A Disturbing Report from Michigan

Allan Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Assistant Professor of Mathematics and Coordinator of Computer Mathematics at York College of the City University of New York. Send problems, solutions, and comments to him at the Department of Mathematics, York College, Jamaica, N.Y., 11451.

Three items of interest have come this month:

William Butler reports a misprint on his problem, JUN/JUL 1. See the "Solutions" section below.

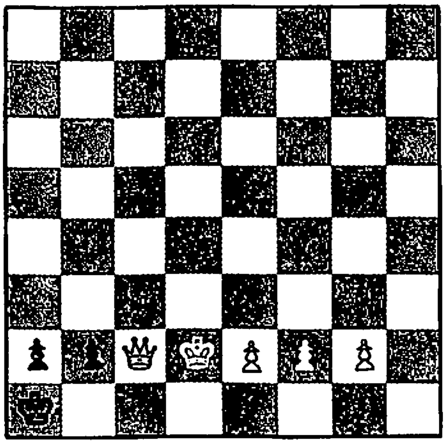
Dr. Frank Rubin reports, "I am embarking upon a new project which I think will be quite exciting. Starting next September, I will be running a mathematics contest for high school classes. The contest will be designed so that entire classes will be the entrants. The problems will be large enough so that they can be solved by dividing the work up among many students working in parallel. I am seeking both problems and judges for the contest. If any 'Puzzle Corner' readers wish to help, they should contact me directly" (59 De Garmo Hills Road, Wappingers Falls, N.Y. 12590).

Finally, J. Coble, of the Population Studies Center of the University of Michigan, sends a disturbing report: "A group of us here waste inordinate amounts of time solving your silly problems. We thought you should know that there are many from whom you may never hear whose careers are being jeopardized in this manner. Keep up the good work."

Problems

NS 13 This one was first published as 1974 JUNE 5, submitted by Gary Ford:
Two large coins and six small coins are placed on a table, each just touching its neighbors as shown in the sketch at the right. What are the relative diameters of the coins?
When this problem appeared in 1974, excellent approximations (using Newton-Raphson) were given. The proposer has subsequently asked for an *exact* solution.

NOV 1 We begin this month's fare with a chess problem from Steve Slesinger.

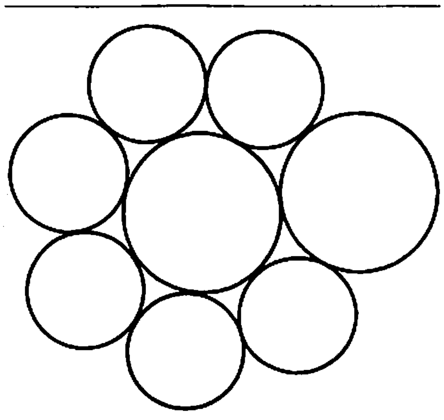


White, who moves down the page, is to move and win.

NOV 2 Thomas Mahon wants to know all positive integer solutions of $A^2 + (A + 1)^2 = C^2$.

NOV 3 Erwin Strauss (who claims to have been known as "Filthy Pierre" while at M.I.T.) offers us a problem in tennis theory:
In tennis, to win a game you must meet two conditions: a) win four or more points, and b) win two more points than your opponent — thus (dispensing with the fancy terminology of tennis scoring) winning scores are 4-0, 4-1, 4-2, 5-3, etc. If one player's probability of winning any given point is p, what is his probability of winning any given game? If winning a set requires winning at least six games, and at least two more games than your opponent, what is this player's probability of winning any given set?

NOV 4 The following problem is from Emmet J. Duffey:
The figure as shown at the right has been divided into five similar trapezoids by drawing lines parallel to the base. What are the heights of the trapezoids?



NOV 5 Dave Rabinowitz is interested in English words in which consecutive letters appear alphabetically. An example with two consecutive letters is "k*no*w." But "t*on*" is not valid as the "o" and "n" are out of order. What is the best word you can find — i.e., the largest number of consecutive letters?

Speed Department

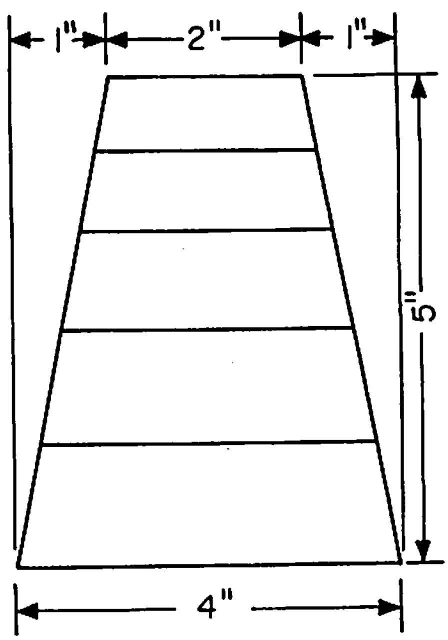
NOV SD 1 Our first speed problem is from R. Crandall:
The hour, minute, and second hands of a clock are *all three* coincident at noon and at midnight. Is this true at any other time(s) of the day?

NOV SD 2 We close with a quickie baseball problem from Mark Astolfi:
In a nonforfeited, nonsuspended, nine-inning baseball game, what is the *smallest* number of runs a team could score given that there exists a player "X" such that X is a member of the team and X gets to bat in all nine innings?

Solutions

The following solutions are to problems published in the June/July issue.

JUN/JUL 1 In the problem as published, we gave North only 11 cards. Please give North the ♥3 and ♥2 as well and try again. The solution will appear, with those for the November problems (*above*), in the March/April issue. The correct statement of JUN/JUL 1 is at the top of the next column:



♠7 3
♥Q J 3 2
♦5 2
♣A K 5 3 2

♠A Q 2
♥A K 7 6 5 4
♦A Q
♣J 6

With South the declarer at six hearts, West leads the ♥10. South thinks a moment or two and plays a heart from the dummy. East discards a diamond. What is the best play to make six hearts? If possible, supply the probability of success.

JUN/JUL 2 Part way along the 25 miles from Aardvark to Zymurgy, I begin to cross a railroad bridge. When I have gone three miles along the bridge, I hear a diesel train leaving the Aardvark station. I could exactly escape the train by running to either end of the bridge. But instead I cross over to the other track and walk four miles further. Now I see a steam train leaving the Zymurgy station on my track. Again, I could exactly escape by running to either end of the bridge. But instead, since the diesel train has just passed, I switch back to the first track. I reach the end of the bridge just as the steam train reaches the start of the bridge (in my direction). One hour later I reach Zymurgy. How long is the bridge?

The following solution is from Irving Hopkins:

There are six unknowns:

L, length of bridge, miles
a, distance from Aardvark to bridge
v, walking speed, m.p.h.
R, running speed
V, speed of diesel
W, speed of steamer

The six equations derived from the given information are:

$a/V = 3/R$ (1)
$(a + L)/V = (L - 3)/R$ (2)
$(25 - a - L)/W = (L - 7)/R$ (3)
$(25 - a)/W = 7/R$ (4)
$(25 - a)/W = (L - 7)/V$ (5)
$a + L + v = 25$ (6)

Dividing (2) by (1), we eliminate V and R and get $a = 3L/(L - 6)$. (7)
Dividing (3) by (4), we eliminate W and R and get $(25 - a)/(25 - a - L) = 7/(L - 7)$. (8)
Substituting (7) in (8), we get the quadratic $29L^2 - 500L + 2100 = 0$ (9)
from which L = 10 or 7.2413793.

The full list of parameters is *(see the list at the top of the next column)*:

L	7.2413793	10
a	17.5	7.5
v	0.2586207	7.5
R	7.5	17.5
V	43.75	43.75
W	8.0357146	32.75

Choosing L = 10 seems reasonable, except perhaps for the extreme demands on the pedestrian.

Also solved by Danny Mintz, Raymond Gaillard, Winslow Hartford, Andrew Purbrick, Douglas Szper, Avi Ornstein, Gerald Blum, Mike Bercher, Richard Kandziolka, Jacob Bergmann, Yale Zussman, William Butler, John Sutton, Alan LaVergne, Frank Rubin, John Rule, Neil Hopkins, Norman Wickstrand, and Richard Shetron.

JUN/JUL 3 Solve the following pair of equations for x and y:

$1 - xy = x + y^2$
$1 - xy = y + x^2$

Carl King had little trouble with this: At first glance, you might think these interesting equations were independent and related to the conic sections, but you would be wrong on both counts. Closer inspection reveals that they are symetrical, i.e., interchanging x and y in one equation produces the other. Hence any property that we discover of one can be translated to the other by interchanging the variables.

Rearranging the first equation we have:

$$xy + x + y^2 - 1 = 0$$

which can be factored:

$$x(y + 1) + (y - 1)(y + 1) = 0$$

hence: $y + 1 = 0$

and: $x + y - 1 = 0$ (1)

So, the equation turns out to be not a curve, but the product of a pair of linear equations.

The second given equation, by symmetry, gives:

$$x + 1 = 0$$

and: $y + x - 1 = 0$

which, in turn, is equivalent to: $x + y - 1 = 0.$ (2)

Equations (1) and (2) are identical. Thus the two given equations are equivalent to just three linear equations, one of which is a polynomial factor common to both of them; whereupon the required "solutions" correspond to the infinite set of points on the line defined by the said linear equation, as last displayed above. In addition to that infinite set, there is a particular solution, which is at the intersection of the two dissimilar equations, occurring at x = −1 and y = −1. A small sample of points that also satisfy the given equations includes:

x	y
−1	+2
− ½	+1½
0	+1
+ ½	+ ½
+1	0
+1½	− ½
+2	−1
+2½	−1½
etc.	etc.

Also solved by Jordan Wouk, Robert Creek, Karl Pfeifer, Herbert Fox, Richard Shetron, Jack Parsons, Edwin McMillan, Alfred Emslie, Norman Wickstrand, William Stein, Charles Beer, John Prussing, F. Steigman, Raymond Gaillard, Will Liddel, William Hartford, Andrew Purbrick, Douglas Szper, Avi Ornstein, Gerald Blum, Mike Bercher, Richard

Kandziolka, Jacob Bergmann, Yale Zussman, William Butler, Alan LaVergne, John Sutton, Frank Rubin, John Rule, Irving Hopkins, Robert Granetz, Merrill King, Ray Kinsley, Emmet Duffy, Harvey Elentuck, Allan MacLaren, Everett Leroy, Sidney Kravitz and the proposer, Eugene Sard.

JUN/JUL 4 Find the telephone number with the following property:

$$myphone = (my)^*(hy)^*(p)^4.$$

Each letter represents a different digit (none repeats), and neither of the first two digits can be 0 or 1 (it's a telephone number, remember).

Several readers programmed computers to do exhaustive searches, and everyone agrees that the unique answer is 2374589. Emmet Duffy found this solution (and showed uniqueness) without a computer and his solution is reprinted below. First, here is a comment from Dennis Sandow which was written on A.T. & T. stationery:

You unnecessarily constrained the problem by suggesting that M and Y $\neq 0$ or 1. By your rule, an Area Code could accommodate 640 ($8 \times 8 \times 10$) exchanges. As that limit is approached, the Area Code must be split into two. Half the subscribers in the Area Code (state, etc.) get a new Area Code. This last occurred in Virginia in 1973. The process of splitting Area Codes unsettles customers and may cause dialing problems for customers outside the affected area who "don't get the word." As a result, the Bell System decided to "unblock" the second digit of the exchange number. This adds 160 ($8 \times 2 \times 10$) more exchanges in each Area Code. These new exchanges are used only after the first 640 exhaust, because they could be confusing to some customers (with a 0 or 1 in the second digit, they look like an Area Code). The first such code was authorized in early 1974. As of mid-1978, there are about ten exchanges in service — all in Los Angeles (Area Code 213). Running the problem without that constraint doesn't change the answer. No solution is found when Y = 0 or Y = 1.

Mr. Duffy's solution follows:
If $myphone = (my)^*(hy)^*(p)^4$, then dividing by my will result in $100{,}000 + phone/my = (hy)^*(p)^4$. As phone cannot be larger than 98765 and my cannot be smaller than 23, the maximum value of phone/my is 98765/23 or 4293. Then $(hy)^*(p)^4$ will equal a number greater than 100,000 but not over 104293. There are only 5 cases as shown:

p	p^4	hy	$(hy)(p^4)$
8	4096	25	102400
7	2401	42	100842
7	2401	43	103243
6	1296	78	101088
6	1296	79	102384

The number 102400 can be eliminated as multiplying by any number will result with double zero at the end. For the other 4 cases multiplying by the possible values of my gives only one correct answer: myphone is 2374589, which is $(23)^*(43)^*(7^4)$.

Also solved by Steven Radtke, Andrew Purbrick, Joseph Bergmann, Yale Zussman, William Butler, Alan LaVergne, Robert Granetz, Richard Shetron, Merrill King, Neil Hopkins, Ray Kinsley, John Sutton, Timothy Maloney, and the proposer.

JUN/JUL 5 Given a line and two points on one side of the line, construct the smaller circle which passes through the points and is tangent to the line.

Ray Kinsley finds the radius, r, of the required circle algebraically. But the formulas which result are all geometrically constructable. Note that the arbitrary points A and B on his arbitrary circle are to be the points mentioned in the problem:
The following relationships must be established first. Construct a circle with arbitrary radius r tangent to line CD. Select two arbitrary points on the circle and label them A and B. Draw a line through A and B to intersect line CD and label the point of intersection E. Construct the perpendicular bisector of chord AB extending it until it intersects line CD. Label this point F. Label the midpoint of the chord G and the angle between line CD and line FG label ϕ. Construct perpendiculars to