"Too Obvious . . . to Make 'm Think, I think"

Allan Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Associate Professor of Mathematics and Coordinator of Computer Mathematics at York College of the City University of New York. Send problems, solutions, and comments to him at the Department of Mathematics, York College, Jamaica, N.Y., 11451.

Randy Wright has written to me from prison asking for people to correspond with him to help break the boredom of prison life. Although I have never met Mr. Wright and do not know why he is in prison, his letter leads me to believe that he is articulate and genuinely interested in self-improvement. His address is:
146106, P.O. Box 45699
Lucasville, Ohio, 45648.

Tomorrow, February 5, is a big day at the Gottliebs'. Alice is to give her public thesis talk, and we are both rather excited. She has worked very hard on her project, and I am pleased to dedicate this column to that imminent doctor, Alice Gottlieb.

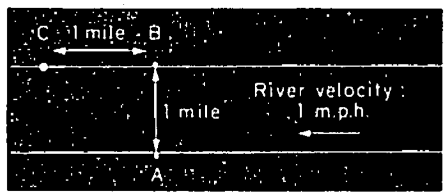
Problems

NS 15 This problem, which first appeared as 1976 DEC 5, was submitted by Norman Wickstrand:
There are 4n tennis players who wish to play (4n − 1) doubles matches, where n equals any positive integer. How can the matches be arranged so that all players play in every match with the limitation that each player plays with each other player once only and against each other player the same number of times? When n = 1 the solution is easy and quite obvious. Is there a general solution or formula or system? Is it limited to perhaps n = 5 or n = 6?

Some partial results were obtained in 1977. The late R. Robinson Rowe presented a solution for n = 2 in the March/April, 1977, issue. Later in the year (July/August), Morton Mathew outlined a solution for n = 5 and claimed to have solution up to n = 6, and Harry Zaremba (October/November) gave a solution for n = 4 and conjectured that solutions exist for 1, 2, 4, 8, 16, . . .

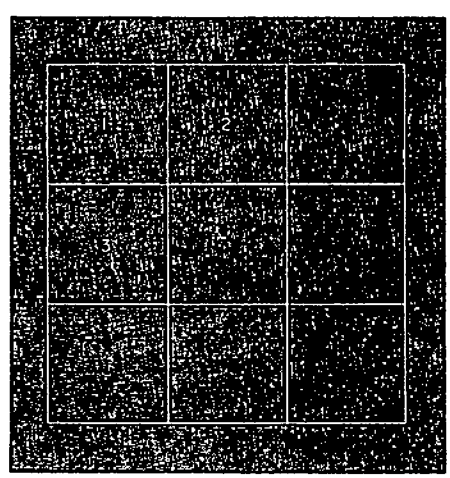
MAY 1 Our first problem this month is from Elliot Roberts, who claims that this is the most beautiful chess problem he has ever seen:
White has four pieces — king, queen, and both bishops — all in starting position on the first rank. Black has a lone king on Black's K4. White is to mate in three.

MAY 2 Arthur Hovey poses a variant on 1977 JAN 4. The original problem was:
A swimmer, who swims at a constant rate of two miles per hour relative to the water, wants to swim directly from point A to a point C, which is one mile downstream and on the other side of a river one mile wide which flows one mile per hour. At what angle should the swimmer point himself, relative to the line AB (perpendicular to the river)?



Now Mr. Hovey wishes to permit the swimmer to swim to any point between B and C and then walk from that point to C. For a given walking speed (which is greater than the swimming speed), you are to find the optimum angle — i.e., the angle for which the travel time is minimized. (As before, obtaining the right equation is a sufficient solution, since tables are needed to obtain the final answer.)

MAY 3 Our final regular problem this month is from Peter Groot, who wants you to complete the following 3 × 3 magic square. That is, fill in the remaining six boxes so that the sum of the entries along any row, column, or (main) diagonal is the same.



MAY 4 Ken Austin wants you to find a continued fraction expansion for the square root of x. That is a continued fraction written in terms of x that, for each positive x, converges to the square root of x.

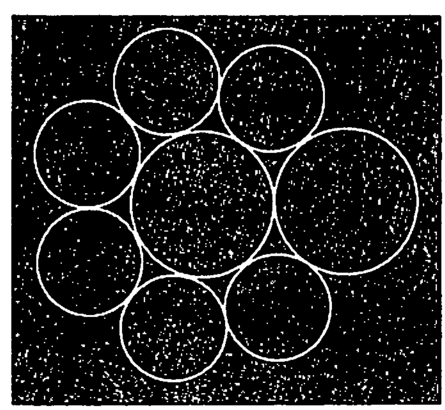
Speed Department

MAY SD 1 Our first speed problem first appeared in *Technology Review* in January, 1940. However, the problem seems particularly timely:
Compare an electric clock, requiring 2 watts power, with an old-fashioned grandfather clock driven by a descending ten-pound weight. How far must the weight drop in 24 hours to furnish power equal to that used by the electric motor?

MAY SD 2 We end with a commuting problem from William McGuinness:
A certain businessman customarily arrives at his home station from work at 5 p.m. sharp. His chauffeur, through long practice, times his run so as to arrive at the station exactly on time. But one day the businessman arrives at his station an hour earlier, at 4 p.m. Not having phoned, he is not surprised to find that his chauffeur is not there. Rather than phone from the station, he decides to start walking home. On the way he meets his surprised chauffeur, who is proceeding on his normal schedule to meet the 5-p.m. train. Together they drive home and arrive ten minutes earlier than usual. How many minutes did the businessman walk?

Solutions

NS 13 (first published as 1974 JUN 5) Two large coins and six small coins are placed on a table, each just touching its neighbors as shown in the sketch. What are the relative diameters of the coins?



When this problem was originally published, several readers found a polynomial

Continued on page 90

Continued from p. 88
equation that the solution was required to satisfy. Approximate solutions to the equation were also given. This time, we were asking for an exact solution. Several readers have responded, and lower-degree polynomials have been obtained (eight); but still no exact solution has been found. Solomon Golomb has written a paper entitled, "Circles Surrounded by Circles," that discusses this and related problems; anyone interested should write Dr. Golomb at the Department of Electrical Engineering, University of Southern California, University Park, Los Angeles, Calif., 90007. Although no one mentioned it, I must point out that there is no reason to believe a "closed form" solution exists. There are polynomials of all degrees greater than four for which the roots are not "solvable by radicals" (a technical term meaning that the roots are *not* of the form

$$\sqrt{63 + (9\sqrt[4]{5})}$$

or any more complicated form involving just addition, subtraction, multiplication, division, and roots). However, no reader has shown that the polynomial in question has this property. Thus the problem is still open.

Responses have been received from Randall Rathbun, Emmet Duffy, Judith Longyear, Temple Patton, Dave Mohr, Matthew Fountain, and Winthrop Leeds.

D/J 1 With both sides vulnerable, and the bidding as shown, how can the contract be defeated?

North:
♠A J 5 2
♥K 3
♦6 4 2
♣K J 7 6

West:
♠8 7 4
♥Q 6
♦9 3
♣A Q 9 8 4 2

East:
♠K Q 10 9 6 3
♥7 5
♦K 10 7
♣10 5

South:
♠—
♥A J 10 9 8 4 2
♦A Q J 8 5
♣3

North	*East*	*South*	*West*
1 club	1 spade	2 hearts	2 spades
—	3 spades	4 diamonds	—
4 hearts	—	6 hearts	—
—	—		

As many readers noted, if West plays the ♣A and a trump, declarer has only one entry to the board and thus cannot take the two required diamond finesses. Elmer Ingraham had so little trouble with this one that he decided to compose his answer as a poem:

Dear Allan, Hi! Your Dee/Jay one distressed my eye, for you should see as well as I, it should have been a quickie.

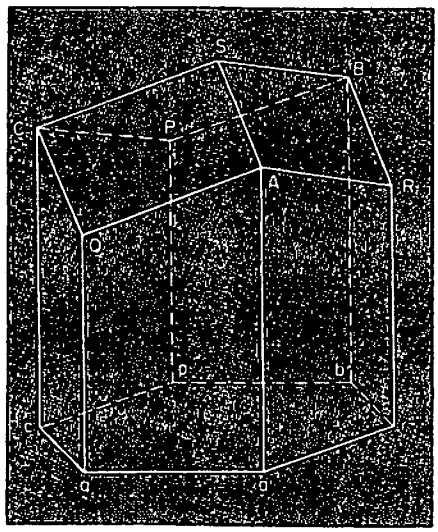
South will never make six hearts, his book, if West lead ace of club before he look and after looking shift to either trump.

A hand spectacular no doubt in Alan Truscott's nook where other Wests choose other leads and South makes book or overtrick with ease.

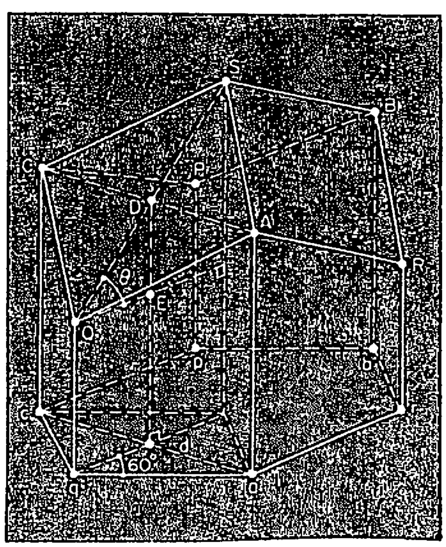
Too obvious, I think, for Allan Gottlieb's book where double dummy spoils the odds but puzzlers need the challenge of design to make 'm think, I think.

Also solved by Peter Steven, Robert Brown, Jonathan Hardes, R. Bart, Michael Kay, Robert Simon, Edwin McMillan, Gordon Bart, Winslow Hartford, John Chandler, Richard Hess, Michael Tersoff, James Prigoff, Hans van Gelder, Scott Nason, and the proposer.

D/J 2 Given a prism with a hexagon base and a roof of three congruent rhombuses CSAQ, ASBR, and BSCP all tilted at the same angle with respect to a horizontal plane, what angle yields minimum surface area for a fixed volume?



Sidney Self, who assumes that Emmet Duffy is a fellow bee-keeper, enclosed a copy of an entry from the 1895 edition of *ABC of Bee Culture* by A. I. Root. Winslow Hartford noted that the shape is identical to a calcite crystal he owns. And Harry Zaremba sent the following solution:



The base of the prism is a regular hexagon in view that the rhombi are congruent. If we let s equal the length of each side of the base, then ad $= \sqrt{3}s/2$, and gd $= s/2$. The area of the base will equal 12 times the area of triangle gda, or,

$A_1 = 12 \cdot (ad \cdot gd)/2 = (3\sqrt{3}s^2/2)$.

The areas of the rhombi are equal to the base area projected on their planes inclined at angle θ with the horizontal; hence the total area of the rhombi is

$A_2 = A_1/\cos\theta = (3\sqrt{3}s^2)/2 \cos\theta$.

In the figure, QE = gd = s/2. Hence, in triangle QED, DE = $(s \tan\theta)/2$.
If we let Qq = h, then Dd = Qq + DE = $h + (s \tan\theta)/2$.
The area of trapezoid QDdq is equal to

$A_T = qd(Qq + Dd)/2$, or

$A_T = sh/2 + (s^2 \tan\theta)/8$.

This area projected on face AQqa results in a face area of

$A_3 = A_T/\cos 60° = sh + (s^2 \tan\theta)/4$.

The total surface area of the prism becomes,

$A = A_1 + A_2 + 6A_3 = (3\sqrt{3}s^2)/2$
$+ (3\sqrt{3}s^2)/2 \cos\theta$
$+ 6[sh + (s^2 \tan\theta)/4]$.

Because of the symmetry of the prism about its vertical axis through point S, the mean height of the figure is,

$Aa = Dd = h + (s \tan\theta)/2$.

The volume of the prism will equal the product of its mean height and base, thus:

$V = [h + (s \tan\theta)/2\,]\,(3\sqrt{3}s^2)/2$.

Solving for h from the prism's volume,

$h = 2V/3\sqrt{3}s^2 - (s \tan\theta)/2$.

Substituting h into the expression for A results in

$A = [3\sqrt{3}s^2(1 + 1/\cos\theta)]/2$
$+ 4\sqrt{3}/3 \cdot v/s - (3s^2 \tan\theta)/2$.

Differentiating A partially with respect to θ, setting equal to zero, and simplifying yields

$\sin\theta = \sqrt{3}/3$.

Hence, the angle which gives the minimum surface area for a fixed volume of the prism is,

$\theta = \sin^{-1}\sqrt{3}/3 = 35.26438968°$,
or $\theta = 35°15'51.8''$.

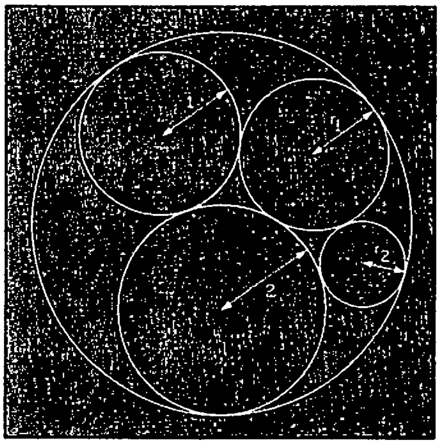
Also solved by Peter Steven, Arthur Hovey, R. Bart, Robert Simon, Edwin McIntee, John Chandler, Richard Hess, and the proposer, Emmet Duffy.

D/J 3 The King wanted to find out who was the wisest among his three Grand Counselors. He blindfolded the three and then announced that he had eight Great Seals, four purple and four gold. He would place two upon the forehead of each counselor, so they could see the four seals on their neighbors but not their own.

have a doubleton of A's color, and A's "pass" told B that he did not have a doubleton of the other color. Therefore, B knew that he must have one seal of each color and that the chest held the same. It remains to show that, given this configuration (doubletons of opposite color on A and C, and mixed colors on B), C with the help of B's "pass," or A with the help of "passes" from B and C, could not have solved the problem before B's second try. Consider C: he knows from B's "pass" that he has not a doubleton matching A's, but he still must "pass" since he could have mixed colors or a doubleton of opposite color and B still would have "passed." Consider A: the information he got from B's "pass" was the same as C got, and it was not enough; and A got no information from C's "pass" since B's mixed colors precluded C from reaching a solution no matter what he saw on A. Therefore, neither A nor C could have solved the problem before B's second try. Note that this proof does not utilize two given facts: that one of A's seals was G and one of B's was P. These facts are redundant to the rest of the statement. The problem would be more elegant if it merely said that the King put two seals on the head of each counselor. Perhaps this redundancy was deliberately introduced to confuse.

Responses were also received from Avi Ornstein, Peter Steven, Stuart Kreiger, William Naylor, Elmer Ingraham, Jonathan Hardis, Dorothy Hirchen, Timothy Wheeler, William McGuiness, R. Bart, Matthew Marcou, Anne Symanovich, Robert Simon, John Prussing, Edwin McIntec, P. Jung, Jim Bier, Robert Shoashan, John Chandler, Richard Hess, Michael Tersoff, Frederick Nash, Naomi Markovitz, Scott Nason, and Herb Riddle.

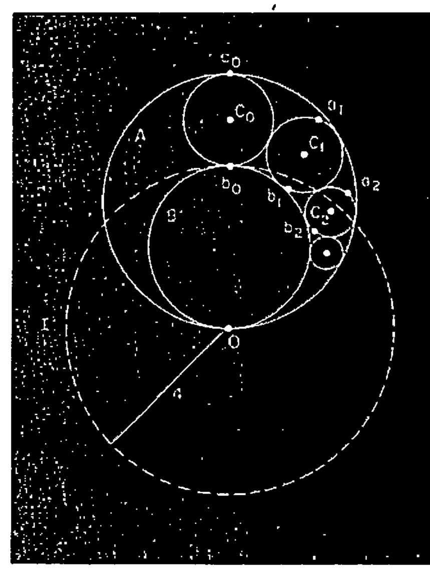
D/J 4 Given three circles of radii 1, 2, and 3, the largest circle that can be drawn inside the biggest circle and outside the other two is $r_1 = 6/7$. Find r_2; and see if there is a general formula for r_n.



Three different methods of solution were found. Fortunately, all lead to the conclusion that the radius of the nth circle is $6/(6 + n^2)$.

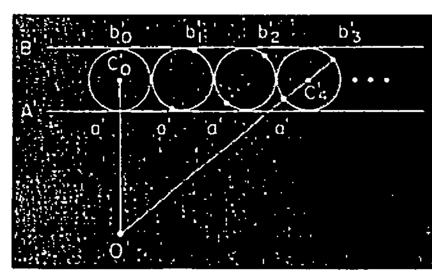
Most readers went in there and got their hands dirty with the necessary algebra and trigonometry. Leon Bankoff applied a theorem of Pappus relating the location of the center of the nth circle to its radius. Interested readers are referred to Martin Gardner's column in the January (1979) issue of *Scientific American.* Chuck Livingston's solution utilized complex analysis. Finally, Richard Hess sent in the following solution involving inversions:

1. Form the inversion circle I, as shown.



In normal space

2. In inversion space, the problem looks as shown below with distances $r' = 16/r$ relative to center 0'.



In inversion space

3. $O'C_0' = (O'a_0' + O'b_0')/2$
$= (16/6 + 16/4)/2$
$= 10/3.$

4. $C_1'C_n' = 4n/3$; thus
$O'C_n' = (2\sqrt{25 + 4n^2})/3$
$O'a_n' = 2(\sqrt{25 + 4n^2} - 1)/3$
$O'b_n' = 2(\sqrt{25 + 4n^2} + 1)/3$

5. $(Oa_n - Ob_n)/2 = 12/(\sqrt{25 + 4n^2} - 1) - 12/(\sqrt{25 + 4n^2} + 1) = 6/(6 + n^2).$
Therefore $r_n = 6/(6 + n^2)$.

Also solved by Peter Steven, Mary Lindenberg, Harry Zaremba, R. Bart, Robert Simon, Jeffrey March, R. Howell, P. Jung,

Continued on page 95

(Continued from p. 92)
Winslow Hartford, Norman Wickstrand, Emmet Duffy, Michael Tersoff, Naomi Markowitz, and the proposer, Robert Lutton.

Better Late Than Never

NS 10 B. Laporte has a "Pascal triangle" formulation to calculate the sum, but still no closed form has been found.
NS 12 E. Phillips has responded.
M/A 5 Irving Hopkins notes that we still have the wrong formula for (cos 18°). The formula should be $5 + \sqrt{5}/8^{\frac{1}{2}}$.
A/S 2 Hugh Covington feels that we should include the host and/or hostess in this problem, thus raising the number of handshakers to 11 or 12.
OCT 4 J. Kramer has responded.
OCT 5 Irving Hopkins has responded.
NOV 2 Joseph Keilin and J. Kramer have responded.
NOV 3 Joseph Keilin and Irving Hopkins have responded.
NOV 4 Joseph Keilin has responded.

Proposers' Solutions to Speed Problems

SD 1 More than two miles!
SD 2 If the chauffeur arrived home ten minutes earlier than normal, he must have saved five minutes on the station-ward leg of his trip. Therefore, he met the businessman at five minutes to 5 p.m. Thus the businessman walked 55 minutes.

Cowen

(Continued from p. 11)

ing work in the Soviet Union. For over a decade, some Soviet mineral geologists have been using dowsing, reportedly successfully, as a supplement to conventional prospecting techniques. N. N. Sochevanov and V. S. Mateev of the All-Union Scientific Research Institute of Hydrology and Engineering Geology in Moscow have described how contour maps of the intensity of dowsing reactions show maximal coinciding with geochemical anomalies and have sufficient detail to guide successful explorational drilling. Commenting on this work in *New Scientist,* Dr. Williamson points out that geological analomies related to mineral deposits or underground flowing water may well induce slight changes in the geomagnetic field.

Here again is a relatively clear-cut physical question. Do humans sense small geomagnetic changes or not? If they do, is this related to dowsing ability? There is so much bunkum going around about softening experimental standards and dispensing with scientific method to accommodate the paranormal that such things should be vigorously investigated. As Dr. Williamson aptly observes: "... if the solution of an age-old mystery, frequently linked with clairvoyance and the occult, were shown after all to be merely a question of thorough analysis it would be a timely demonstration of the value of scientific method." □

Bankers Discriminate Against People, not Places

Most observers believe that "redlining" — the denial of mortgages or loans to certain property-owners — is a function of the location of property: banks don't like to lend money on houses in slum areas, for example, even though the specified return is as high as that from any other possible loan.

But that's wrong, says Harvard's Professor Robert Schafer, reporting a study at the Harvard-M.I.T. Joint Center for Urban Studies. "Individuals, more than neighborhoods, are the object of discrimination."

Professor Schafer studied five metropolitan areas from New York City to Buffalo; he found discrimination against individual black loan applicants in four of the five. Black applicants have twice as great a chance of being turned down for a mortgage as do white applicants with similar socio-economic, property, and neighborhood characteristics.

Professor Arthur Solomon, director of J.C.U.S., says the study "clarifies a national problem," and he thinks it should be the basis for "constructive action by both the public and private sectors." □

Geology Under the Atlantic

The mid-Atlantic ridge, that crack in the sea floor between the Eastern and Western Hemispheres where new crust is formed as the plates which make up the earth's continents are pulled apart, held three surprises for scientists on the deep-diving submarine *Alvin* last year:

□ Tilting, rather than faulting, is the major form of displacement along the rift valleys in the mid-Atlantic; that observation imposes some new conditions on hypotheses of geological action at the ridge.

□ The mid-Atlantic rift valleys have a common evolutionary history: all show evidence of filling up and breaking down as new ocean floor is formed.

□ Sheet flows of lava, as well as bulbous "pillow" lava flows, appear in the mid-Atlantic areas; how the sheet flows occur isn't yet clear.

Professor Tanya M. Atwater of M.I.T. was chief scientist on the expedition, and scientists from six institutions were on board under National Science Foundation sponsorship. □

No lens and no camera were needed to make this silhouette photograph of an eel larva, copepods, shrimp, and plankton in water from the Sargasso Sea. Resolution was "more than adequate . . . to make generic and, occasionally, specific identifications of specimens," say the developers of the system.

A Lensless Photo System to Capture Sub-Sea Life

You're in the middle of the Sargasso Sea on a rolling oceanographic research vessel. Someone has handed you a sample of sea water just brought over the rail; you're asked, "What's in it?"

Until now, your only alternatives were to peer through a microscope at a small sample of the water, moving as the ship moves, and identify and count the organisms at best you can; or to tell your colleagues to be patient, preserve the sample (but plankton species often deteriorate in preservatives), and make the examination with the more efficient techniques available at shore.

Now there's a better way: bring the sample into your darkroom, put a sheet of unexposed film on the bottom of a plastic tray, cover the film with sea water from the sample, and flash a very fast strobe lamp — three-millionths of a second — at a distance of one meter to expose the film. When processed, the film will show silhouettes of all the creatures, great and small, between film and light in the sea water sample. The result is "a permanent record of a live sample unobtainable at present by any other means," say Professor Harold E. Edgerton and Jeffrey S. Wilson of M.I.T. and three scientists at N.O.A.A.'s Atlantic Oceanographic and Meteorological Laboratories, Miami — "the first adequate method available to characterize a sample of zooplankton quickly and accurately." *(Nature, January 4, 1979, pp. 80-81)* □