Back to Aardvark and Zymurgy

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In October I asked readers to report which types of problems are most (and least) favored. The response, although light, has been interesting. The chess and bridge problems received almost all the attention. But there was no consensus: about half of the respondents considered chess and bridge problems to be among the best while the other half placed them among the worst. Far fewer people mentioned the "finite" problems — i.e., those where it is possible (with a calculator or computer, perhaps) to try all the possibilities: most of the comment on these was favorable. Geometry problems were also mentioned, but — like chess and bridge — I can't read any clear preference.

Leon Bankoff has kindly sent me a photograph of R. Robinson Rowe taken at a recent conference. Regular readers will appreciate how pleased I am to have it.

The solution given in October to NS12 contains an inaccuracy. See "Better Late Than Never" for details.

Problems

Y1979 This being the first issue for the new year, we begin with our yearly problem: form as many as possible of the integers from 1 to 100 using the digits 1, 9, 7, and 9 once each (thus 9 will be used twice) and the operators +, −, * (multiply), / (divide), and ** (exponentiation). We desire solutions containing the minimum number of operators; and, for a given number of operators, solutions using the digits in the order 1, 9, 7, and 9 are preferred. The solutions to Y1978 are given below.

NS 14 This problem was first published as 1976 FEB 2, and Eric Jamin is still waiting for an answer: How many sequences can be formed using the 28 dominoes? We require legal (in the domino-theoretic sense) sequences. The proposer feels that the answer is 7,959,229,931,520, and that an important quantity to calculate is the number of complete closed routes that exist on a complete 7-gon (a heptagon with all its diagonals). If you wish to reduce the problem to a complete 7-gon, that is fine. But you must show us how to calculate the number of complete closed routes.

FEB 1 We begin this month's regular selection with a very famous chess problem from Steven Ross, who asks you to find a knight's tour. That is, with one knight placed on an empty board, make 63 moves that result in the knight having been on each square once. I might add that you could try to have the knight end on a square from which it can go to its original square; or for even more fun try to avoid having the knight cross its own path (but in this variation don't expect to be able to cover all 64 squares).

FEB 2 Next we have a geometry problem from Norman Spencer: Given an equilateral triangle ABC with an interior point P located 3, 4, and 5 inches from A, B, and C, respectively, what is the length of a side of the triangle?

FEB 3 Victor Sauer submitted the following "entertaining" problem, which he credits to someone else; but unfortunately I cannot read his handwriting at that point. There are nine suspects in a certain crime. When questioned, each answers as follows:

John: "Elvis is guilty."
George: "It was not Elvis."
Ringo: "I did it."
Paul: "It was either Ringo or Tommy."
Elvis: "George isn't telling the truth."
Fabian: "Ringo is guilty."
Chubby: "It was not Ringo?"
Tommy: "It was neither Ringo nor I."
Ricky: "Tommy is telling the truth, and it wasn't Elvis either."

Only three of these nine are telling the truth. That being so, who committed the crime?

FEB 4 Here is a real calculator problem from Bruce Andeen:

An interesting problem to try on scientific calculators is the generation of integers without the use of integer keys, the arithmetic operators (+, −, ×, ÷), or summation keys. For example, on an HP-45 one is excluded from using any of the bottom four rows of keys. Several problems can be posed:

- ☐ The number of different ways of generating a particular integer.
- ☐ The minimum number of keystrokes necessary to generate a particular integer.

Two very intriguing ones are:

- ☐ The minimum number of keystrokes necessary to generate the numbers 1 through 10 (not necessarily in sequence).
- ☐ The largest integer of a sequence that starts from 1 that can be generated.

Three comments are necessary for clarification: obviously, solutions vary with the calculator used; on machines with dual function keys, use of a "gold" (or whatever) key and another key constitutes two keystrokes; and numbers must be generated such that the round-off capability of the machine is not used — i.e., the numbers must be integers within the total display capability of the machine.

FEB 5 We close with a solid geometry problem from Eric Osman. Consider n points on the surface of a sphere, free to move anywhere on the sphere's surface. The problem: if n = 5 and the points repel each other (that is, they assume positions that maximize the minimum distance between any two of the n points), where will the points go? For n = 2, they go to the ends of a diameter. For n = 3, they wind up on a great circle and form an equilateral triangle. For n = 4, the points move to the vertices of a tetrahedron. But what about n = 5?

Speed Department

FEB SD 1 Ronnie Rybstein has A, B, and C line up so that A can see B and C, B can see C, and C can see no one. Mr. Rybstein then announces that he has three black hats and two white ones. While A, B, and C are not looking, Mr. Rybstein places one hat on each of their heads. He then asks each what color his hat is. A reports that he cannot determine this, and B gives the same answer. What is the color of C's hat?

FEB SD 2 Mr. and Mrs. D. Szper have two poles in their yard (and neither one is the Pope — ed.) with 150 feet of rope strung from the top of one to the top of the other. Each pole is 100 feet tall. How far apart are the poles if the lowest point of the rope is 25 feet from the ground?

Solutions

Y 1978 Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 7, and 8, according to the same rules which are given above for Y 1979. There is agreement that 24 of the numbers cannot be formed. An optimal solution for the 76 possible numbers is:

1. 1**978	51. ——
2. 81 − 79	52. ——
3. 9 − 7 + 1**8	53. ——
4. 91 − 87	54. 71 − 9 − 8
5. (91/7) − 8	55. (8*9) − 17
6. 7 − (1**98)	56. 1 + (9*7) − 8
7. (71 − 8)/9	57. (1**9) + (7*8)
8. (1**97)*8	58. (8 − 1)*7 + 9
9. (1**97) + 8	59. 78 − 19
10. (79 + 1)/8	60. ——
11. 1 + 9 − 7 + 8	61. 79 − 18
12. (97 − 1)/8	62. (1 + 9)*7 − 8
13. 91 − 78	63. 7*81/9
14. 7*18/9	64. (17 − 9)*8
15. 1 + 98/7	65. 81 − 9 − 7
16. 97 − 81	66. 1 + 9 + (7*8)
17. 17*(9 − 8)	67. ——
18. 89 − 71	68. 87 − 19
19. 19*(8 − 7)	69. 78 − (1*9)
20. 19 − 7 + 8	70. 1 − 9 + 78
21. (91/7) + 8	71. 79 − (1*8)
22. ——	72. 89 − 17
23. 9 + 7 + 8 − 1	73. (1 + 9)*8 − 7
24. 1*9 + 7 + 8	74. 9*(8 + 1) − 7
25. 1 + 9 + 7 + 8	75. 19 + (7*8)
26. ——	76. 91 − 8 − 7
27. 98 − 71	77. 87 − 9 − 1
28. ——	78. (1**9)*78
29. ——	79. 97 − 18
30. ——	80. 79 + (1**8)
31. ——	81. 98 − 17
32. ——	82. 89 − (1*7)
33. ——	83. 89 − 7 + 1
34. 19 + 7 + 8	84. ——
35. 91 − (7*8)	85. ——
36. 18*(9 − 7)	86. 79 + 8 − 1
37. (7*8) − 19	87. (1*9) + 78
38. ——	88. 1 + 9 + 78
39. (7 − 1)*8 − 9	89. (8*9) + 17
40. (8 − 1)*7 − 9	90. 1 + 97 − 8
41. ——	91. 98 − (1*7)
42. ——	92. 98 − 7 + 1
43. ——	93. ——
44. ——	94. ——
45. (7*9) − 18	95. 87 + 9 − 1
46. (7*8) − 9 − 1	96. (19 − 7)*8
47. (7*8) − (1*9)	97. 19 + 78
48. 1 − 9 + (7*8)	98. 98*(1**7)
49. 7**(18/9)	99. (18 − 7)*9
50. ——	100. ——

Responses were received from Warren Lane, Harry Zaremba, David Schoengold, Mike Bercher, John Rudy, Harry Hazard, Avi Ornstein, Gardner Perry, A. Holt, and Charles Rivers.

OCT 1 Conventional point counting (in bridge) gives four points for each ace, three per king, two per queen, and one per jack. The average bridge hand has ten such points. What is the probability of receiving a hand with exactly ten points?

Everyone agrees that the probability is about 9 per cent. Several computer programs confirm Harry Zaremba's careful calculation that the exact answer is 59,723,754,816 / 635,013,559,600. His solution follows:

Let the letters A, K, Q, J, and X designate an ace, king, queen, jack, and any card which has no points, respectively. There will be 52 − 4 · 4 = 36 cards which have zero points. In the table are the card distributions in a bridge hand that will total exactly ten points and the number of combinations for each distribution:

Number	*Distribution*	*Combinations*
1	2A + Q + 10X	6,100,484,544
2	2A + 2J + 9X	3,389,158,080
3	A + 2K + 10X	6,100,484,544
4	A + K + Q + J + 9X	24,100,679,680
5	A + K + 3J + 8X	1,936,661,760
6	A + 3Q + 9X	1,506,292,480
7	A + 2Q + 2J + 8X	4,357,488,960
8	A + Q + 4J + 7X	133,562,880
9	3K + J + 9X	1,506,292,480
10	2K + 2Q + 9X	3,389,158,080
11	2K + Q + 2J + 8X	4,357,488,960
12	2K + 4J + 7X	50,086,080
13	K + 3Q + J + 8X	1,936,661,760
14	K + 2Q + 3J + 7X	801,377,280
15	4Q + 2J + 7X	50,086,080
16	3Q + 4J + 6X	7,791,168
Total combinations of C =		59,723,754,816.

The total number N of different card combinations which could occur in a bridge hand is

$$N = \binom{52}{13} = 635,013,559,600.$$

Thus, the probability of receiving a hand with exactly ten points is

P = C/N = 59,723,754,816 / 635,013,559,600 = 0.09405.

Also solved by Tom Weddell, Scott Nason, Judith Longyear, Raymond Kinsley, S. Turner Smith, Gerald Blum, Michael Jung, Richard Hess, Alan LaVergne, Dennis Kluk, and the proposer, William Butler.

OCT 2 Find two positive rational numbers the sum of whose cubes is 6. In other words, find positive integers a, b, c, and d satisfying $(a/b)^3 + (c/d)^3 = 6$.

Despite one reader's proof of nonexistence, everyone else found the same answer. Although many of the solutions were based on a computer search, Richard Hess was able to avoid using either a computer or a great many trials by utilizing modulo arithmetic. His answer follows:

Assume a/b and c/d are reduced to lowest terms.

Multiply both sides by b^3d^3 => $a^3b^3 + c^3b^3 = 6b^3d^3$

Take the equation modulo d^3 => $c^3b^3 \equiv 0 \pmod{d^3}$

Take the equation modulo b^3 => $a^3d^3 \equiv 0 \pmod{b^3}$

$(2) \Rightarrow \left.\begin{matrix} d \text{ divides } b \\ d \text{ divides } d \end{matrix}\right\} \Rightarrow b=d \Rightarrow a^3 + c^3 = 6b^3$ with b relatively prime to a and c.

a and c must be odd and relatively prime =>

$\left.\begin{matrix} a = p + q \\ c = p - q \end{matrix}\right\}$ p and q relatively prime

$a^3 + c^3 = 6b^3 \Rightarrow 2p(p^2 + 3q^2) = 6b^3 \Longrightarrow p(p^2 + 3q^2) = 3b^3$

Take each side modulo 3 ==> $p \equiv 0 \pmod 3$ ==> $p/q\ \{3(p/q)^2 + q^2\} = 9(b/3)^3$

Take each side modulo 3 ==> $p/3 \equiv 0 \pmod 3$ ==> $p/q\ \{27\ (p/q) + q^2\} = 3(b/3)^3$

Take each side modulo 3 => $p/q \equiv 0 \pmod 3$ ==> $p/27\{243\ (p/27)^2 + q^2\} = (b/3)^3$

Note that p = 27, q = 10, b = 21 works.

$(37/21)^3 + (17/21)^3 = 50653/9261 + 4913/9261 = 6.$

Also solved by Steven Feldman, Avi Ornstein, R. Smith, Bill Wilson, Mike Younkin, Eric Piehl, William Butler, Harry Zaremba, Raymond Kinsley, Judith Longyear, John Rule, Maury Goodman, Jack Crawford, Alan Lavergne, Michael Jung, Paul Dieges, Randall Rathbun, Dennis Kluk, Dennis Sandow, and Leon Bankoff.

OCT 3 The following cryptarithmetic problem consists of two mathematical statements which are correct in base 10 when digits are substituted for letters and are also true as read for modulo 9 mathematics:

SIX + TWO + TWO = ONE
SIX + SIX = TWO + ONE.

There was some confusion about the meaning of "is also true as read for modulo 9 mathematics." This is not a requirement for the values substituted to form a valid equation base 9. First of all, base 9 is not modulo 9. Secondly, the phrase "as read" precludes the substitution of digits for the letters. So this is just a normal base 10 problem with a coincidental truth when read modulo 9. Eric Piehl, however, was not confused by the wording and sent us the following solution: As this is base 10 arithmetic, we can substitute the first into the second:

SIX + SIX = TWO + SIX + TWO + TWO
or SIX = 3 · TWO.

Substituting this into the first equation

ONE = 5 · TWO.

As ONE is a three-digit number, T must be 1. We can test all possible values for TWO and check to see that all letters correspond to different numbers. Note that the "O" on ONE. Testing all possibilities:

(Continued on p. 94)

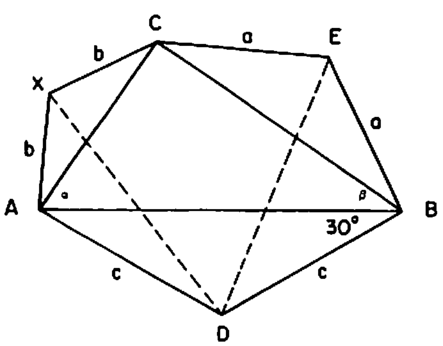
two ↓	two ↓				
105	105	126	126	136	136
× 5	× 3	× 5	× 3	× 5	× 3
525	315	630	378	685	308
↖ one	↖ six				
147	147	157	157	168	168
× 5	× 3	× 5	× 3	× 5	× 3
735	381	785	471	846	504
178	178	189	189	199	199
× 5	× 3	× 5	× 3	× 5	× 3
890	534	945	567	995	597

The only system that satisfies the above constraints is that in which

TWO = 178, ONE = 890, SIX = 534
giving 534 + 178 + 178 = 890
and 534 + 534 = 178 + 890.

Also solved by Judith Longyear, Raymond Kinsley, Harry Zaremba, William Wagner, William Butler, Harry Hazard, R. Smith, Winthrop Leeds, Bill Wilson, Mike Younkin, Eric Ranboy, Blaine French, Alan LaVergne, Jack Crawford, Emmet Duffy, Richard Hess, Gerald Blum, Maury Goodman and the proposer, Avi Ornstein.

OCT 4 Starting with *any* triangle, construct three exterior triangles having base angles of 30° and vertices at D, E, and X, as indicated in the diagram. If the distance DE is taken as 100, what is the distance DX?



Norman Wickstrand sent me a beautiful solution. Nothing unusual about that; but he added, as a final comment, that his solution is from his February, 1938, notes. As I mentioned in October, this problem first appeared in the *Review* in 1938, and Mr. Wickstrand, apparently anticipating my actions, saved his notes. Here they are:

$DE^2 = a^2 + c^2 - 2ac\cos(60° + \beta)$

$DX^2 = b^2 + c^2 - 2bc\cos(60° + \alpha)$

$DE^2 = a^2 + c^2 - 2ac(\cos 60° \cos\beta - \sin 60° \sin\beta)$

$= a^2 + c^2 - 2ac(\frac{1}{2}\cos\beta - \sqrt{3}/2 \sin\beta)$

$= a^2 + c^2 - ac\cos\beta + \sqrt{3}\sin\beta$

$DX^2 = b^2 + c^2 - bc\cos\alpha + \sqrt{3}\sin\alpha$

$\cos\alpha = [(2b\cos 30°)^2 - (2c\cos 30°)^2 - (2a\cos 30°)^2]/[2(2b\cos 30°)(2c\cos 30°)]$

$= (b^2 + c^2 - a^2)/2bc$

$\cos\beta = (a^2 + c^2 - b^2)/2ac$

$DE^2 = a^2 + c^2 - ac(a^2 + c^2 - b^2)/2ac - ac\sqrt{3}\sin\beta$

$= a^2/2 + c^2/2 + b^2/2 - ac\sqrt{3}\sin\beta$

$DX^2 = a^2/2 + c^2/2 + b^2/2 - bc\sqrt{3}\sin\alpha$

$(\sin\alpha)/(\sin\beta) = (2a\cos 30°)/(2b\cos 30°) = a/b$

$\sin\alpha = a/b \sin\beta$

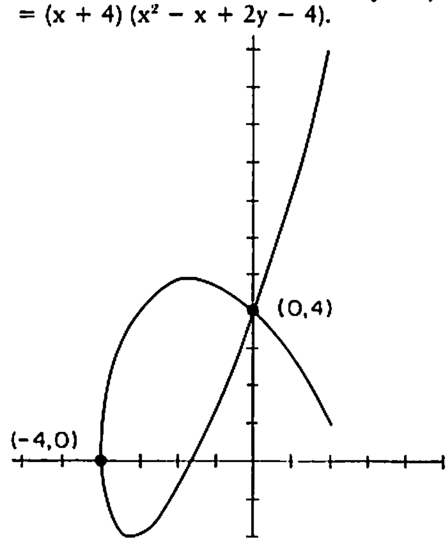
$DE^2 - DX^2 = \sqrt{3}(bc\sin\alpha - ac\sin\beta)$

$= \sqrt{3}(bc\, a/b \sin\beta - ac\sin\beta)$

$(DE^2 - DX^2)/\sqrt{3}\sin\beta) = ac - ac = 0;$ therefore $DE = DX = 100$.

Also solved by Eric Ramboy, Paul Perkins, Harry Hazard, Norman William Butler, William Wagner, Edwin Comfort, Harry Zaremba, Smith Turner, Raymond Kinsley, John Rule, Maury Goodman, Gerald Blum, Richard Hess, Jack Crawford, Alan Lavergne, Sidney Shapiro, John Oehrle, M. Jung, Maomi Markovitz, Leon Bankoff and the proposer, J. Friedman.

OCT 5 Find the area of the loop of $y^2 = (x + 4)(x^2 - x + 2y - 4)$.



Our final solution is from Charles Rozier:
The equation can be rearranged into a quadratic in $(y - x)$:

$(y - x)^2 - 8(y - x) - (x^3 + 4x^2 - 16) = 0.$

Solving the quadratic,

$y = x + 4 \pm x\sqrt{x + 4}.$

The loop lies between the two values of x for which y is single-valued, 0 and −4. If we let z equal the distance from the lower branch to the upper branch of y, then the area A is given by

$$A = \int_{-4}^{0} z\,dx = -2\int_{-4}^{0} x\sqrt{x+4}\,dx$$

Integrating (by parts with $u = x$, $dv = \sqrt{x + 4}\,dx$) yields

$$A = \left[-\frac{4x}{5}(x+4)^{3/2} - \frac{32}{15}(x+4)^{3/2}\right]_{-4}^{0}$$

$= 256/15.$

Also solved by Raymond Kinsley, Harry Zaremba, Eric Piehl, Norman Wickstrand, William Butler, Alan LaVergne, Kyle Roberson, Jack Crawferd, Richard Hess, Gerald Blum, Winthrop Leeds, Rob Newman, Michael Jung and the proposer, Harvey Elentuck.

Better Late Than Never

NS12 A. Walther's claim that the answer is approximately the fourth power of (1/e) is incorrect. His analysis of the one-suit case is correct and so is his answer that the value is approximately (1/e). But now his generalization to n suits is invalid (i.e., you cannot just raise the answer to the nth power). An easy way to see this is to take a simpler case having only two suits with three cards each and actually count the possibilities. You will see that Walther's formula for one suit (with 13 changed to 3) works fine but that the square is not the right answer for two suits. Comments were received from Ron Graham, Jack Parsons, M. Fountain, Edwin McMillan, and John Oehrle.

1977 J/A 4 John Oehrle has responded.

1978 JAN 3 Walter Delashmi has responded.

MAY 1 John Oehrle wonders if Black can draw by capturing the knight and allowing White to Queen. The answer is no since Queen verses Rook is a standard win.

M/A 1, M/A 3, and M/A 5 John Oehrle has responded.

MAY 2 Samuel McCluney has responded and Charles Blake has submitted the following Greek-theoretic comment:
I was interested in one word; I assume that it is supposed that the words repre-

sent pairs with two members being literally different in meaning. If that is the case, then one word does not qualify: Hedriophthalmous. The beginning of the word is from the Greek word "hedra" meaning "seat." However, since the "h" is not represented by a letter in Greek but merely by a diacritical mark, the word is more often spelt without the initial "h" but the two spellings are identical in meaning.

MAY 4 John Oehrle has responded.

J/J 2 The solution published has been severely criticized by Lloyd Furthmyer and a team consisting of Richard Pavelle and LeRoy Sievers. Begin with Mr. Furthmyer: No less than 19 persons submitted what you apparently regarded as solutions to this problem. But how, may I please inquire, do these people get off with completely ignoring the statement that the diesel train "has just passed" at the point seven miles out on the bridge as the pedestrian steps back to the first track? This statement is fully as worthy to generate a seventh equation as are most of the others, if we may allow the length of the train to be negligible. It is not, of course; but then, we did not know if either train had its locomotive at some precise point at the Aardvark or the Zymurgy stations. If we had taken the length of the diesel, or at least the time it took to pass the man, as negligible, then a seventh equation would have had to be reckoned with, namely $4/v = (a + 7)/v$. It is my belief that inclusion of this seventh equation makes the set not soluble. But there is an "out" to this: we simply add another unknown, the length of the diesel train. If we plug this seventh equation and unknown into your published solutions, the diesel train is, I believe, about 652 miles long in the first solution. But let us agree that this is the extraneous root of the equation, which often leads one astray into the hazy realms where small boys fish with poles of negative length while unmowing their neglected lawns. The second solution requires the diesel train to be only around 8.83 miles long — although uncommon, undoubtedly well within our mathematicians' license. What bothered me about this problem was the "inelegance." We are expected to *infer* that all speeds were constant and that the locomotives were precisely 25 miles apart at their respective stations. But this is scarcely reason to infer that we can ignore the seventh equation, even if we choose to ignore the length of the train or the time it takes to pass the man. This is the only badly stated problem I have noticed in your column, which I greatly enjoy.

Now for Messrs. Pavelle and Sievers: Equation (5) should contain a lower case v corresponding to the walking speed rather than a capital V. With this the first set of numbers satisfies the equations but the second set does not. However, even the first set is not a correct solution because there is a seventh equation which was not considered. The equation arises in the problem from the statements, "But instead I cross . . . walk four miles further . . . the diesel has just passed" and is

$$4/v = (a + 7)/V$$

This yields seven equations with only six unknowns and no set of values for the six unknowns will satisfy all seven equations simultaneously. Following a conversation with the proposer, Frank Rubin, it was suggested we take into account the length of the diesel. If T corresponds to this length in miles, then one must modify the seventh equation to read

$$4/v = (a + 7 + T)/V$$

There are now seven equations and seven unknowns which yield the following two non-trivial solutions:

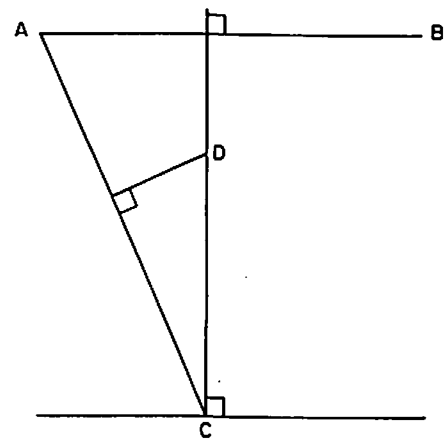
$$\left[A = \frac{15}{2}, V = \frac{175}{4}, R = \frac{35}{2}, L = 10, T = \frac{53}{6}, W = \frac{175}{4}, v = \frac{15}{2}\right]$$

$$\left[A = \frac{35}{2}, V = \frac{175}{4}, R = \frac{15}{2}, L = \frac{210}{29}, T = \frac{3913}{6}, W = \frac{225}{28}, v = \frac{15}{58}\right]$$

although the second should be discarded since the length of the train is greater than 25 miles.

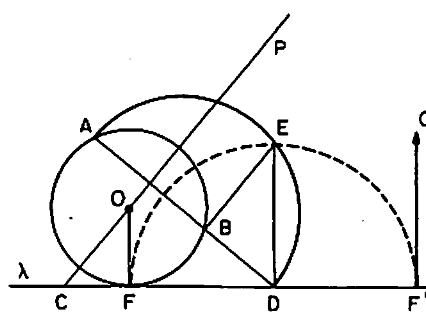
J/J 4 John Oehrle has responded.

J/J 5 John Oehrle and Samuel McCluney have responded, and Emmet Duffy makes the following remark:
The published answer has the same failing as the answer I submitted — there is no answer when a line joining the two given points is parallel to the given line. In the published solution the cosine of theta becomes zero and the radius r becomes indeterminate in the form 0/0. If the two points are so given that a line joining them is parallel to the given line, then the following solution applies:



Draw AB
Draw a perpendicular bisector of AB intersecting line at C.
Draw AC.
Construct a perpendicular bisector of AC intersecting at D. AD is the desired radius.

Finally, Leon Bankoff has sent us the following purely geometric solution:



The required circle can be constructed without recourse to trigonometric and algebraic involvements. Connect the two given points A and B and let CP, the perpendicular bisector of AB, cut the given line λ at C. Extend AB to cut λ at D. Then let the perpendicular to AD at B cut the semicircle on the diameter AD at the point E. With D as center and with DE as radius, describe an arc cutting λ at F. The perpendicular to CD at F will put CP at O, the center of the required circle. (As a gratuitous sidelight, if F' is the second intersection of the arc (D)DE with the line λ, the perpendicular to λ at F' will cut the line CP at O', the center of the larger of the two circles through the two given points and tangent to the line λ.) *Proof:* The center O of the required circle must lie on CP, the perpendicular bisector of what will become the chord AB. From the relation $DE^2 = DB \cdot DA = DF^2$, it is apparent that the required circle will touch λ at F. (By Euclid, Book III, Proposition 36,

(Continued on p. 100)

Classified

Puzzle Corner

(Continued from p. 96)

the square of a tangent from a point to a circle is equal to the product of the secant from the same point and the external segment of the secant.) The same argument applies to the construction of the larger circle centered at O′, for which the square of the tangent DF′ is equal to the product of the secant DA and the outer segment DB.

A/S 1 Alan LaVergne, Nick Michael, Jacob Bergman, Roger Lipsett, John Oehrle, Ben Rouben, and G. Saulnier have responded.

A/S 2 Alan LaVergne, Jacob Bergman, Jack Crawferd, and Roger Lipsett have responded.

A/S 3 John Oehrle, Naomi Markovitz and Alan LaVergne have responded.

A/S 4 Ivor Morgan, Dave Stolfa, Emmet Duffy, Roger Lipsett, Alan LaVergne, and Jacob Bergman have responded.

A/S 5 Alan LaVergne, Naomi Markovitz, Jack Crawferd, Jacob Bergman, Roger Lipsett, and Emmet Duffy have responded.

Proposers' Solutions to Speed Problems

SD 1 Since A cannot determine his hat color, either B or C must have a white hat. B knows this but still cannot determine his hat color. Thus C must have a white hat. (Answer courtesy of the editor.)

SD 2 Zero feet. (Answer courtesy of the editor.)