Shirley Wilson forewoman of the jury; here is the jury's verdict and Ms. Wilson's analysis:

The statements of John and George have opposite truth values, as do those of Ringo and Chubby and those of Paul and Tommy. The statements of Ringo and Fabian have the same truth values. Since there are exactly three true statements, which must be included in the three pairs of opposite valued statements, Elvis and Ricky are lying.

(1) Elvis lying ⇒ George is truthful ⇒ Elvis is not guilty. Hence, John is lying and George is telling the truth.

(2) Ricky is lying ⇒ Tommy is lying ⇒ Paul is truthful. Hence, either Ringo or Tommy is guilty.

(3) Since there are only three true statements, Ringo and Fabian must both be lying.

(4) Ringo is lying ⇒ Chubby is truthful ⇒ Ringo is not guilty.

(5) Tommy is guilty.

Also solved by Raphael Justewicz, Scott Nason, Monroe Kaufman, Peter McMenamin, Avi Ornstein, L. Marden, William Katz, Jordan Wouk, Eric Rayboy, Harry Zaremba, Frank Carbin, Mary Lindenberg, Gerald Blum, Robert Kimble, Richard Marks, Ben Ackerman, Naomi Markovitz, Winthrop Leeds, Ken Zeger, Charles Rivers, Smith Turner, Richard Hess, Margaret Marcou, Brad Balfour, Ron Smirlock, Don Trumpler, James Shearer, Frank Rubin, R. Terry, and the proposer, Victor Sauer.

FEB 4 An interesting problem to try on scientific calculators is the generation of integers without the use of integer keys, the arithmetic operators (+, −, ×, ÷), or summation keys. For example, on an HP-45 one is excluded from using any of the bottom four rows of keys. Several problems can be posed:

☐ The number of different ways of generating a particular integer.

☐ The minimum number of keystrokes necessary to generate a particular integer.

Two very intriguing ones are:

☐ The minimum number of keystrokes necessary to generate the numbers 1 through 10 (not necessarily in sequence).

☐ The largest integer of a sequence that starts from 1 that can be generated.

Three comments are necessary for clarification: obviously, solutions vary with the calculator used; on machines with dual function keys, use of a "gold" (or whatever) key and another key constitutes two keystrokes; and numbers must be generated such that the round-off capability of the machine is not used — i.e., the numbers must be integers within the total display capability of the machine.

First of all, Roger Powell sends us the following controversial solution for the HP-45:

The solution of the shortest number of keystrokes necessary to produce the integers 1 through 10 is either three or five, depending on one's definition of keystroke. By pressing first RCL and then CHS, STO, and R↓ simultaneously the display will now show

00.00 00 00

indicating that one has successfully accessed the hidden "timer" on the 45 that was not made explicitly available until the 55. By now pressing CHS, the timer will begin to run and in due course will produce 01.00 00 00, albeit rather briefly, and then later 02.00 00 00, etc. The ambiguity in the number of keystrokes, then, is due to the possibility that three keys pressed at the same time constitute only one logical keystroke. At any rate, even five will easily be the optimum solution.

Smith Turner's favorite is the TI-SR50:

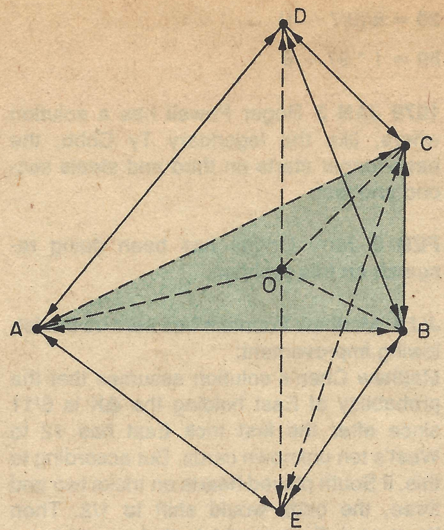
1	e or !							
2	e	e	x^2	1_n				
3	E	π	x^2	x^2	x^2			
4	e	e	x^2	x^2	1_n			
5	E	π	e	x^2	x^2			
6	E	π	e	1/x	x^2	x^2		
7	E	π	x^2	x^2	x^2	x^2		
8	e	e	x^2	x^2	x^2	1_n		
9	E	π	log	log	x^2	x^2	x^2	x^2
10	E	π	e	x^3	x^3	x^3		

Finally, Joe Sansone submitted the solution in the box at the bottom of this page for the HP-35. All solutions start from 0 (power on), except as noted. He describes this as a problem in learning how your particular model rounds off numbers and then learning how to get around it, i.e. getting 3. rather than 2.999999999. Evidently, he never did solve 7. By the way, getting 3. in 16 steps as shown above was the toughest of the bunch.

FEB 5 Consider n points on the surface of a sphere, free to move anywhere on the sphere's surface. The problem: if n = 5 and the points repel each other (that is, they assume positions that maximize the minimum distance between any two of the n points), where will the points go? For n = 2, they go to the ends of a diameter. For n = 3, they wind up on a great circle and form an equilateral triangle. For n = 4, the points move to the vertices of a tetrahedron. But what about n = 5?

We have two different arguments for believing the following 3-2 solution. Harry Zaremba's argument is physical in flavor whereas Avi Ornstein sends us a chemist's solution. We begin with Mr. Zaremba's diagram at the top of the next column:

Let the five points be designated by A through E. Three points, A, B, and C, will be the vertices of an equilateral triangle within a great circle, and the other two points D and E will be at the ends of a diameter perpendicular to the plane of the triangle at 0. The repellent forces which will act on any point will be symmetrically disposed with respect to a radius of the sphere passing through the point, and will maintain the point in equilibrium. The direction of the resultant of these force components will coincide with that of the radius. The resultant force at D will equal the force at E, and the resultant at A will equal each of those at B and C. The



five points will form two identical pyramids having a common base ABC.

Mr. Ornstein's solution (he explains that he teaches honors chemistry and a course in pure mathematics) follows: First, n = 2 has the same position as an sp hybrid bond, such as in $CaCl_2$. When n = 3, the solution is the same as an sp^2 hybrid bond, such as in $AlCl_3$. When n = 4, the solution is the tetrahedral structure of the sp^3 hybrid bond, such as CH_4 or CCl_4. For n = 5, I believe the answer would be the combination of n = 2 and n = 3. Three of the points would be 120° apart from each other on the equator of the sphere, while an additional point would be at each of the poles, 90° from the equator. This matches an sp^3d hybrid bond, as in PCl_5. The same positioning is also supported by some other bonding which includes filled orbitals. In $XeCl_2$ the chloride ions are at opposing poles while the filled orbitals are on the equator. Likewise, the filled orbitals are at the poles and the chloride ions are at the equator in ICl_3.

Also solved by Raphael Justewicz, Eric Rayboy, Winslow Hartford, Harry Zaremba, Gerald Blum, Michael Tersoff, Irving Hopkins, Richard Hess, Jerome Shipman, and Rob Cave.

FEB SD 1 Three people, A, B, and C, are lined up so that A can see B and C, B can see C, and C can see no one. While A, B, and C are not looking, one of five hats, three black and two white, is placed on each of

1	2	3	4	5	6	8	9	10
e^x	ARC	ARC	ARC	ARC	start	ARC	start	ARC
	COS	COS	COS	COS	from 3	COS	from 3	COS
	TAN	TAN	TAN	TAN	e^x	TAN	STO	TAN
	LOG	LOG	LOG	$\sqrt{x}$	STO	STO	e^x	LOG
	LOG	LOG	STO	$\sqrt{x}$	R↓	$\sqrt{x}$	RCL	$\sqrt{x}$
		STO	LOG	LOG	ARC	$\sqrt{x}$	x⌒y	
		1/x	RCL	$\sqrt{x}$	COS	$\sqrt{x}$	x^y	
		ARC	x^y		TAN	LOG	LN	
		SIN	LOG		LOG	1/x		
		TAN			LOG	RCL		
		RCL			RCL	x^y		
		x⌒y			x^y	LOG		
		x^y			LN	1/x		
		e^x				1/x		
		LN						
		1/x						

their heads. A, B, and C are then asked what color each's hat is. A reports that he cannot determine this, and B gives the same answer. What is the color of C's hat?

As you know, solutions to speed problems are given at the end of the column in which the problem appears. But this time the given solution was incorrect, as Turner Gilman explains:

If A sees two white hats he knows that his is black. So B and C either both have black hats or one is black and one is white. Knowing this, if B sees a white hat he knows his is black. So C must have a *black* hat.

The error was also found by Elliot Roberts, Robert Logcher, Warren Dietz, Jerry Bozman, Raphael Justewicz, Shirley Wilson, Avi Ornstein, William Katz, L. Rosmussen, Parker Chapman, John Moulson, Jordan Wouk, Eric Rayboy, J. Friedman, Gerald Blum, Elmer Ingraham, Jeff Wisnia, Gregg Bemis, Leslie Toth, Paul Manoogran, David Mackapetris, Joel Ackerman, Hugo Mayer, James Shearer, R. Terry, and Robert Granetz.

Better Late Than Never

Y1978 Several readers found solutions which used 1, 9, 7, and 8 in order but required more operators than the published solution. This is not desired. There were, however, legitimate improvements found by Charles Rivers, Lou Cesa, and Clark Baker:

$3 = 1 + (9 + 7)/8$

$28 = 8/(9/7 - 1)$

$89 = 1 * 97 - 8$

1978 JAN 3 Roger Powell has a solution where, like the legendary Ty Cobb, the base-runner starts on third and steals second and first.

FEB 3 Jerry Griggs has been doing research on this problem.

J/J 1 Matthew Fountain has sent us the following improvement:

Matthew Chen's solution assumes that the probability of East holding the ♠K is 6/11 since after the first trick East has 12 to West's ten unknown cards. But according to this, if South played hearts on tricks two and three, the odds would shift to 1/2. Then West and East would both have ten unknown cards. Here is a somewhat similar case: I take 26 cards at random from a deck. Before I look at them I may persuade someone to wager at even money on whether or not I have a ♠K. But would anyone offer me better odds if I exposed 25 of my cards after looking at them all? They would suspect my last card had 26 times as much chance of being the ♠K as any of the individual unknown cards still in the deck. I believe the correct probability for East holding the ♠K is 13/23, provided West is committed to always leading hearts when he has three hearts and North and South have the given holdings.

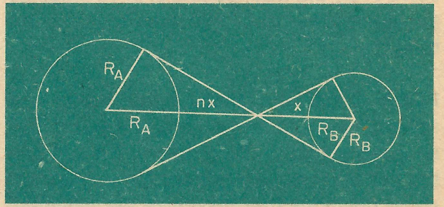
NOV 4 Craig Murphy has responded.

NOV 5 Tom Weil has found some "pseudowords" that have five consecutive letters.

D/J 3 Thomas Harriman has responded and Frank Rubin, the proposer, informs me that a sentence was omitted. The questioning was supposed to start with Adak. He is not sure that the "truncated problem" as published is solvable. Since a solution was given in the last issue there apparently is no trouble.

Proposers' Solutions to Speed Problems

SD 1 By similar triangles (see diagram), $RA/(RA + nx) = RB/(RB + x)$. Thus the radius of A is n times the radius of B.



SD 2 In the case of A the answer is 99.99+ per cent, since numbers with many digits are "bound" to have least one 3 and "essentially all" positive integers have "arbitrarily many" digits. In the second case the answer is 33⅓ per cent, since in base three there is no digit 3.