An Oscillating Piston Since 1940

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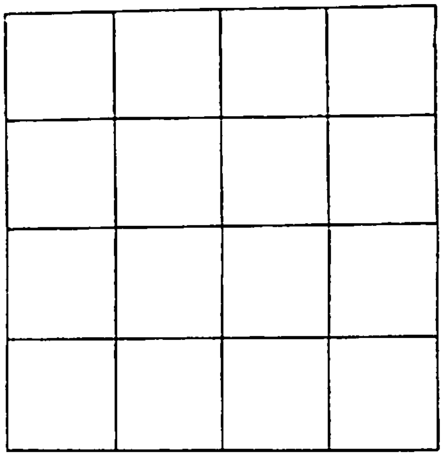
Though at times last year some skeptics doubted it could happen, C.U.N.Y. classes have begun this fall, and York College has continued its policy of accepting unusual freshmen: for some reason their average age remains constant, but every year they look younger. I wonder how that happens.

Several readers have submitted interesting letters. Neil Cohen reports that a letter of his to columnist D. Keith Mano concerning a marriage problem in "Puzzle Corner" resulted in a column on stable marriages. Jerome J. Taylor asks some common questions about computer chess, and since they appear to be of general interest, let me answer them here. A computer program as world chess champion is not imminent. I expect this to happen in my lifetime (a controversial expectation), but I am watching my diet and not smoking to increase the likelihood. Specifically, humans will easily hold on for ten years; but this century will most surely be the last without a computer champion — unless it is the first century to have one. However, neither computers nor humans will be able to "solve" chess (i.e., consider all possible outcomes) before the sun "burns" out! Judith Q. Longyear destroys NS 8 (see solutions section) and reports the pleasant news that she has received tenure at Wayne State. Congratulations!

Problems

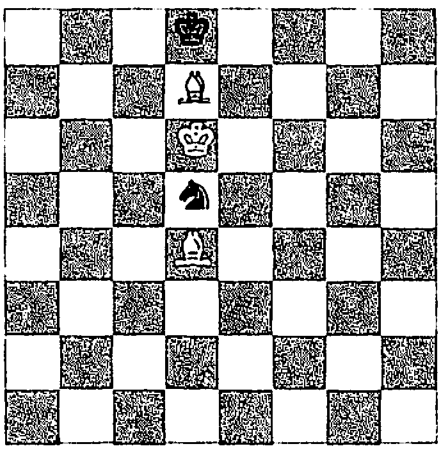
NS 9 This problem, from George L. Uman, originally appeared as 1972 DEC 3. There was no solution then; can we have one now?

In each of the 16 squares of the figure, place a *different* letter, selected so each row, column, and long diagonal will spell



a *different* four-letter word when the letters are selected consecutively in one or the other of the only two possible directions, as we do with numbers. There will be a total of ten different words, all of which must be defined in any one edition of Webster's dictionaries.

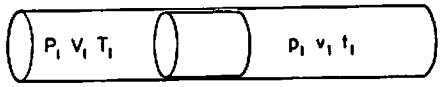
DEC 1 Our first new offering is a fairly simple chess problem from Glen Ferri:



White to play and mate in two.

DEC 2 The following from J. L. Friedman appeared in a series of advertisements published in *Technology Review* by Calibron Products, Inc., of West Orange, N.J., back in 1940:

The ends of a closed cylinder, fitted with a leakproof, frictionless piston, are filled with perfect gases having the initial pressures, volumes, and temperatures indicated. If all of the walls are perfect heat insulators, where will the piston finally stop? Three students propose three answers: A says it will stop where P = p, using adiabatic processes. B says the piston will oscillate perpetually. C says: even though heat does not flow *through* the piston, the piston itself will act like a big molecule, and (after many oscillations) the pressures *and temperatures* will equalize. Who is right? Other possibilities?



DEC 3 Frank Rubin wants you to replace each letter by a unique digit to obtain a valid addition:

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   F I V E
     T W O
 +   O N E
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 E I G H T
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DEC 4 Peter Hadley has an interesting result in number theory for you:

It is well known and easily proved that the differences between consecutive perfect squares are always odd numbers and that the difference between two consecutive differences will always be 2, which equals 2! It is also true that the differences of the differences of the differences of consecutive perfect cubes is always 6 = 3! In fact, this pattern holds for all natural numbers and zero; i.e., (the differences of)^n consecutive perfect nth powers is n! Below are several arrays which attempt to demonstrate this more clearly. The bottom line consists of consecutive integers all raised to the same power. Each higher line consists of numbers each the difference of the two numbers beneath it. Note that after making the same number of subtractions as the power in the bottom row, we obtain a row all of whose elements are the factorial of that power. Prove that this is so.

```
      2   2   2   2   2      2! = 2
    1   3   5   7   9   11
  0   1   4   9  16  25   36

        6   6   6   6        3! = 6
      6  12  18  24  30
    1   7  19  37  61  91
  0   1   8  27  64  125  215

         24  24  24          4! = 24
       36  60  84  108
     14  50  110  194  302
    1  15  65  175  369  671
  0   1  16  81  256  625  1296
```

DEC 5 John T. Rule offers the following geometry problem: Given two sides, construct a parallelogram whose angles are equal to the angles between its diagonals.

Speed Department

DEC SD1 Leo Santori wants you to draw eight equal-length non-intersecting lines which define two squares and four triangles.

DEC SD2 Joe Horton submits a quickie I often use at cocktail parties:

Add a single straight line to the following "equation" transforming it to a valid equation (*not* an inequality).

1 = V1

Solutions

JUN 1 You are South, the declarer, with a contract of three no-trump. West opens with ♣6. How do you play the hand?

North:
♠ Q J 10 9 4
♥ Q 5
♦ A J 10
♣ J 10 3

West:
♠ K 7 6 3 2
♥ 6 3
♦ 3
♣ Q 9 7 6 2

East:
♠ 8 5
♥ K J 9 7
♦ Q 9 8 7
♣ 8 5 4

South:
♠ A
♥ A 10 8 4 2
♦ K 6 5 4 2
♣ A K

Apparently the key to this is to discard the ♣A on the fourth trick; after that, it's not too hard. The following is from R. C. Evans:

South takes the opening lead of ♣6 with ♣K. He plays the singleton ♠A at trick 2. He then crosses to the dummy's ♦A, lays down the ♠Q and jettisons his ♣A. The opponents can take the ♠K, ♣Q, one heart, and one diamond for a total of four tricks. Declarer takes nine tricks to fulfill the contract.

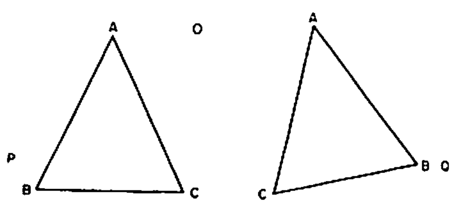
Also solved by Hilda Gilinson, Emmet J. Duffy, R. Robinson Rowe, Smith D. Turner, Stuart Schulman, M. F. Colaco, Mark Chen, William J. Butler, Jr., Elmer Ingraham, Peter Groot, Dave Windsor, and the proposer, Russell A. Nahigian.

JUN 2 APPEARS NEXT ISSUE

JUN 3 Prove that the base angles of an isosceles triangle are equal without constructing an angle bisector.

The following is from Frederick Hooven (with help from Marc Kac and an unnamed computer):

Professor Kac was reporting, along about 1964, the results of having programmed a computer to perform the proofs of Euclid's propositions, which it performed in more or less the conventional way until it came to this one. He also reported that this proof had been found later in some old texts. In the diagram below the plane OPQ contains the isosceles triangle ABC. ABC is inverted by rotation through 180° about an axis contained in the plane OPQ, producing the triangle ACB. Since the two sides AB and AC are equal and the included angle A is common to both, the two triangles ABC and ACB are congruent. Therefore, angles C and B are equal. Q.E.D.



Also solved by Stuart Schulman, William J. Butler, Jr., Peter Groot, Naomi Markovitz, Frank Rubin, Winslow Hartford, R. Robinson Rowe, Smith D. Turner (Jdt), Robert Saunders, James Shearer, Raymond Gaillard, Farrel Powsner, Mary Lindenberg, Jonathan Luke, Alexander Slocum, Raymond Kinsley, M. Capobianco, Leon Bankoff, Emil Deeg, Elliott Roberts, Hal Shane, Bruce Fleischer, Daniel Grunberg, Philip Martel, Harry Zaremba, Verona Winn, Robert Pogoff, and Avi Orstein.

JUN 4 Using a computer with a four-register stack plus one storage register, numerical entries, clear and print instructions, and nine operating keys, generate 355/113, a rational number which is a close approximation to pi, in the minimum number of operations. The nine operating keys have the following functions, starting from the following initial condition of the registers:

M DCBA

The "add" instruction yields:

M ODCA + B

The "subtract" instruction yields:

M ODCB − A

The "multiply" instruction yields:

M ODCA · B

The "divide" instruction yields:

M ODCB/A

The "square root" instruction yields:

M DCB$\sqrt{A}$

The "repeat" (or "enter") instruction yields:

M CBAA

The "interchange" instruction yields:

M DCAB

The "recall" instruction yields:

M CBAM

The "store" instruction yields:

A DCBA

The shortest solution is from the proposer, B. W. Latourneau, who offers a 21-step calculation as shown at the top of the next page.

Albert A. Mullin notes that he posed a similar problem in *American Mathematical Monthly* in February, 1975, relating to the design of electric circuits; thus diophantive equations are applicable to both computer hardware and software.

Also solved by William J. Butler, Jr., Harvey Greenberg, Jacob Bergmann, Philip Martel, and Peter Groot.

0	0	0	0	1	
0	0	0	1	1	RPT
0	0	1	1	1	RPT
0	0	0	1	2	ADD
0	0	1	2	2	RPT
0	0	0	1	4	ADD
0	0	1	4	4	RPT
0	1	4	4	4	RPT
0	0	1	4	16	MUL
16	0	1	4	16	STO
16	1	4	16	16	RPT
16	0	1	4	32	ADD
16	0	0	1	128	MUL
16	0	0	0	129	ADD
16	0	0	129	129	RPT
16	0	129	129	16	RCL
16	0	0	129	113	SUB
16	0	0	0	129/113	DIV
16	0	0	129/113	16	RCL
16	0	0	129/113	4	ROOT
16	0	0	129/113	2	ROOT
16	0	0	0	129/113	ADD

JUN 5 Six width gauges are permanently mounted on a ring. They can measure any width from 0.001" to 0.031" in 0.001" steps by using individual gauges or by sliding two, three, four, five, or six consecutive gauges together. Find five ways to do this, giving the widths of the gauges and their order on the ring.

The following is from Peter Groot:

Since we can start with any of six gauges and count 1, 2, 3, 4, or 5, we have 30 measurements, plus a measurement of .031" with all six. Thus each individual gauge and each combination must be unique. There must then be a .001 and a .002, and if not together there must be a .003 in the set. From the requirement for uniqueness and sum we can construct the 21 possible sets. After some trial and error, the following solutions emerge:

.001,	.003,	.002,	.007,	.008,	.010
.001,	.003,	.006,	.002,	.005,	.014
.001,	.007,	.003,	.002,	.004,	.014
.001,	.002,	.005,	.004,	.006,	.013
.001,	.002,	.007,	.004,	.012,	.005

Also solved by William J. Butler, Jr., R. Robinson Rowe, James W. Shearer, Harry Zaremba, Winslow Hartford, Stuart Schulman, Dennis Sandow, Kenneth Wise, and the proposer, Emmet J. Duffy. Mr. Sandow reports that his SR-52 found four solutions after 93 hours, and Mr. Rowe comments that this might be considered a particular case for a set of n gauges making all incremental thicknesses from 1 to $n^2 - n + 1$ mils; and he wonders for what n besides 6 the general case has solutions.

NS 7 Let N be some fixed positive integer. Show that there exist positive rational numbers $a_1, \ldots, a_N$ such that for any m, $1 \leq m \leq N$

$$S(m) = \sum_{i \leq m} a_i^3$$

is the square of a rational number, and $S(N) = 1$.

Alan LaVergne has supplied a long and complex proof that no such sequence of rationals exists. Unfortunately, space doesn't permit its publication; readers who wish a copy should write to the Editors at Room 10-140, M.I.T., Cambridge, Mass., 02139. William J. Butler and Charles Richards have also responded.

NS 8 Under what additional conditions is it true that $6N + 1$ or $6N - 1$ is prime, where N is a counting number?

Judith Longyear writes:

Ha! The twin primes conjecture is that there exist infinitely many pairs of numbers x, x + 2 both prime. Past x = 3, they must be $x = 6n - 1$, $x + 2 = 6n + 1$, so this is probably an NS for a long time. Bombieri recently received a field medal for coming close.

Better Late Than Never

NS 4 John Stillwell solved the problem, noticed the solution in Spivak's book mentioned in June, and traced the problem to a 1923 book of Kerekjarto.

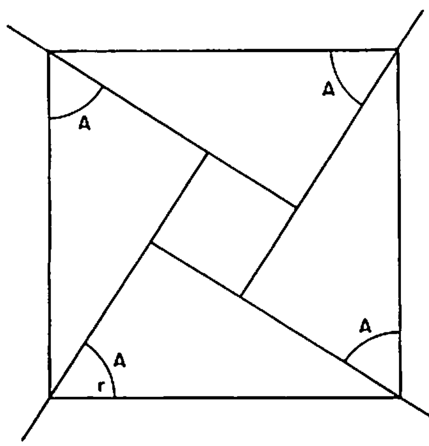
NS 5 James Shearer noticed that B should be connected to every C in the non-polar projection.

NS 6 William McGuinness has responded.

PERM 2 Morrie Gasser has responded.

Proposers' Solutions to Speed Problems

SD1 Let A be any angle between 45° and 90°:



SD2 1 = VT.