# How Kissinger Gained His Power

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Congratulations to Karpov and Levy for their victories; but Korchnoi and Chess 4.7 certainly made it interesting. Karpov defeated Korchnoi six wins to five (with over 20 draws) to retain the world chess championship. David Levy had an easier time with 4.7 scoring three wins to one (one draw) to clinch his 1968 bet with several illustrious computer scientists that no computer could beat him in ten years. The play showed a familiar pattern for computer-human play. The two tactical games favored the computer. In game one, 4.7 defended brilliantly to reach a won endgame, which it could not win. In game four, Levy, up 2½ to ½, sportingly played the ultra-risky Latvian gambit to try to beat the computer at its own game, but 4.7 prevailed. The three positional games were fairly easy wins for Levy. I stick with my prediction that a computer world champion in the 20th century is conceivable, whereas a human world champion in the 22nd century is not. I am hopeful that with good medical care, I will live long enough to see the transition.

Problem NS 10 had appeared in the *American Mathematical Monthly* during the 1940s. Apparently no closed formula was found there either.

In the October issue I pointed out that NS9 is reopened due to an error in the published solution. But for some reason we did not print the error; it is that the 16 letters in the square must be distinct.

## Problems

D/J 1 The Editor of *Technology Review* suggested our possible interest in the following bridge problem from Alan Truscott's column in the *New York Times*. Mr. Truscott describes it as "the most dramatic deal" in the final session of the Tenth Grant National Match between Texas and Chicago in 1977:

North:
♠A J 5 2
♥K 3
♦6 4 2
♣K J 7 6

West:
♠8 7 4
♥Q 6
♦9 3
♣A Q 9 8 4 2

East:
♠K Q 10 9 6 3
♥7 5
♦K 10 7
♣10 5

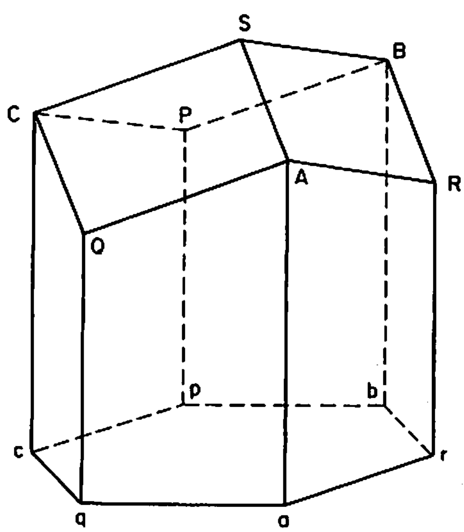
South:
♠—
♥A J 10 9 8 4 2
♦A Q J 8 5
♣3

Both sides were vulnerable. The bidding:

| *North* | *East* | *South* | *West* |
|---|---|---|---|
| 1 club | 1 spade | 2 hearts | 2 spades |
| — | 3 spades | 4 diamonds | — |
| 4 hearts | — | 6 hearts | — |
| — | — | | |

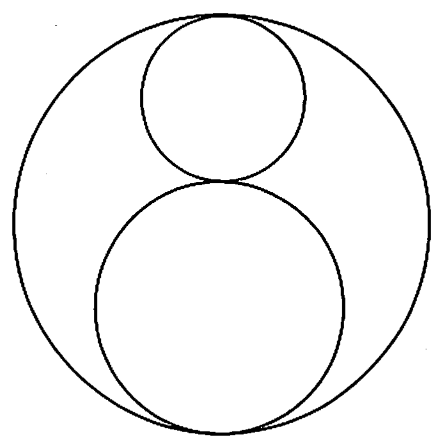
How can the contract be defeated?

D/J 2 Emmet Duffy has a problem concerning the geometry of honeybee hives: Given a prism with a hexagon base and a roof of three congruent rhombuses CSAQ, ASBR, and BSCP all tilted at the same angle with respect to a horizontal plane, what angle yields minimum surface area for a fixed volume?

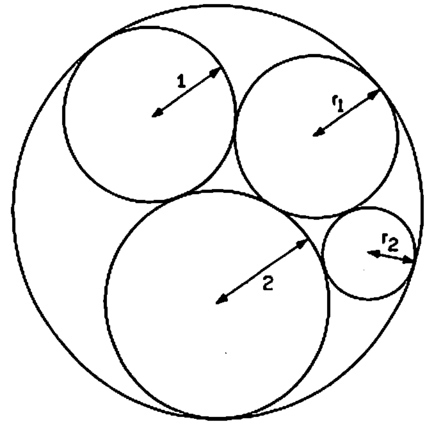



D/J 3 A cute problem from Frank Rubin: The King wanted to find out who was the wisest among his three Grand Counselors. He blindfolded the three and then announced that he had eight Great Seals, four purple and four gold. He would place two upon the forehead of each counselor, so they could see the four seals on their neighbors but not their own. The last two he would place in a locked chest. The first counselor who determined the colors of the seals in the chest through correct reasoning would become Supreme Counselor, but if he guessed the answer, rightly or wrongly, he would be beheaded. The King then put a purple seal and another on the head of Adak, a gold seal and another on the head of Baraz, and two more seals on the head of Cabul. After placing the last two seals in the chest, he removed the blindfolds and began questioning the counselors. "What seals are in the chest, Baraz?" "I do not know." "What seals are in the chest, Cabul?" "I do not know." Disappointed, he tried again. "What seals are in the chest, Adak?" "I do not know." "What seals are in the chest, Baraz?" And this time Baraz answered correctly, explained his logic, changed his name to Kissinger, and became Supreme Counselor. What seals are in the chest? Answer carefully.

D/J 4 Robert Lutton has submitted an extension of 1974 O/N 4.



For those whose memory fades after four years, let me remind you: that problem gave three circles of radii 1, 2, and 3 and asked you to determine how large a circle could be drawn inside the biggest circle and outside the other two. The answer was $r_1 = 6/7$. Mr. Lutton wants you to continue and find $r_2$; and he wonders if a general formula exists for $r_n$.



D/J 5 Richard Orr has a generalization of 1975 JAN SD 1. A game is played by N players where the loser doubles the money of the other N-1 players. After N games, each player has lost once and each has D dollars. Find $d_i$, the number of dollars the ith player began with.

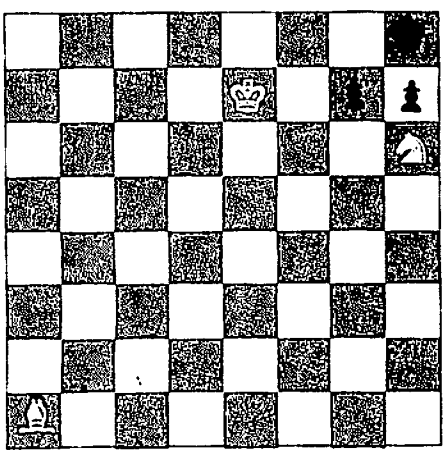
Speed Department

D/J SD1 Mitz Hofferfelt wants you to provide letters missing from the following sequence: — TTFFSSE — ETTFFSS

D/J SD2 Laurent Hodges submitted the following "piece of whimsy" to test your familiarity with the metric system:
On a recent cross-country trip in a VW bus pulling a heavily-laden trailer, my fuel consumption averaged 0.110 $mm^2$. Was the engine properly tuned? What was the fuel consumption in acres?

Solutions

A/S 1 White to play and mate in three moves, both up the board and down.



This unusual problem surprisingly resulted in a near-unanimous response. Let me paraphrase everyone. If White moves up the board, he mates with

| | |
|---|---|
| 1 B — B6 | P × B |
| 2 K — B8 | P — B4 |
| 3 N — B7. | |

If White moves down, he starts with
1 K — B3,
forcing
P — N8.
If Black chooses a rook, White mates with
2 N — B7.
If Black chooses a queen,

| | |
|---|---|
| 2 N — B7 ck | Q × N |
| 3 K × Q mate. | |

A bishop is similar. Finally,

| | |
|---|---|
| 1 | P — N8 (N) ck |

leads to

| | |
|---|---|
| 2 K — B2 dis ck | N — B6 |
| 3 B × N mate. | |

Solved by Ron Adelsman, Steve Slesinger, Eric Piehl, Mayer Margolis, Harry Movitz, Eric Rayboy, Frank Rubin, William Butler, Richard Hess, Glenn Iba, Phillip Feuerwerger, and the proposer, Abe Schwartz.

A/S 2 Consider a cocktail party to which Allan Gottlieb and his wife and four other married couples were invited, all of whom were introduced and shook hands. No one shook hands with him- or herself, or with his or her spouse, and no one shook hands with the same person more than once. When asked how many people he/she had shaken hands with, each guest had a different answer. How many hands did Allan's wife shake?

Eric Ramboy notes that we should have asked how many people my wife shook hands with. The following is from David McArthur:

Each person shook hands with either 0, 1, 2, 3, 4, 5, 6, 7, or 8 other people. Since the 9 people other than Allan each gave a different answer and there are 9 possible answers, the answers the 9 people gave must have been 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Now, the person that shook hands with 8 people shook hands with everyone at the party except his or her spouse. Now, everyone except the spouse of the person that shook hands with 8 people has shaken hands with at least 1 other person. So, the spouse of the person that shook hands with 8 people is the person that shook hands with 0 people.

Now, the person that shook hands with 7 people shook hands with everyone at the party except his or her spouse and the person that shook hands with 0 people. Now, everyone except the spouse of the person that shook hands with 7 people has shaken hands with at least 2 other people except the person that has shaken hands with 0 people. So, the spouse of the person that shook hands with 7 people is the person that shook hands with 1 person.

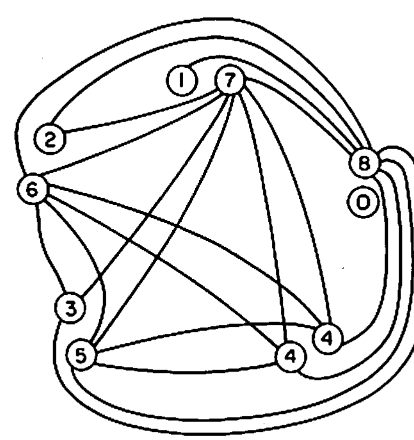
Now, the person that shook hands with 6 people shook hands with everyone at the party except his or her spouse, the person that shook hands with 0 people, and the person that shook hands with 1 person. Now, everyone except the spouse of the person that shook hands with 6 people has shaken hands with at least 3 other people except the person that has shaken hands with 0 people and the person that has shaken hands with 1 person. So, the spouse of the person that shook hands with 6 people is the person that shook hands with 2 people.

Now, the person that shook hands with 5 people shook hands with everyone at the party except his or her spouse, the person that shook hands with 0 people, the person that shook hands with 1 person, and the person that shook hands with 2 people. Now, everyone except the spouse of the person that shook hands with 5 people has shaken hands with at least 4 other people except the person that has shaken hands with 0 people, the person that has shaken hands with 1 person, and the person that has shaken hands with 2 people. So, the spouse of the person that shook hands with 5 people is the person that shook hands with 3 people.

Now, the person that shook hands with 4 people shook hands with everyone at the party except his or her spouse, the person that shook hands with 0 people, the person that shook hands with 1 person, the person that shook hands with 2 people, and the person that shook hands with 3 people. Now, everyone except the spouse of the person that shook hands with 4 people has shaken hands with at least 5 other people except the person that has shaken hands with 0 people, the person that has shaken hands with 1 person, the person that has shaken hands with 2 people, and the person that has shaken hands with 3 people. So, the spouse of the person that shook hands with 4 people also shook hands with 4 people.

Now, Allan must have shaken 4 hands, otherwise he would have received two answers of 4. *So, Allan's wife shook 4 hands.*

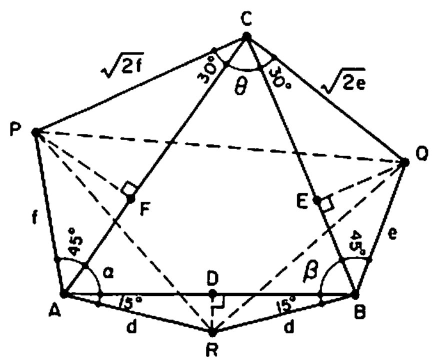
Below is a diagram:




Also solved by Scott Nason, Laureat Broca, Ray Hardin (who noticed that a similar problem appeared in the London Sunday Times), Ronnie Rybstein, Avi Ornstein, Steve Feldman, W. McGuinness, Eric Piehl, Biff Gout, Glenn Iba, Beverly Jamison, Mark Del Giorno, Frank Rubin, William Butler, Richard Hess and the proposer, Michael Auerbach.

A/S 3 Given any triangle ABC, construct outside it the three triangles BCP, CAQ, and ABR such that

<PBC = <CAW = 45°,
<BCP = <QCA = 30°, and
<ABR = <RAB = 15°.




Show that <QRP = 90° and that RQ = RP.

The following is from Harry Zaremba:

In the figure, let BC = a, AC = b, AB = c, PR = f, QB = e, and AR = BR = d. From triangle ABC,

$$b \sin \alpha = a \sin \beta; \text{ or } b/a = (\sin \beta)/(\sin \alpha). \quad (1)$$

Also,

$$c = b \cos \alpha + a \cos \beta. \quad (2)$$

Substituting a or b from (1) into (2),

$$c = [\sin (\alpha + \beta)/\sin \beta]b = [\sin (\alpha + \beta)/\sin \alpha]a. \quad (3)$$

From similar triangles APC and BQC (equal angles),

$$f/e = b/a = \sin \beta/\sin \alpha. \quad (4)$$

From isosceles triangle ARB,

$$c = 2d \cos 15° = \sqrt{2 + \sqrt{3}}\, d. \quad (5)$$

In triangle APF, AF = PF = f cos 45°; in triangle CPF, FC = PF/tan 30° = (f cos 45°)/tan 30°. Thus

AC = b = AF + FC = f cos 45° + (f cos 45°)/tan 30°, or

$$b = \sqrt{2 + \sqrt{3}}\, f. \quad (6)$$

In a similar manner from triangles BQE and CQE,

$$a = \sqrt{2 + \sqrt{3}}\, e. \quad (7)$$

From triangle CPF, PC = PF/sin 30° = (f cos 45°)/sin 30°, and

$$PC = \sqrt{2}\, f. \quad (8)$$

Similarly from triangle CQE, $QC = \sqrt{2}\, e$ (9)

Substituting c, b, and a from (5), (6), and (7) into expression (3) gives

$$d = [\sin (\alpha + \beta)/\sin \alpha]\, e = [\sin (\alpha + \beta)/\sin \beta]\, f. \quad (10)$$

From triangle PAR,

$$PR^2 = f^2 + d^2 - 2fd \cos (60° + \alpha). \quad (11)$$

Using relations (6), (10), and (1), equation (11), after lengthy algebraic manipulation, can be reduced to

$$PR^2 = (a^2/[(2 + \sqrt{3})\sin^2\alpha])[\sin^2\beta + \sin \alpha \sin (\alpha + \beta)(\cos \beta + \sqrt{3} \sin \beta)]. \quad (12)$$

From triangle QBR,

$$QR^2 = e^2 + d^2 - 2ed \cos(60° + \beta). \quad (13)$$

With use of relations (7), (10), and (1), equation (13) can be reduced such that $QR^2 = PR^2$, proving the sides are equal. Also, from triangle PCQ,

$$PQ^2 = (\sqrt{2}f)^2 + (\sqrt{2})e^2 - 2\sqrt{2}f\sqrt{2}e \cos (60° + \theta). \quad (14)$$

Use of relations (6), (7), (1), and θ = 180° − (α + β) reduces equation (14) to $PQ^2 = 2PR^2 = 2QR^2$. The latter relationship is characteristic of the relations between the hypotenuse and sides of an isosceles right triangle; hence <QRP = 90°. The algebraic and trigonometric reduction process to reach the results above is quite long and as a consequence has not been included.

Also solved by Richard Hess, Robert Simon, Glenn Iba, and the proposer, Eric Jamin.

A/S 4 Consider the number 153. It turns out to be equal to the cubes of its digits — i.e., $153 = 1^3 + 5^3 + 3^3$. Find three other such numbers. This property can be extended to four-digit numbers — for example, $1634 = 1^4 + 6^4 + 3^4 + 4^4$. Find two other such four-digit numbers. In the same way, the property can be generalized for numbers of any size. Find a proof that there are an infinite number of such numbers, or else prove that only a finite number exist.

Richard Hess gives complete solutions to the three questions asked and a partial solution to one question not asked.

(a) $407 = 4^3 + 0^3 + 7^3$
$371 = 3^3 + 7^3 + 1^3$
$370 = 3^3 + 7^3 + 0^3$

(b) $8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

(c) There are only a finite number of such numbers.

*Proof:* All n-digit numbers are $\geq 10^{n-1} \equiv a_n$.

The sum of the nth powers of the digits of an n-digit number is always $\leq n \cdot 9^n \equiv b_n$. For $n > 60$, $b_n < a_n$ because

$$\left.\begin{array}{l}\left(\frac{10}{9}\right)^{61} = 618.31 \\ 10 * 61 = 610\end{array}\right\} \begin{array}{l} = 10 * 61 < \left(\frac{10}{9}\right)^{61} \\ 61 * 9^{61} < 10^{60}\end{array}$$

Thus $b_n$ will never have enough digits if n > 60 and there are a finite number of n digit numbers equalling the sum of the nth powers of their digits. What is the largest of these numbers? A slight modification of the above proof shows that $n_{max} \leq 58$. It is probably tough to find the largest number.

Also solved by William Butler, Avi Ornstein, Ken Zeger, Thomas. Peterson, Glenn Iba, Harry Zaremba, Jack Crawford, Phillip Werger, Frank Rubin, and the proposer. Harry Nelson sent a Lawrence Radiation Laboratory report of his on this subject.

A/S 5 Replace each letter by a unique decimal digit:

```
     H E
  ×  I S
   H U B
   O F
   L A B
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Harry Garber sent us the solution:

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     1 8
  ×  3 9
   1 6 2
   5 4
   7 0 2
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Also solved by Harry Hazard, Gerald Blum, Edwin McMiller, Eric Ramboy, Glenn Iba, Steve Feldman, Ronnie Rybstein, Dan Sheingold, Lester Glickman, Norman Wickstrand, James Cooney, Winthrop Leeds, Abe Schwartz, Richard Hess, William Butler, Avi Ornstein, and Harry Zaremba.

Better Late Than Never

NS9 The problem is still open. The solution published was incorrect, as several letters were used more than once. We thank Werner Glass, Mary Hazard, and John Rule for pointing this out.

1978 JAN 1 There have been no solutions to this problem until now. Alan La Vergne has submitted a fairly convincing argument that no contract higher than one no-trump is possible. In addition, he has supplied a hand that shows that one no-trump itself is possible. He writes:

It is extremely unlikely that the handicap of the opening lead should cost both sides three tricks. Three-trick end-plays are not unknown, but they involve forcing a defender to provide transportation to already-established but otherwise inaccessible tricks. However, this is ruled out by the conditions of the problem — when the "inaccessible" hand is opening leader, the defenders will have too many tricks. It's not exactly a proof, but if you buy that argument, the maximum contract makeable from four positions is one no-trump. And here is a hand to do it:

```
              ♠ A x x
              ♥ 4 3 2
              ♦ Q x
              ♣ 10 8 7 6 5
♠ —                          ♠ Q J x x x x
♥ 10 8 7 6 5                 ♥ —
♦ A 10 x                     ♦ K 9 x x
♣ K Q J 9 4                  ♣ A 3 2
              ♠ K 10 x x
              ♥ A K Q J 9
              ♦ J x x x
              ♣ —
```

Both North-South and East-West have six tricks off the top. It is pretty clear that whoever has the opening lead is endplayed into providing the opponents' seventh trick. Only a couple of slightly tricky aspects should be pointed out: If East-West are defending, South sluffs a diamond and a spade on the first two clubs. South's sluff on the third club trick is a heart if East is winning the trick with the ♣A and a spade if East has already played the ♣A, so that West is winning the trick. Regardless of which one is on lead, the defenders are out of leads which do not surrender a trick. On the other hand, if North-South are defending, East obviously has no trouble sluffing on four rounds of hearts. If North's opening lead is a spade, East splits his honors and West sluffs a club. If South is opening leader and plays a low spade, West can sluff a club since East can now set up a spade trick.

Phillip Feuerwerger has also responded.

JAN 3 Apparently a baseball technicality invalidates James Coomey's solution: the sixth batter is not given a hit. Mark Astolfi and Hal Ostrander pointed this out, and Noel Perry has also responded.

FEB 1 Harry Shershow noticed that the black king belongs on QR8.

FEB 2 Dermott Breault has responded.

1978 M/A 1 Gary Schwartz notes that West is squeezed twice.

1978 M/A 2 Henry Curtis, Edwin McMillan, Allen Tracht, Winslow Hartford, Manjeet Karra, Jerome Shipman, and Theodore Engle found a smaller solution. Mr. Engel also included some additional comments:

Minimil written out is 278,914,005,382,139,703,576,000. Its structural formula is $2^6 \times 3^5 \times 5^3 \times 7^2 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$. Minimil is number 212 in the list of Factor Champions, if you count the number 4 (one factor) as number one. Minibil, the smallest integer with a billion or more distinct divisors, is number 444 in the same list, and written out appears thus: 474,386,058,264,023, 040,500,951,689,662,949,694,304,000. Its prime composition is $2^8 \times 3^5 \times 5^3 \times 7^2 \times 11^2 \times 13 \times 17 \times 19 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53 \times 59 \times 61 \times 67 \times 71 \times 73 \times 83 \times 89$. With great effort I have compiled a list of 3,388 Factor Champions, where the definition is simply "any number which has more factors than any number smaller than itself." The first members of the list are 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720 and 840. Number 3308 in the list is Minioct, the smallest number with over an octillion (American style) factors. When I first began working on this list eleven years ago I had no inkling of how easy the task was to become as I became aware of the nature of numbers. I quickly calculated the numbers given above, and also minitril, and soon after miniquad, but I had not yet discovered that the classes of numbers (early discovered) could be arranged into series and then "Families of Classes." Only after this discovery could I be sure no member of the series of Factor Champions was left out. One of my early lists, in fact, contained four omissions and included two numbers which were not actual Factor Champions.

Once I assembled the table of Families, I could be sure that there were no omissions. Yet for years I despaired of ever determining the value of Minisex because I thought it would take too much trial and error. But after I bought my first SR-50 calculator, and especially after I discovered the Families, it became so easy that I coasted right on past Minisex to Minisept and then Minioct. The last number contains 182 digits, and the logarithm of its number of factors is 27.00301. Of course I work only with logarithms when I work with very large numbers, and I have devised a shorthand code for designating them.

Minimil, for example, is 6532-0, where the first four digits are the exponents of the first four primes, and 0 is capital letter O and not zero, and stands for 47, the 15th prime, as O is the 15th letter of the English alphabet. In giving letter assignments to the primes, I omit no letter, so that Z stands for the 26th prime. For higher primes I just repeat the alphabet and append a prime stroke after it. Primes greater than the 52nd are designated by letters with a double-prime stroke, also available on the typewriter. Still higher primes I would designate by the usual letter followed by a number indicating which round of the alphabet was being used. Fortunately for my list as far as I have gone no prime higher than the 78th (Z") has been necessary.

MILFAC, my code word for the smallest number with exactly one million different integer factors, is readily calculated in this way. In order to have a million factors exactly, a number must have a Gross Number of Factors (hereinafter called GNOF) of exactly 1,000,002. This number can be factored into primes as follows: 166,667 × 3 × 2. Since the method of obtaining the GNOF of a number is to multiply together the exponents of all of its primes, each augmented by one, we must now diminish each of these numbers by one, to arrive at the needed exponents for our primes. Hence the number we are seeking is $2^{166666} \times 3^2 \times 5$. This number has exactly 50,174 digits in it, since its log is 50173.11846 98468 65225 17238 (to twenty decimal places). Since my calculator handles only thirteen significant figures, I can determine that this huge number starts with 1313620286926, and can therefore be expressed as $1.313620286926 \times 10^{50173}$.

M/A 3 W. Shelton, Dermott Breault, Shirley Wilson, Jerome Shipman, and Winslow Hartford have responded. Neil Hopkins reports that as of *June 18*, Tim Lefever had received two phone calls from *Review* readers.

M/A 4, M/A 5 Winslow Hartford has responded.

M/A 5 Irving Hopkins notes that cos(18) $= \sqrt{5 + \sqrt{5}/2}\sqrt{2}$

MAY 2 Harry Hazard informs me that Ralph Beaman is the author of the *Word Ways* article. Phillip Feuerwerger has responded.

MAY 4 Mark Browning has responded.

MAY 5 Walter Delashmit and J. Coble have responded.

J/J 2 Naomi Markovitz has responded.

J/J 3 Two long-distance responses from Naomi Markovitz (Israel) and T. Ryan (Kenya).

J/J 4 Naomi Markovitz has responded.

J/J 5 Zoltan Mester has responded

J/J SD 1 The proposer points out a missing minus sign in his solution. The correct solution is $b^2/d^2 = -a/c$.

A/S SD1 As you know, comments about speed problems are rarely printed. This time two such rarities occur. The answer should be $1 + \sqrt{3}$ as pointed out by John Padolsky, Robert Simon, J. Price, Bob Pease, Arthur Ballato, Eric Piehl, Gerald Blum, Donald Savage, Stuart Flockencier, Steve Brown, A. Gray, Winthrop Leeds, Eric Ramboy, Art Delagrange, Victor Newton, Irwin Tessman, Turner Gilman, George Piotrowski, T. Harris, Bob Montante, Abe Schwartz, Andy Foster, Jack Crawford, and Irving Hopkins.

OCT SD2 The first two lines should have been "A dozen, a gross, and a score; plus three times the square root of four; . . ." I'm embarrassed to have had this wrong, as I've known the limerick since high school. The improvement came from Chuck Cegielski, R. Smith, Bill Wilson, Mike Younkin, Avi Ornstein, T. Harris, Virginia Merrill, J. O'Connor, Anne Symanovich, and Chester Sandberg (a classmate of mine at M.I.T. who actually contributed the same problem to "Puzzle Corner" six or seven years ago).

**Proposers' Solutions to Speed Problems**

**SD1 ZOTTFFSSENTETTFFSS**

The sequence is formed from the initial letters of the words for the consecutive numbers 0 through 17 (zero, one, two . . . sixteen, seventeen). It is devious in that all the letters given are doubles and the missing ones are not.

SD 2 Yes. 0.110 $mm^2$ corresponds to 1 gallon per 21.4 miles — not bad under the circumstances. This also comes to 27.2 trillionths of an acre.