The Editor in Ashes and Sackcloth

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Hi. I hope that your summer was as enjoyable as mine. In addition to a one-week vacation, I attended the International Computer Conference in Toronto which included the World Computer Chess Championship. The winning program, Chess 4.6 (Aiken and Slate), played very fine chess. Its tournament play is at the expert level, and the consensus is that Chess 4.6 plays speed chess as well as a master (U.S.C.F ratings). If you have a chance to see a computer chess tournament, don't fail to take it; for one thing, you will not be asked to keep quiet!

As a new volume begins, let me answer some procedural questions about this column as well as explain the ground rules for new readers. Each issue I publish five "regular" problems (the first of which concerns either bridge or chess) and two of the "speed" variety. All problems are submitted by you, the readers. For the regular problems, I will publish someone's solution, chosen from among readers' responses, three issues later along with the names of everyone else who responded. (That means that solutions to the problems given below will appear in the February, 1978, issue of the *Review*, my deadline for which will occur early in December.) For the "speed" problems, which are supposed to be cute and easy, the procedure is different: if the author supplies a solution it appears at the end of the column containing the problem.

As you can see, this column is only as good as its readers. Currently I have a two-year backlog of ordinary problems and about a one-year backlog for chess, "speed," and bridge problems.

Finally, Bob Ferrara — an M.I.T. classmate — writes, "You may be the first member of the Class of 1967 to have his own puzzle column, but you are no longer the only one." He has had variants of acrostic puzzles published in the *Boston Herald American*.

Problems

O/N 1 We begin this month with a bridge problem from Winslow H. Hartford: given the following hands, show how South can make six spades.

North:
♠ Q J 10 9 8
♥ A 5
♦ A 10
♣ A K 9 3

West:
♠ A K 4 3 2
♥ K 4 3
♦ 8 3
♣ 6 5 2

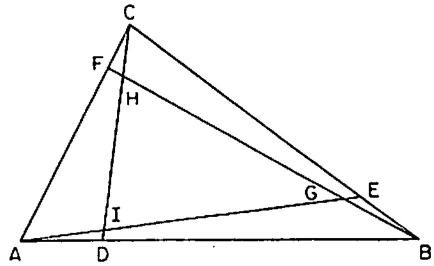
East:
♠ 7 6 5
♥ —
♦ J 9 7 6 5 4 2
♣ Q J 10

South:
♠ —
♥ Q J 10 9 8 7 6 2
♦ K Q
♣ 8 7 4

O/N 2 Eric Jamin wants you to find three perfect squares such that the sum of any two is also a perfect square.

O/N 3 Naomi Markovitz is interested in trapezoidal representations; she writes: Let n be a positive integer. A trapezoidal representation of n is a decomposition of n into a sum of consecutive positive integers — e.g., 15 = 15, 15 = 7 + 8, 15 = 4 + 5 + 6, 15 = 1 + 2 + 3 + 4 + 5. How many distinct trapezoidal representations does n have? Hint (courtesy of the editor): the answer is in terms of another, far more familiar, decomposition of n.

O/N 4 A geometry problem from Harold Heins: Given triangle ABC such that AB/AD = BC/BE = CA/CF = n. Draw AE, BF, and CD intersecting at points G, H, and I. What is the area of triangle GHI?



O/N 5 I consider problems like the following, from William J. Butler, Jr., to be combinatorial and not bridge problems: if a bridge foursome plays one hand every five minutes, how long will they have to play to have a 1 per-cent chance of a hand repeating? We require that each person has the same hand that he or she did on any previous deal. The two deals in question need not be consecutive.

Speed Department

O/N SD1. The editor knows a common English nine-letter word that contains only one vowel. Can you find one?

O/N SD2. Emmet J. Duffy submits the following: A post in a rectangular field is 17 feet from one corner and 33 feet from the diagonally opposite corner. If the post is 37 feet from the third corner, what is the distance from the post to the fourth corner?

Solutions

JAN 1 (as modified in May) With a no-trump contract and the following hands, how can South, who is on lead, make four of the remaining five tricks against any defense?

North:
♠ 8
♥ Q 7
♦ 9
♣ 10

West:
♠ A
♥ 10 9 6
♦ 8
♣ —

East:
♠ —
♥ —
♦ A 7
♣ Q 9 6

South:
♠ —
♥ J
♦ J 5
♣ A 8

The following solution is from Scott Byron with help from Steve Weisman: The opening lead can quickly be narrowed down to a couple of choices. The right choice is the ♦5. If East uses his ♦A now, you will set up a squeeze in West. No matter what East returns, you win your remaining diamond and the ♣A in South. This forces West to make two discards in addition to the ♦8. Having discarded the ♣10 and the ♦9 already, your third discard from North will be the ♥7 if West discarded a heart and a spade, or it will be the ♠8 if he discarded the two hearts. You then lead the ♥J to the ♥Q and take your remaining winner.

If East ducks the diamond lead, you then put the squeeze on him. Lead back the ♠8 to West's ♠A, and he must return a heart. You had discarded the ♥J on the ♠8. You win the heart return with the ♥Q in North. If East discards two clubs, discard your ♦J. If he discards a club and a diamond, discard your ♣8. Play the ♣10 to your ♣A and take the remaining winner.

Also solved by Edwin S. Strauss, Edwin

H. Koehler, Mark Chen, Joe Feil, Bas Leeuwenburg, Bo Jansen, Alan Lemmon, Roger Yaseen, George Holderness, Sr., James W. Poynter, Michael A. Kay, Steven J. Projan, Willaim C. Everett, Ronnie Rylstein, Willaim J. Butler, Jr., Jerry Grossman, Warren Himmelberger, D. Pratt, David E. Romm, Howard Stevens, Thomas Mauthner, Bob Hess, D. Pomerantz, Mrs. D. S. Floyd, Paul M. Knopf, Elmer C. Ingraham, Ron Adelsman, Bowman Cutter, Anthony J. Albanese, Richard I. Hess, Albert J. Fisher, N. Neuberger, R. Robinson Rowe, Jacob Bergmann, Jeffrey M. Bowen, an anonymous employee of General Mills Chemicals Inc., and the proposer, Emmet J. Duffy.

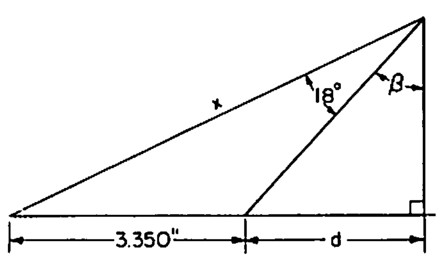
MAY 1 White begins a chess game with the usual set-up, while Black has only his king (in the usual position). What is the minimum number of moves White needs to achieve mate, assuming Black tries to avoid it? Is the first man moved by White the same in all minimal solutions?

The responses from William J. Butler, Jr., and Jeffrey M. Bowen are quite similar. Both use the queen and one rook to mate in eight moves and in both solutions, Black's moves are immaterial. One solution is:

1	P-K4	
2	Q-R5	Black's king now confined to three ranks
3	P-KR4	
4	R-R3	
5	R-KN3	
6	R-N6	Black's king now confined to two ranks
7	Q-R7	Black's king now confined to one rank
8	R-N8	Mate

Since the first and third moves may be permuted, the first move is not unique.

MAY 2 A typical drill-pattern problem; solve for x:



This problem was well received, and nearly everyone's solution is correct. The following is from Robert A. Stairs:

$d/5 = \tan\beta$ (1)

$(d + 3.350)/5 = \tan(\beta + 18°) = (\tan\beta + \tan 18°)/(1 - \tan\beta \tan 18°)$ (2)

In (2) substitute d/5 for tan β from (1). The result is a quadratic in d which may be solved. Then, by Pythagoras,

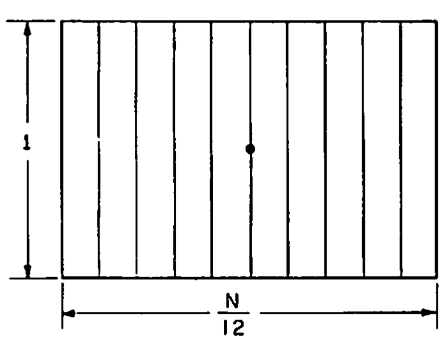
$x^2 = 5^2 + (3.350 + d)^2$,
and $x = 8.678$ inches.

Also solved by William J. Butler, Jr., Michael A. Kay, Erwin S. Strauss, Lindsay Faunt, Morrie Gasser, Gerald Blum, David C. Allen, Arthur L. Mavis, Serge Loussarian, John E. Prussing, Mr. and Mrs. D. Szper, Richard Berkof, Eugene McManus, Roy Sinclair, S. K. Bhalla, Winslow H. Hartford, Ed Parks, Norman M. Wickstrand, Joe Lacey, W. M. Leeds, Thomas Parrish, David Emmes, Frank Rubin, Norman Spencer, W. M. Cheung, Arthur L. Kaplan, Emmet J. Duffy, Frank Carbin, William Schoenfeld, Harry Zaremba, Avi and Bernice Ornstein, Rich Clark, Warren Lane, P. Michael Jung, John E. Morse, Carl M. King, and Henry Fleischer.

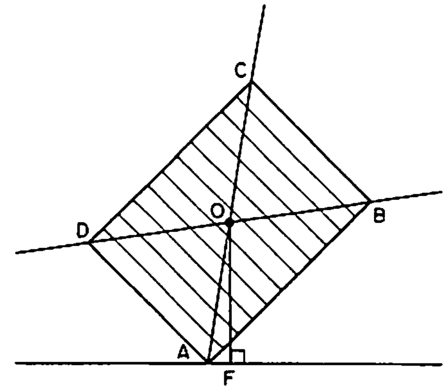
MAY 3 Given a "super-coin" made of N coins epoxied together in a cylindrical form. Assuming that the diameter of a coin is 12 times its thickness, what is the nearest integer N for (a) the "super-coin" with equal chances of heads, tails, or edge? (b) the "super-coin" with equal chances of face or edge?

I may be a stickler but I am not completely satisfied with any of the solutions submitted. My complaint is that no one's analysis includes the rotation of the coin. Several people just ignored the issue and many explicitly assumed that the rotation is not to be considered (certainly better than ignoring it). A few gave some sort of argument, but I found none of these convincing. In particular, saying that the various contributions will cancel out begs the question. The proposer claims that for a "super-coin" the angular velocity will be small for a normal flipping action. True enough, but the angular momentum will not be small. Once you decide to ignore rotations, the analyses look much better. The following is from Ed Parks:

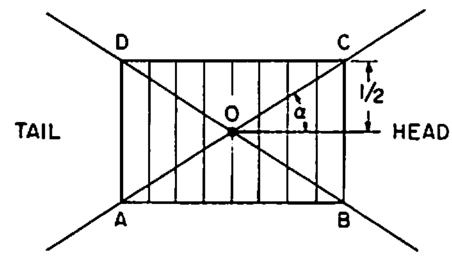
The chances of getting a head, tail, or edge depend on how the center of gravity of the "super-coin" is oriented over the sides when the coin hits the ground. The problem can be completely posed in two dimensions due to the rotational symmetry of the third dimension:



Here we are looking at the coin with the edges of the N pennies facing us. Let the diameter of a coin be one unit; then the dimensions of the "super-coin" are 1 x N/12.



When the coin hits the ground, it will tip towards the side which is crossed by a perpendicular to the ground, OF, through the center of gravity O. If we assume that all orientations of the coin are equally likely when it first touches the ground, then the probability of the coin landing on a given side is proportional to the angle formed by connecting the endpoints of the side to the center of gravity.



Thus:
$Pr(H) = (\text{Angle } BOC) / 360°$

$Pr(T) = Pr(H) = (\text{Angle } AOD) / 360°$

$Pr(E) = 2(\text{Angle } DOC) / 360° = 1 - 2 \cdot Pr(H)$

Let $\alpha = (\text{Angle } BOC) / 2$. Then $\tan\alpha = ½/(N/24) = 12N$.

The first problem is to find N such that $Pr(H) = Pr(T) = Pr(E) = ⅓$:

$Pr(H) = (\text{Angle } BOC) / 360° = \alpha / 180° = ⅓ \Rightarrow \alpha = 60°$

$\tan 60° = 12/N \Rightarrow N = 12/(\tan 60°) = 6.93$.

The nearest integer N is thus seven pennies.
The second problem is to find N such that $[Pr(H) + Pr(T)] = Pr(E) = ½$:

$Pr(H) = (\text{Angle } BOC) / 360° = \alpha / 180° = ¼ \Rightarrow \alpha = 45°$

$\tan 45° = 12/N \Rightarrow N = 12/(\tan 45°) = 12$ pennies.

Also solved by P. Michael Jung, Harry Zaremba, Frank Rubin, Winslow H. Hartford, Roy Sinclair, Eugene McManus, Mr. and Mrs. D. Szper, John E. Prussing, Erwin S. Strauss, William J. Butler, Jr., and R. Robinson Rowe.

MAY 4 Is there a proof for the proposition that if you reverse a number and add it to itself, and repeat that process, you will eventually get a palindrome?

It is fairly easy to reduce the problem to one of possible carries. The following demonstration is from Harry Zaremba: The process is basically palindromic in the first step. The palindrome is disguised only by a carry-over when the original number is added to its reversal, and each is written with its digits in the normal positional form of the decimal system.

If, for example, as shown below, a three-digit number with digits A, B, and C is written with digital position values and is added to its reversal with digits also written with their positional values, the sum of the two numbers is always a palindrome when carry-over is ignored:

100 A	10 B	C
100 C	10 B	A
100 (A + C)	10 (2B)	(A + C)

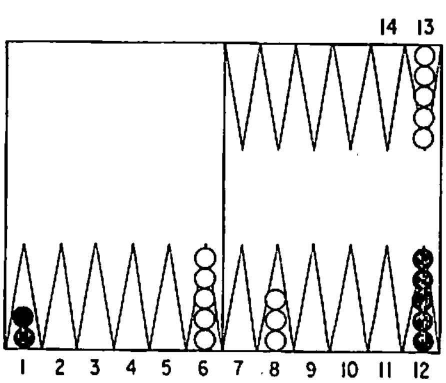
The palindrome will not be apparent until a point is reached in the process when no carry-over is existent.

But now, going beyond Harry Zaremba's analysis, comes the interesting part: Will we necessarily reach a point where there is no carry? On this there is no definitive word. Carl M. King sent me an HP67 program on a mag card, but — as he noted — the calculator is limited to ten digits. Alan Lemmon believes the problem is open and that 196 has never yielded a palindrome. Frank Rubin claims that 196 and nine other three-digit numbers will never yield a palindrome; he did not supply a proof (but claims one is known).

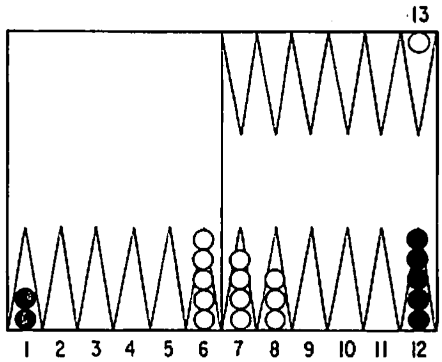
Also solved by Winslow H. Hartford, Erwin S. Strauss, and John E. Prussing.

MAY 5 Given the original backgammon board set-up, play any three initial dice of your choosing such as 6-6, 5-5, 6-1, etc., to lock up the two black pieces in the corner of your home table with a perfect prime. That is, the black pieces in the corner are locked in such that they cannot move even one space (or point) with the six openings in front blocked by closed points. (Assume the black corner pieces stay in one place.)

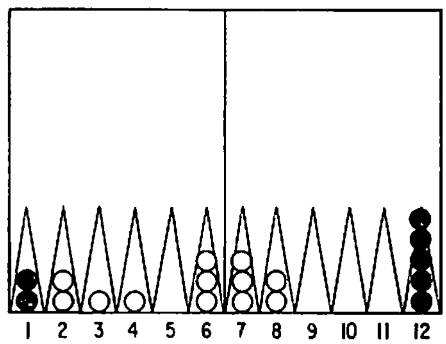
As I mentioned in May, I am not a backgammon expert by any means. I was therefore pleased to note that the solutions to this problem are not controversial. The following is from Bill Blake: This is the position at the start of the game:



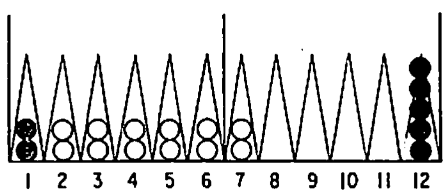
First roll 6-6. Move four men from space 13 to space 7. The result looks like this:



Second roll 4-4. Move two men from space 6 to space 2, one from space 7 to space 3, and one from space 8 to space 4. The result looks like this:



Last roll 3-3. Move two men from space 8 to space 5, one from space 6 to space 3, and one from space 7 to space 4. The result looks like this:



Also solved by Jeffrey M. Bowen, W. Everett, Jacob Bergmann, David Heller, Steve Altchuler, Stephen H. Gersuk, G. Raymond, Brown, and the proposer, Russ Nahigian.

Better Late Than Never

1975 JUN 1 Richard L. Hughes and Frank Rubin have responded.

1975 DEC 1 and 5 Frank Rubin has responded.

1976 DEC 2 Manuel R. F. Moreira, Edward Lunch, Benjamin Rouben, and Gerald Blum have responded.

DEC 3 Benjamin Rouben and Gerald Blum have responded.

1976 DEC 5 Harry Zaremba has submitted the following solution for n = 4: There are 15 different rounds of doubles matches with four matches in each round. Every player is paired only once with each other participant and opposes each player twice. The players are identified by numbers 1 to 9 and letters A to G:

Round	*Doubles matches*			
1	12:34	56:78	9A:BC	DE:FG
2	13:57	24:68	9B:DF	AC:EG
3	14:58	23:67	9C:DG	AB:EF
4	15:AE	26:9D	37:CG	48:BF
5	16:BG	25:CF	38:9E	47:AD
6	17:CE	28:BD	35:AG	46:9F
7	18:AF	27:9G	36:CD	45:BE
8	19:4C	2A:3B	5D:8G	6E:7F
9	1A:6D	29:5E	3C:8F	4B:7G
10	1B:7D	2C:8E	39:5F	4A:6G
11	1C:2B	3A:49	5G:6F	7E:8D
12	1D:3F	2E:4G	59:7B	6A:8C
13	1E:69	2D:5A	3G:8B	4F:7C
14	1F:2G	3D:4E	5B:6C	79:8A
15	1G:89	2F:7A	3E:6B	4D:5C

The attack for the solution was begun by arranging the pairs in the form of an upper triangular matrix of which the first-row elements consisted of player 1 matched consecutively with players 2 to G, the second row had player 2 matched with players 3 to G, the third row had player 3 matched with players 4 to G, etc. The pairs for each round were selected in a pattern symmetrical about the partial main diagonal upward to the right, with the last round having elements of the diagonal. The pair elements of each round then were arranged in columns in which the pairs were placed from top to bottom in a progressively increasing order of magnitude represented by the number or letter assigned to the player on the left of the pair symbol. The topmost elements of each column corresponded to the first row of the original matrix. From this point, the pairs in each column were formed into doubles matches. The matches in the first eight rounds were chosen in an orderly pattern without any difficulty; however, in subsequent rounds, the orderliness vanished, and the solution was completed by sheer illogical logic, mental muscle, and a bit of luck. It is my conjecture that solutions are possible for all n which are in the geometric progression 1, 2, 4, 8, ... Oddly enough, the solution for n = 3 by the same procedure outlined above proved futile. The pairs for each round of play can be established in several ways; however, the assembly of the doubles matches in the rounds to meet the problem requirements has eluded me. I hesitate to conjecture that it cannot be done.

Y 1976 Harry W. Hazard.

1977 JAN 2 Harry Garber, Jacob Bergmann, and George H. Ropes have responded.

JAN 3 Frank Rubin and Richard I. Hess.

JAN 4 Joseph E. Keilin points out that there is another solution with the swimmer heading obliquely upstream. I am not sure that this answers the question, "At what angle should he point himself?" Morton Hecht, Serge Loussarian, and Richard L. Schapker, Gerald Blum, Mario Diquilio, Henry Heiberg, and Richard Hess have found alternate methods of solution.

JAN 5 C. M. Delaney gets the same ratio as published but believes that t and d should be 1.5844×10^{-5} cm. and 4.826×10^{-4} cm., respectively. Gerald Blum and Richard Hess have also responded.

FEB 1 Ted Mita, Frank Rubin, and Gerald Blum have responded.

FEB 3 Bob Lutton and I hang our heads in shame. The following blast is from Leo G. Chouinard II:

Let me add a few more ashes to the sackcloth I hope you are already wearing as a result of the "solution" you published. In Nebraska, at least, the system

$$3x_1 = a^2 + b^2 + c^2 - 2d^2$$

$$3x_2 = a^2 + b^2 - 2c^2 + d^2$$

$$3x_3 = a^2 - 2b^2 + c^2 + d^2$$

$$3x_4 = -2a^2 + b^2 + c^2 + d^2$$

has a solution with x_1, x_2, x_3, and x_4 integers if each of the right sides is congruent to 0 (mod 3). But since $-2 \equiv 1$ (mod 3), each of the right sides reduces to

$$a^2 + b^2 + c^2 + d^2 \pmod 3.$$

Since $y^2 \equiv 1$ (mod 3) if 3 does not divide y, and $y \equiv 0$ (mod 3) if 3 divides y, the solutions to

$$a^2 + b^2 + c^2 + d^2 \pmod 3 \equiv \pmod 3$$

are all integer values of a, b, c, and d such that either all four are divisible by 3 or exactly one of the four is divisible by 3. As your solution (implicitly) noted, if we assume all integers used are positive, then the condition that the x's be distinct is just the assumption that a, b, c, and d are distinct, and if by symmetry we also assume $a < b < c < d$., i.e., $x_1 < x_2 < x_3 < x_4$, our requirement that the x_i's are positive is just $2d < a^2 + b^2 + c^2$. In particular, $a = 8$, $b = 9$, $c = 10$, $d = 11$ gives the solution $x_1 = 1$, $x_2 = 22$, $x_3 = 41$, $x_4 = 58$, which is 1/9 of your "smallest" solution. Better luck next time.

R. V. Mullikin, L. F. Howard, L. D. Woodruff, Frank Rubin, Gerald Blum, and Charles F. Rozier have also responded.

FEB 4 Ted Mita, Frank Rubin, and Gerald Blum have responded.

M/A 1 Jeffrey M. Bowen, Ted Mita, and Mr. and Mrs. D. Szper have responded.

M/A 2 James T. Aslanis, Jr., and John E. Prussing have responded.

M/A 3 Jonathan G. Bressel, Raymond Gaillard, James T. Aslanis, Jr., Mr. and Mrs. D. Szper, and Ted Mita have responded.

M/A 4 Ted Mita and Mr. and Mrs. D. Szper have responded.

M/A 5 Ted Mita, Mr. and Mrs. D. Szper, Jonathan G. Bressel, and Raymond Gaillard have responded.

NS 6 John and Karen Terrey, Charlie Bahne, Harry Zaremba, and Ted Mita have responded.

JUN SD1 Joseph Feil points out that the width should be 34 feet 8 inches.

PERM-2 Andy Egendorf notes that 99 = 4! /. (4!) − 4/4. (This use of . is all right; .4 = .4444 . . . is not.) So up to 100 we are missing only 73, 77, 87, and 93. The rules are given in March/April. Pooling the responses received from H. W. Hazard, Harry Zaremba, Robert Roth, George H. Ropes, Greg Schaffers, and Edward Lynch, I present the following extension up to 130. We use ** for exponentiation. Beyond 130 the density of solutions drops markedly.

$101 = \sqrt{4}/.4 + (4!)*4$

$102 = (4!)*4 + 4 + \sqrt{4}$

$103 =$

$104 = (4!)*4 + 4 + 4$

$105 = (44 - \sqrt{4})/.4$

$106 = (4! + \sqrt{4})*4 + \sqrt{4}$

$107 =$

$108 = (4! + \sqrt{4})*4 + 4$

$109 = (44 - .4)/.4$

$110 = (4! - \sqrt{4})*(\sqrt{4}/.4)$

$111 = 444/4$

$112 = 4!*4 + 4*4$

$113 =$

$114 = 44/.4 + 4$

$115 = (4!*\sqrt{4} - \sqrt{4})/.4$

$116 = (4!*\sqrt{4})/.4 - 4$

$117 =$

$118 = (4!*\sqrt{4})/.4 - \sqrt{4}$

$119 = (4!*\sqrt{4} - .4)/.4$

$120 = (4!/.4)*4/\sqrt{4}$

$121 = (4!*\sqrt{4} + .4)/.4$

$122 = (4!*\sqrt{4})/.4 + \sqrt{4}$

$123 = \sqrt{\sqrt{\sqrt{(\sqrt{4}/.4)**(4!)}}} - \sqrt{4}$

$124 = (4!*\sqrt{4})/.4 + 4$

$125 = (4!*\sqrt{4} + \sqrt{4})/.4$

$126 = (4!/.4)/.4 - 4!$

$127 = (4**4 - \sqrt{4})/\sqrt{4}$

$128 = 4*4*4*\sqrt{4}$

$129 = (4**4 + \sqrt{4})/\sqrt{4}$

$130 = (4!*\sqrt{4} + 4)/.4$

Proposers' Solutions to Speed Problems

O/N SD1 Strengths

O/N SD2 Since the sum of the squares of the distances to two diagonally opposite corners is the same as the sum of the squares of the distances to the other two corners, then if x is the unknown distance,

$x^2 + 37^2 = 17^2 + 33^{22}$; and $x = 3$.