Baseball: Action But No Score

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Happy New Year. I have received several comments concerning our latest addition to Puzzle Corner — the Editor's picture. The consensus seems to be that as a photographer's model I make a good computer scientist-mathematician. But every engineer knows that one can learn to make do with the material available.

Problems

This being the first issue for the new year, we begin with our yearly problem: Y1978. Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 7, and 8 once each and the operators +, −, * (multiply), / (divide), and ** (exponentiation). We desire solutions containing the minimum number of operators. For a given number of operators, solutions using the digits in the order 1, 9, 7, and 8 are preferred. The solution to Y1977 is given below.

JAN 1 We start this month's regular problems with a bridge question from Kenneth Lebensold:
Included with his solution to 1975 FEB 1 were two related questions: What is the largest contract that can be made from three positions on the same deal? From four positions? Assume best defense.

JAN 2 J. Kleilin and Bob Martin can find, for each n and each j, integers $X_1, X_2, \ldots, X_j$, W, and Z such that $X_1^n + X_2^n + \ldots + X_j^n = W^{n-1} = Z^{n+1}$. Can you?

JAN 2 Seville Chapman writes:
Nearly 40 years ago Ripley had a problem which yielded me and 102 others in the country his autographed book: How can a baseball team make three triples, one double, two singles, and steal two bases in one inning without scoring a run?

JAN 4 Bruce Hannon wants us to prepare for a tornado attack:
Suppose you suddenly realize that you are (on foot) directly in the path of a midwestern tornado which is approaching with a speed of V_T. The tornado has a lethal radius of R_L. You wish to take a path which maximizes your chance of survival. Your speed is $a - bt$ (in which a and b are positive constants). In which direction should you head, and will you escape?

JAN 5 A space-age problem from Harvey Elentuck:
A satellite is orbiting Earth and is now directly above Chicago. How far from the center of the earth is the satellite if New York City and Los Angeles bound the horizon? You are given the following information:

	Latitude	*Longitude*	*Altitude*
New York	40°45'06"N	73°59'39"W	55 feet
Chicago	41°52'28"N	87°38'22"W	595 feet
Los Angeles	34°03'15"N	118°14'28"W	340 feet

The diameter of Earth is 7926.69 miles.

Speed Department

JAN SD 1 This quicky is from Glenn Rowsam:
Two men were playing chess. They played five games and each man won the same number of games. How is this possible if no draws occurred?

JAN SD 2 We end with a question from David and Aviva Eichler:
A shopper who is in a rush and is carrying heavy bags is going from Point A to Point B on a route which includes an escalator. He is tired and must rest. Should he rest on or off the escalator to save time? Or does it matter? (If he rests off the escalator he then actively walks up the moving escalator.)

Solutions

Y1977 Take the digits 1, 9, 7, and 7; and the operators +, −, * (multiply), / (divide), and ** (exponentiation); and form the integers from 1 to 100 using each digit once (7 is thus used twice) and the fewest possible number of operators. Parentheses may be used to indicate the order of operations and, in the case of a tie, the solution using 1, 9, 7, and 7 in order is to be favored.

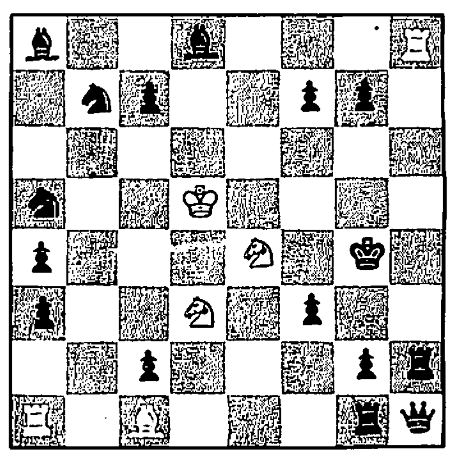
The problem is more difficult than usual due to the repeated digit. There are apparently 26 unattainable numbers. I can hardly wait for 1999 (or 2222). Two readers sent in computer printouts, and others included acknowledgements to H-P, T.I., etc.

An optimal solution is:

1	1 ** 977	51	9 − [(1 − 7) * 7]
2	1 + [9 ** (7 − 7)]	52	−
3	[(1 ** 7) + 9] − 7	53	−
4	(1 + 7)/(9 − 7)	54	9 * [7 − (1 ** 7)]
5	19 − 7 − 7	55	71 − 9 − 7
6	(91/7) − 7	56	(17 − 9) * 7
7	(1 ** 97) * 7	57	1 + [(9 * 7) − 7]
8	79 − 71	58	77 − 19
9	(1 ** 77) * 9	59	1 + 9 + (7 * 7)
10	(1 ** 77) + 9	60	−
11	1 + 9 + (7/7)	61	[9 * (7 − 1)] + 7
12	(91 − 7)/7	62	79 − 17
13	[(9 − 7) * 7] − 1	63	[(1 + 9) * 7] − 7
14	91 − 77	64	(1 + 7) ** (9 − 7)
15	(17 − 9) + 7	65	[(1 + 7) * 9] − 7
16	[(1 ** 7) * 9] + 7	66	−
17	[(1 ** 7) + 9] + 7	67	77 − 1 − 9
18	19 − 7/7	68	19 + (7 * 7)
19	19 + 7 − 7	69	(1 − 9) + 77
20	19 + 7/7	70	[(1 * 9) * 7] + 7
21	[(1 + 9) − 7] * 7	71	79 − 1 − 7
22	[(9 − 1) + 7] − 7	72	(1 * 79) − 7
23	[(1 * 9) + 7] + 7	73	(1 + 79) − 7
24	1 + 9 + 7 + 7	74	−
25	−	75	−
26	97 − 71	76	77 − (1 ** 9)
27	−	77	(1 ** 9) * 77
28	−	78	(1 ** 9) + 77
29	−	79	(1 ** 7) * 79
30	7 * 7 − 19	80	97 − 17
31	−	81	9 ** 1 + (7/7)
32	−	82	−
33	19 + 7 + 7	83	−
34	17 * (9 − 7)	84	(19 − 7) * 7
35	−	85	9 − 1 + 77
36	(7 − 1) ** (9 − 7)	86	(1 * 9) + 77
37	−	87	1 + 9 + 77
38	−	88	−
39	(7 * 7) − 1 − 9	89	97 − 1 − 7
40	(1 * 7 * 7) − 9	90	(1 * 97) − 7
41	(1 − 9) + (7 * 7)	91	(1 + 97) − 7
42	91 − (7 * 7)	92	91 + 7/7
43	−	93	−
44	−	94	−
45	—	95	−
46	(9 * 7) − 17	96	19 + 77
47	[(1 + 7) * 7] − 9	97	(1 ** 7) * 97
48	[7 ** (9 − 7)] − 1	98	(1 ** 7) + 97
49	[(1 ** 9) * 7] * 7	99	−
50	1 + [7 ** (9 − 7)]	100	−

Solutions from Avi Ornstein, H. W. Hazand, Howard Reuter, Igor Limansky, W. V. McGuinness, Richard Rudel, John Richards.

J/A 1 White forces a draw, White to move.



This was a good problem. The following solution is from Randy Kimble:

1 N—K5 ch K—B4 forced
2 N—N3 ch K—B3 forced
3 N—N4 ch K—K2
4 N—B5 ch K—Q2 forced
5 N—K5 ch K— B1 forced

(If Black's response in 3 is K—N3, then
4 N—K5 ch K—B3
5 N—N4 ch
and Black must eventually play K—K2 or the game is drawn by repeating the same position three times with the same player to move.)

6 N—K7 ch K—N1 forced
7 N—Q7 ch K—R2 forced
8 N—B8 ch K—R3 forced
9 N—N8 ch K—N4 forced
10 N—R7 ch K—N5
11 N—R6 ch K—B6

(If Black's response in 10 is K—N3, then
11 N—B8 ch K—N4
12 N—R7 ch
and Black must eventually play K—N5 or draw by repeated position.)

12 N—N5 ch K—Q6

(If Black's response in 11 is K—N6, then
12 R x P mate.)

13 N—N4 ch K—K7

(If Black's response in 12 is K—N6, then
13 R x P mate.)

14 N—B3 ch K—B7
15 N—Q3 ch K—N6

(If Black's response in 14 is K—K8 or K—B8, then
15 B—K3 ch P—B8(Q)
16 R x Q mate.)

16 N—K4 ch K—N5

(If Black's response in 15 is K—B8, mate results.)

After 16 moves, the original position is restored. White can chase Black around the board again and then claim a draw when the position reappears for the third time.

Also solved by William Butler, Robert Saunders, Bill Camperlino, Steve Grant, Jeffrey Miller, Jerome Taylor, Bill Wilbur, Martin Donovan, John McCauley, John Chandler, and the proposer, Bob Kimble.

J/A 2 Find the smallest number which can be partitioned in six distinct positive integers such that the sum of any five of these six is a perfect square.

Adding like terms:
$r^2 - 30r + 55 > 0$,
or $4(r - 30) + 55 > 0$.
The smallest r satisfying this relationship is 29 since
$-29 + 55 > 0$, but $28(-2) + 55$ is -1 and < 0.
Also, any r larger than 29 will also satisfy the relationship.
Diminishing the size of any or all of the five squares other than r^2 does not permit a smaller r:

1. Diminishing the smallest of the squares, $(r-5)^2$, to $(r-6)^2$ and leaving the other squares the same changes the inequality to $r(r - 32) + 66 > 0$; and 30 is the smallest r that will satisfy it since $(30)(-2) + 66 > 0$, but $29(-3) + 66$ is $(-87 + 66)$, which is negative.
2. Retaining $(r-6)^2$ and diminishing $(r-4)^2$ to $(r-5)^2$ changes the inequality to $r(r-34) + 75$; and 32 is the smallest r that will satisfy it, since $32(-2) + 75$ is $+11$ while $31(-3) + 75$ is -18 and <0.
3. Other changes diminishing the size of any of the squares obviously, therefore, increase the size of the r that will satisfy the arrangement.

It follows that $(29^2 + 28^2 + 27^2 + 26^2 + 25^2 + 24^2)$ is the smallest sum of six different squares that will be larger than $5 \cdot 29^2$. While this sum satisfies this requirement of the problem, it does not satisfy the requirement that the sum be divisible by 5. In the series of perfect squares, not only are the squares of 5 and its multiples divisible by 5; between each such square and the next higher such square, there are always two pairs of squares the sum of each of which is divisible by 5. Therefore, to be divisible by 5, a sequence of six squares consisting of $r^2 + (r-1)^2 + \ldots + (r-5)^2$ must comprise the square of each of two adjacent multiples of five and the four intervening squares. The next higher value of r which satisfies this requirement is 30. As shown above, there are only two sequences of squares with $r = 30$ which satisfy the requirement that the sum of the squares $> 5 \cdot 30^2$. They are
(1) $30^2 + 29^2 + 28^2 + 27^2 + 26^2 + 25^2$, and
(2) $30^2 + 29^2 + 28^2 + 27^2 + 26^2 + 24$.

The following detailed solution is from Frederick Nash:
Let x = the desired number; A, B, C, D, E, and F the positive distinct integers into which x is to be partitioned; and m, n, o, p, q, and r the numbers which are to be squared. Then

$$\begin{array}{lll}
x = A + B + C + D + E + F, & \text{and} & \\
m^2 = A + B + C + D + E & x - m^2 & = F \\
n^2 = A + B + C + D + E + F & x - n^2 & = E \\
o^2 = A + B + C + E + F & x - o^2 & = D \\
p^2 = A + B + D + E + F & x - p^2 & = C \\
q^2 = A + C + D + E + F & x - q^2 & = B \\
r^2 = B + C + D + E + F & x - r^2 & = A
\end{array}$$

$x - m^2 + x - n^2 + x - o^2 + x - p^2 + x - q^2 + x - r^2$
$= A + B + C + D + E + F = x$,
or $6x - (m^2 + n^2 + o^2 + p^2 + q^2 + r^2)$
$= x$,
or $5x = m^2 + n^2 + o^2 + p^2 + q^2 + r^2$,
$x = (m^2 + n^2 + o^2 + p^2 + q^2 + r^2)/5$.

By hypothesis, x is an integer. Therefore $m^2 + n^2 + o^2 + p^2 + q^2 + r^2$ must be evenly divisible by 5. Assuming that r^2 is the largest of the squares, the next problem is to find the smallest r^2 such that $x - r^2 > 0$, for the smallest r^2 will yield the smallest x.
Since $x = (m^2 + n^2 + o^2 + p^2 + q^2 + r^2)/5$, the problem may be restated as finding an $[(m^2 + n^2 + o^2 + p^2 + q^2 + r^2)/5] - r^2 > 0$,
or $m^2 + n^2 + o^2 + p^2 + q^2 - 4r^2 > 0$,
or $m^2 + n^2 + p^2 + q^2 + r^2 > 5r^2$.
If r^2 is the largest of the squares and all squares are different (as they are, by hypothesis), then the other five squares cannot be larger than $(r - 1)^2$, $(r - 2)^2$, etc. Substituting these in the above inequality and expanding:
$(r^2 - 2r + 1) + (r^2 - 4r + 4) + (r^2 - 6r + 9) + (r^2 - 8r + 16) + (r^2 - 10r + 25) - 4r^2 > 0$.
But only the first is divisible by 5, and it is the smallest sequence of squares that satisfies both requirements. The answer to the problem is, therefore, obtained by adding this sequence, dividing the sum by 5, and partitioning the result in accordance with the equations shown in italics in my statement of the problem,

$25^2 = 625$
$26^2 = 676$
$27^2 = 729$
$28^2 = 784$
$29^2 = 841$
$30^2 = \underline{900}$
$5 \quad \underline{4555}$

$$\begin{array}{lllll}
x - r^2 & = 911 - 900 = & 11 & = A \\
x - q^2 & = 911 - 841 = & 70 & = B \\
x - p^2 & = 911 - 784 = & 127 & = C \\
x - o^2 & = 911 - 729 = & 182 & = D \\
x - n^2 & = 911 - 676 = & 235 & = E \\
x - m^2 & = 911 - 625 = & \underline{286} & = F \\
 & & 911 &
\end{array}$$

$x = 911 = 11 + 70 + 127 + 182 + 235 + 286$.

Proof:
$A + B + C + D + E = 625 = 25^2$
$A + B + C + D + F = 676 = 26^2$
$A + B + C + E + F = 729 = 27^2$
$A + B + D + E + F = 784 = 28^2$
$A + C + D + E + F = 841 = 29^2$
$B + C + D + E + F = 900 = 30^2$.

Also solved by Edward Parks, Frank Carbin, Steve Grant, Bob Lutton, Jacob Bergman, Neil Cohen, Judith Longyear, Raymond Gaillard, Alan LaVergne, William Rosenfeld, Charles Rozier, David Gluss, Harvey Elentuck, Emmet Duffy, Gerald Blum, Steve Clarke, William Butler, Bill Wilber, John Chandler, Winslow Hartford, Harry Zaremba, and the proposer, P. V. Heftler.

J/A 3 Assume a fixed number of buttons can fit on a calculator. In order to maximize the total number of functions, how many buttons should be "primary" keys and how many should be "shift" keys? (Each "primary" key gives one function if depressed alone and a different one if depressed with different "shift" keys. No more than one "shift" key may be used at once.) After answering the question with those rules, try lifting the last restriction.

The problem did not indicate whether more than one shift key can be depressed at once. If this is allowed, the number of possibilities is $2^x y$, where x is the number of shift keys and y the number of function keys. As Frank Rubin points out, this is maximized when there is exactly one function key. Most likely the intent was to allow only one shift key to be depressed at one time (as in current calculator designs). In this case, the number of possibilities is $(x + 1)y$. Daniel Cheng found the maximum to occur when $x = \frac{1}{2}(b - 1)$, where b is the number of buttons. When the latter is non-integral, you may round it either up or down. Maximums were obtained by the usual derivative test.

Also solved by Morrie Gasser, William Rosenfeld, Charles Rozier, David Gluss, Steve Clarke, William Butler, John Chandler, Frank Carbin, Michael Kennedy, Naomi Markovitz, Raymond Gaillard, Neil Cohen, Steve Grant, Edward Parks, Jordan Wouk, Robert Saunders, and the proposer, Joe Horton.

J/A 4 How many times must a deck of cards be shuffled before it returns to the same order? Assume 52 cards and a perfect, nonconservative shuffle — one in which the first card on shuffle j is the second on shuffle j + 1.

The following is from Gordon Cochrane: A deck of cards must be shuffled 52 times before it returns to the same order. It is necessary to assume a perfect, nonconservative shuffle each time in which the deck is divided in half and the cards from one half are interspersed uniformly with the cards from the other half, with the first card in shuffle j being the second card in shuffle j + 1. The position of any card in shuffle j + 1 is determined according to the following rule:

If the position of any card, P, (counting from the top of the deck) on shuffle j is equal to or less than 26, the position in shuffle j + 1 is 2P. If the position P is greater than 26 in shuffle j, then the position in shuffle j + 1 is equal to (P − 26) 2 − 1. The card which is initially at the top of the deck takes the following positions in the 52 shuffles:

1 2 4 8 16 32 11 22 44 35 17 34 15 30 7 14 28 3 6 12 24 48 43 33 13 26 52 51 49 45 37 21 42 31 9 18 36 19 38 23 46 39 25 50 47 41 29 5 10 20 40 27 1

Also solved by Randy Kimble, M. G. Settle, Bruce Andeen (who sent in some computer runs for a more general problem), Alan LaVergne, Bob Lutton, Steve Grant, Neil Cohen, Raymond Gaillard, Michael Kennedy, John Chandler, William Butler, Steve Clarke, Charles Rozier, and Frank Rubin.

J/A 5 Find an integer solution to:

$$\frac{A^4 + B^4 + C^4}{A + B + C} = 39.$$

All respondents found the solution 1,2,4; but Steve Grant was among the few to prove it unique. His analysis follows: Let m be the maximum of |a|, |b|, |c|. then

$m^4 \leq a^4 + b^4 + c^4$, and $3m \geq a + b + c$. Thus

$m^3/3 \leq (a^4 + b^4 + c^4) / (a + b + c) = 39$, and hence $m \leq 4$.

This shows that a, b and c must be 0, ±1, ±2, ±3, or ±4.

x	x^4	x^4 *(mod 39)*
0	0	0
±1	1	1
±2	16	16
±3	81	3
±4	256	22

Since $a^4 + b^4 + c^4 \equiv 0 \pmod{39}$, the above table shows that the only possibilities (up to permutations) are $(a^4, b^4, c^4) \equiv (0, 0, 0) \pmod{39}$ or $(a^4, b^4, c^4) \equiv (1,16,22) \pmod{39}$. But (0,0,0) gives a zero denominator in the original problem. Thus (a,b,c) = (±1, ±2, ±4). Since $a + b + c = (a^4 + b^4 + c^4)/39 = 273/39 = 7$, (a,b,c) = (1,2,4) is the unique solution (up to permutations).

Also solved by Randy Kimble, Bob Lutton, Robert Saunders, Jordan Wouk, Edward Parks, John Chandler, Alan LaVergne, Neil Cohen, Frank Carbin, David Gluss, William Rosenfeld, Michael Kennedy, William Butler, Steve Clarke, Morrie Gasser, Charles Rozier, Frank Rubin, Judith Longyear, Harvey Elentuck, Gerald Blum, Winslow Hartford, John Podolsky, Harry Zaremba, Raymond Gaillard, P. V. Heftler, and Avi Ornstein.

Better Late Than Never

1976 J/A 1 John Podolsky points out that the simplification given recently is unattainable since the White KB cannot reach Q1 without movement of the KP or QBP.

1977 JAN 1 James Prigoff and Smith Turner have responded.

FEB 2 Judith Longyear notes that a solution by Meredith and Lloyd appears in the *Journal of Combinatorial Theory (B)* 15 (1973), page 161.

FEB 3 Peter Groot has responded.

MAY 1 One of the solvers commented that the third move could be placed first, showing that the first move is not unique. Tom Jenkins and your Editor, however, feel that

1	P-KR4	K-B2
2	P-K4	K-N3
3	Q-R5 ck	KxQ

refutes this.

MAY 3 Frederic Vose has submitted the following comments:

This is a variant on a problem dealt with in Frederick Mostleller's *Fifty Challenging Problems in Probability* (Addison Wesley, 1965); Mr. Mosteller says that John von Neumann solved it in his head in 20 seconds. You are correct that the angular momentum of the toss cannot be handled and must be disposed of; Mosteller envisions the coin falling on a viscous bed which absorbs it. But it is not necessary to ignore the angular displacement of the coin, thereby allowing a plane geometric solution to a solid geometric problem. Consider the "super-coin" within a sphere tangent to the perimeters of its faces. The probabilities are proportional to the areas of the three zones into which the sphere is divided, in turn proportional to the thickness of the zones. Working it out to the nearest integer, N = 4 in case (a) and N = 14 in case (b).

JUN 2 What is the maximum possible score for one player on one turn of a legal scabble match? The maximum possible score in a legal match?

Apparently a scrabble match is legal if a player uses non-words but is not challenged. However, I do not believe that was the intended problem and have not allowed such solutions; I apologize for the poor wording. The best (and most unbelievable) solutions were reports on a previous contest concerning this same problem. The following is from Frank Rubin:

This problem has been the subject of lively debate for several years in *Games and Puzzles* magazine. The winner of the first Scrabble Superscore Competition, for a single play, was Ron Jerome (the British R. Robinson Rowe) in the May, 1974, edition. He played BZYCHRS to form the word BENZOXYCAMPHORS as shown below, for 1,961 points.

```
       S      J
       Q      I
       U   EEVN
       A  EE AN
     ADNOTE  GY
      AD     AR
      FET    BI
P     FRO  G OC
RIEW  OM  QUINK
O  E  DA   LODS
V  RUTIN   L AH
E  T  LI   I GA
RI U ELAT  STEW
BENZOXYCAMPHORS
```

The full scrabble game in the second Superscore Competition was also won by Ron Jerome with the key word being ALEXIPHARMAKONS on the bottom row.

But, alas, Jerome's result of 3,881 was passed repeatedly after the close of the competition. The last effort I have seen was submitted by Ralph G. Beaman, below, and scores 4,153 points. The full sequence of moves is not given — only the final position. The game is Solitaire Scrabble, so that all 100 tiles have been used; otherwise, remove the S in "Mitigates," giving 4,142 points. The underlined letters are blank tiles.

```
JACKPUDDINGHOOD
OLEANDRINES
   ETE I
    EU
     MITIGATS
    TA
   VA            B
PREQUALIFYING E
   LU          ARF
   VA            L
   E             O
   TS            W
   EW            E
ONEIROTIC        R
BENZOXYCAMPHORS
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Also solved by Gerald Ruderman, Ted Mita, Stuart Schulman, William J. Butler, Jr., Peter Groot, Donald Oestreicher, Donald Pratt, Nancy Burstein, Robert Moore, Bob Ferrara, and Harry Nelson.

PERM 2 Harvey Goldman noticed that the term $.\sqrt{4} = .2$ opens up new possibilities. For example, let

$$z = \sqrt{\sqrt{\sqrt{(.\sqrt{4} ** (-4!)}}} = 125.$$

Then

$73 = (.\sqrt{4} ** -\sqrt{4}) + 4! + 4!$
$77 = z - 4! - 4!$
$103 = z - 4! + \sqrt{4}$ → up to 130
$113 = z - 4! / \sqrt{4}$
$117 = z - 4 * \sqrt{4}.$ missing 87 93

Frank Corbin has also responded. 107

NS 6 Irving Hale notes that some of the clues are stronger than necessary.

NS 8 Edward Parks, Steve Grant, and Harry Zaremba have responded.

Proposers' Solutions to Speed Problems
JAN SD 1 The men were not playing against each other.

JAN SD 2 All points off the escalator have exactly the same status, so if the shopper rests off the escalator we can assume, without loss of generality, that he is resting just before stepping onto it. But this is silly, in the interest of saving time, he might as well just get on and then rest.