Help Us Find John Prussing's Lost Dog

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A lament from the "Big Apple": New York in February was 12° F. colder than normal and three times as snowy. As a result our streets have not been swept in over 50 days; the previous record for days without sweeping for an entire winter is in the low 30s, and we may well double that mark.

My backlog of regular problems is in excess of two years and still growing. I thank you for the support. On the other hand chess problems are critically short. Unless new ones appear, we will soon be forced to revert to the previous practice of one bridge problem each month.

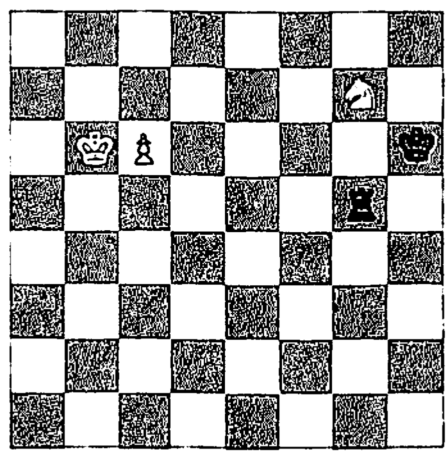
Finally, let me report that NS 8 was prematurely put to rest last December and has been revived. See "Better Late Than Never," in the June/July issue.

Problems

NS 12 This month we again present an old problem that was never solved (completely). Formerly 1974 J/A 4, this problem came from John G. Connine, who describes "a lesser known game of solitaire played in the snow-belt during long winter nights." (It is also played here in the pollution belt.)
A standard deck of 52 cards is shuffled and placed face-down upon the table. The cards are then turned face up one at a time by flipping over the top card of the face-down stack. As this is done, the player simultaneously calls out the sequence A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A, 2, etc., one call being made for each card flipped over. To win the game, one must go through the deck without matching a card flipped over with the card called. Suits don't matter, so, for example, any 4-spot flipped over on the 4th, 17th, 30th, or 43rd turn results in a loss. "Since winter will surely come again," Mr. Connine would like to know what are the chances of winning the game. How about a second solution for the same game with a 48-card pinochle deck?

When this problem first appeared several excellent approximations were published. So this time we are only looking for *exact* solutions.

MAY 1 We begin this month with a chess end-game problem from M. Laufer:
White to play and win:



MAY 2 We next present a word problem from Donald Forman:
"Dentification" and "identification" are both English words. For each English letter α, what is the longest string β such that both β and $\alpha\beta$ (the concatenation of α followed by β) are English words? Pairs such as "allelujah" and "hallelujah" or "enanthaldehyde" and "oenanthaldehyde" are excluded, since they are simply variant spellings or variant pronunciations of the same word.

MAY 3 Sheldon Razin offers us a checkerboard problem:
Given an n-by-n checkerboard and n^2 checkers of n different colors, and given that there are n checkers of each color, is it possible to arrange all the n^2 checkers on the board such that no two checkers of the same color lie in the same row, column, or diagonal? (By "diagonal" is meant *all* the diagonals, not just the two main diagonals.) It turns out that for certain values of n it is possible to so arrange the checkers; in this case we say a solution exists — e.g., n = 1. But for certain other values of n such an arrangement is impossible — i.e., no solution exists. For which values of n does a solution exist?

MAY 4 A base-12 cryptarithmetic problem from William Schumacher, which he says he devised for the Duodecimal Society of America a number of years ago:
Remember, duodecimal notation has two extra digits following 9 before reaching the radix. For convenience and uniformity, call these "dek" (symbol x, numerical value equals decimal 10) and "el" (symbol ϵ, numerical value equals decimal 11). Thus one counts as follows: zero, one, two, three, four, five, six, seven, eight, nine, dek, el, do, . . ." ("Do" is short for "dozen," the radix, and is denoted by 10 — one dozen and no units.) When you get it into your thinking that 8 + 6 = 12, 17 + 14 = 2ϵ, 9 · 5 = 39, and 60 ÷ 16 = 4, the rest comes easy. The problem is to assign numerical values to each letter:

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                 HEN
AARON)PHARAOH
      AARON
        OOYPO
        BBNYZ
        AEYDNH
        ACEPHH
         ERPZC
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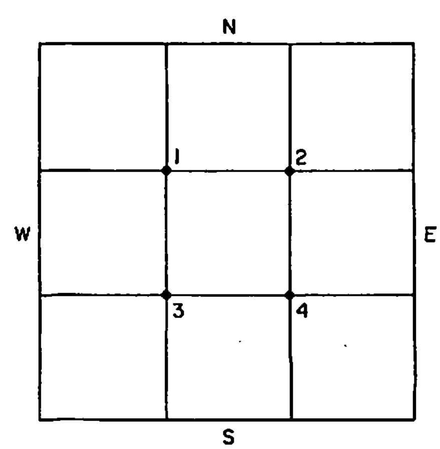
MAY 5 John Prussing needs your help to find his lost dog:
A dog is lost in a square maze of corridors. At each intersection, he chooses a direction at random and proceeds to the next intersection or exits at one of the sides. His walk is over when he reaches one of the sides. What is the probability P_k that the dog, starting at intersection k, will exit at the south side?



Speed Dept.

MAY SD 1 R. Robinson Rowe has sent us a "metric sequel or alternative":
A fence around a circular corral is as long in meters as there are ares contained.. What is the radius of the corral?

MAY SD 2 A Jimmy-the-Greek-theoretic game from William Blake:

Nine football games are being played this weekend; how many predictions (win, lose — no ties) are needed to be *sure one* is correct?

Solutions

NS 9 In each of 16 squares arranged in a four-by-four square, place a *different* letter, selected so each row, column, and long diagonal will spell a *different* four-letter word when the letters are selected consecutively in one or the other of the only two possible directions, as we do with numbers. There will be a total of ten different words, all of which must be defined in any one edition of Webster's dictionaries.

Several readers found this too easy this time and arranged for the letters to be selected in the forward direction. Avi Ornstein sent us the following:

C R A M
L O R E
E A S E
F R E T

Also solved by Mark Green, George Ropes, Raymond Kinsley, Marshall Bern, and Winthrop Leeds.

JAN 1 What is the largest contract that can be made from three positions on the same deal? From four positions? Assume best defense.

No takers so far.

JAN 2 Find, for each n and each j, integers $X_1, X_2, \ldots X_j$, W, and Z such that $X_1^n + X_2^n + \ldots + X_j^n = W^{n-1} = Z^{n+1}$.

The following is from Harry Zaremba: With $W^{n-1} = Z^{n+1}$, we can assume that W and Z are each multiples of some integer A. If we let $W = A^{n+1}$ and $Z = A^{n-1}$, then $W^{n-1} = Z^{n+1} = (A^{n+1})^{n-1} = (A^{n-1})^{n+1} = A^{n^2-1}$.

The problem as stated does not impose the constraint that each integer X_i must be distinct. Consequently, if we assume each X_i equals A,

$$\sum_{i=1}^{j} X_i^n = jA^n = A^{n^2-1}, \text{ or}$$
$$j = A^{n^2-n-1}.$$

Hence, the equation has a solution for any n = 2, 3, 4, . . . and any integer A when $W = A^{n+1}$, $Z = A^{n-1}$, and $X_i = A$ for $j = A^{n^2-n-1}$ number of identical integers. The solution appears trivial but satisfies the problem as given.

Also solved by Frank Rubin, Alan LaVergne, and the proposers, J. Kleilin and Bob Martin.

JAN 3 How can a baseball team make three triples, one double, two singles, and steal two bases in one inning without scoring a run?

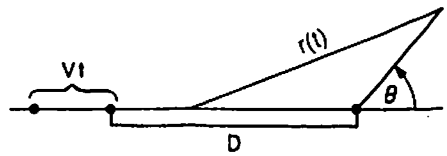
This problem has created some real interest. Someone from the Detroit Tigers telephoned to inquire about it. (Last year, the New York Yankees asked about a previous baseball problem in "Puzzle Corner"; my response inspired them to a "world" — i.e., U.S. — championship.) James Cooney sent us the following solution:

First batter triples and is picked off — one out.
Second batter triples and is picked off — two outs.
Third batter doubles.
Fourth batter singles but no advance by the third batter.
Double steal to 3rd and 2nd base.
Fifth batter singles — no advance — bases loaded.
Sixth batter homers, but runner coming in from third fails to touch home plate and hence is called out — third out.
Sixth batter is credited with a triple.

Also solved by Harry Zaremba, Frank Rubin, Bleiweiss, Robert Sutton, Kenneth Glass, Mark Astolfi, Hal Moeller, Jack O'Neill, John Corrigan, Dick Segers, John Rudy, and Alan LaVergne.

JAN 4 Suppose you suddenly realize that you are (on foot) directly in the path of a midwestern tornado which is approaching with a speed of V_T. The tornado has a lethal radius of R_L. You wish to take a path which maximizes your chance of survival. Your speed is a − bt (in which a and b are positive constants). In which direction should you head, and will you escape?

The following is from Alan LaVergne:



Let F(t) be the distance you have traveled by time t, so that your speed is F(t)/t. Let V be the speed of the storm, and D its distance from you at time t = 0. You will take a straight-line course at an angle θ with the line between you and the center of the storm. You will choose θ so that the minimum (over time) distance between you and the center of the storm is maximized.

Let r(t) be the distance between you and the center of the storm.

Then, by the cosine law,

$$r(t)^2 = (D - Vt)^2 + F(t)^2 + 2(D - Vt)F(t)\cos\theta \qquad (1)$$

This is minimized at $t = \tau$, where

$$\left.\frac{dr^2}{dt}\right|_{t=\tau} = 0, \text{ i.e.,}$$

$$-V(D - V_\tau) - F(\tau)F'(\tau) + (D - V_\tau)F'(\tau)\cos\theta - VF(\tau)\cos\theta = 0 \qquad (2)$$

This τ depends on θ, so let $\rho(\theta) = r(\tau)$ for this τ; that is, $\rho(\theta)$ is the distance of closest approach for the given angle θ. To maximize $\rho(\theta)$, we set $d\rho^2/d\theta = 0$, where

$$\rho^2(\theta) = (D - V_\tau)^2 + F(\tau)^2 + 2(D - V_\tau)F(\tau)\cos\theta.$$

Since

$$\frac{d\rho^2}{d\theta} = \frac{\delta\rho^2}{\delta\theta} + \frac{\delta\rho^2}{\delta\tau}\;\frac{d\tau}{d\theta}$$

and $\delta\rho^2/\delta\tau = 0$ by (2), we must have $\delta\rho^2/\delta\theta = 0$. Therefore $(D - V\tau)F(\tau)\sin\theta = 0$. $\theta = 0$ obviously represents a minimum whenever (as in this case) $Vt - D$ eventually exceeds $F(t)$. Hence the maximum is at $\tau = D/V$. Plugging this into (2), we find $\cos\theta = F'(\tau)/V$. In our particular case $F'(\tau) = a - b\tau = a - bD/V$, so $\cos\theta = a/V - bD/V^2$. You escape if $\rho(\theta) > R_L$, and since $\rho(\theta) = F(\tau)$, this translates to $R_L < a\tau - ½b\tau^2 = a(D/V) - ½b(D/V)^2$. Finally, in order for the solution to make any sense, we must have $F'(\tau) \geq 0$, or $aV \geq bD$.

If $aV < bD$, the maximum is at the "endpoint" or $F(a/b)$ and $\theta = 90°$, since $F'(t) = 0$ for $t \geq a/b$. The escape criterion is then $F(a/B) > R_L$ or $a^2 > 2bR_L$.

Also solved by Harry Zaremba, R. Robinson Rowe, and Winslow Hartford.

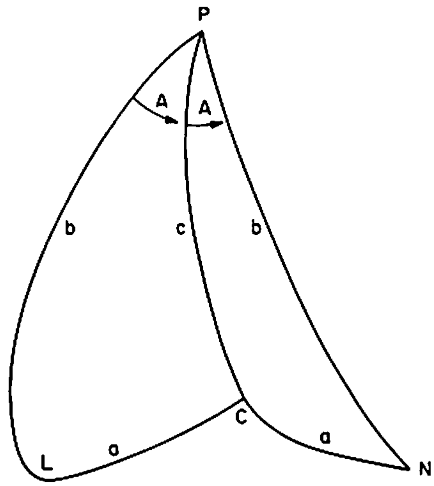
JAN 5 A satellite is orbiting earth and is now directly above Chicago. How far from the center of the earth is the satellite if New York City and Los Angeles bound the horizon? You are given the following information:

	Latitude	*Longitude*	*Altitude*
New York	40°45'06"N	73°59'39"W	55 feet
Chicago	41°52'28"N	87°38'22"W	595 feet
Los Angeles	34°03'15"N	118°14'28"W	340 feet

The diameter of earth is 7926.69 miles.

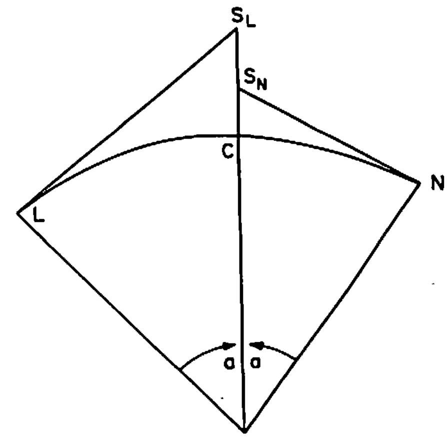
A complete analysis from R. Robinson Rowe:

The first diagram (above) shows two spherical triangles with P at the north pole and N, C, and L at New York, Chicago, and Los Angeles. For each of them we have two sides b and c, the co-latitudes, and A, the angle at the pole, being the difference in longitudes. The figure with two sides and the included angle is solved for the third side by $\cos a = \cos b \cos c + \sin b \sin c \cos A$



For PCN: $A = 13.645\ 277\ 778$
$b = 49.248\ 333\ 333$
$c = 48.125\ 555\ 556$

Whence $\cos a = 0.983\ 886\ 8920$
$a = 10.299\ 405\ 84$



For PCL: $A = 30.601\ 666\ 667$
$b = 55.945\ 833\ 333$
$c = 48.125\ 555\ 556$

Whence $\cos a = .904\ 780\ 2505$
$a = 25.206\ 297\ 23$

The second diagram (above) shows a section (developed) thru N, C, L and the earth's center at O, with the angles *a* now at the center. Using the given earth's diameter and the elevations of N, C and L, the distances in miles are:

ON: 3963.355
OC: 3963.458
OL: 3963.409

If N is on the horizon of the satellite, it is at S_N and ONS_N is a right triangle with

$OS_N = 3963.355/\cos a = 4028.262\ 834$

If L is on the horizon of the satellite, it is at S_L and OLS_L is a right triangle with

$OS_L = 3963.409/\cos a = 4380.521\ 124$

Perhaps Elentuck can explain the apparent discrepancy of the satellite being at two different elevations about 350 miles apart. Or maybe he intended this as a gimmick in the problem for the solver to unravel. The true horizon as seen from an elevated point is a circle, the locus of tangents to the earth thru the elevated point. The center of the locus circle is on the line OS. N and L could not both be on the true horizon because their angles *a* were different. But these tangents "graze" the earth so that landmarks some distance from the true horizon seem to lie on it. Thus if L could be seen and the satellite was at S_L, N could also be seen — with nothing beyond it to the true horizon but the ocean with no landmarks. Also solved by Harry Zaremba and Winslow Hartford.

Better Late Than Never

1977 MAY 2 and J/A 2 Frank Rubin has responded.

O/N 1 Caruthers Coleman has responded.

O/N 2, O/N 3, and O/N 5 Frank Rubin has responded.

DEC 1 Frank Rubin, Harry Hazard, Elliot Feit and Gardner Perry have responded.

DEC 3 Gardner Perry, Elliot Feit, Frank Rubin, and Naomi Markovitz have responded.

DEC 4 Naomi Markovitz, Frank Rubin and Edward Lynch have responded.

DEC 5 Edward Lynch, Frank Rubin, Naomi Markovitz, Mary Lindenberg and P. Jung have responded.

Y 1977 Harry Hazard points out $22 = ((9-1)+7)+7$ and $50 = 1{*}9 + 7\cdot 7$.

1978 JAN SD 2 Ben Suetitsky feels that it is immaterial where the walker stops providing he walks at the same rate both on and off the elevator. If, in addition relativistic considerations are applied, the man should stop while on the elevator.

Proposers' Solutions to Speed Dept.

MAY SD 1 Each meter of fence bounds a one-are sector, with an area of R/2. Since 100 square meters equal an are, the radius must be 200 meters.

MAY SD 2 Two. One says that Team A will lose, the other that it will win.

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