How High Is Your Scrabble Score?

Puzzle Corner
by
Allan J. Gottlieb

I have received several comments on possible sexism in this column, all centered on the use of the term, "old maid," which is apparently insulting to women. As you may recall, this was the term on which Ms. Fester originally commented (*see February, p. 67*). My only defense is that "old maid" is not part of my vocabulary, and my first thought when I hear it is of the children's card game. But this excuse is obviously inadequate, since many other people have different impressions and find the term offensive. It will not appear again.

The backlog of regular problems remains in excess of a year and a half. Backlogs of chess, bridge, and speed problems are much less.

Problems

JUN 1 We begin this month with a bridge problem from Russell A. Nahigian:

North:
♠ Q J 10 9 4
♥ Q 5
♦ A J 10
♣ J 10 3

West:
♠ K 7 6 3 2
♥ 6 3
♦ 3
♣ Q 9 7 6 2

East:
♠ 8 5
♥ K J 9 7
♦ Q 9 8 7
♣ 8 5 4

South:
♠ A
♥ A 10 8 4 2
♦ K 6 5 4 2
♣ A K

You are south, the declarer, at a contract of three no-trump. West opens with ♣ 6. How do you play the hand?

JUN 2 James N. Cawse and Zoltan Mester are avid Scrabble fans, and they wanted to know the maximum possible score in a legal match. This seems too hard to me, so I'll settle for the maximum possible score for one player on one turn. Of course, any solution to the original Cawse-Mester problem would also be appreciated.

JUN 3 Joseph Haubrich knows the standard proof that the base angles of an isosceles triangle are equal. One of his geometry books claims this can be done without constructing an angle bisector. Can anyone help him find this proof?

JUN 4 A computer (really, calculator) problem from B. W. Letourneau, who explains that the basis is an early Monroe electronic calculator design, with a four-register stack plus one storage register. In addition to numerical entries and clear and print instructions, there are nine operating "keys" with the following functions:

From this initial condition of the registers:

M D C B A

The "add" instruction yields:

M O D C A + B

The "subtract" instruction yields:

M O D C B − A

The "multiply" instruction yields:

M O D C A · B

The "divide" instruction yields:

M O D C B/A

The "square root" instruction yields:

M D C B $\sqrt{A}$

The "repeat" (or "enter") instruction yields:

M C B A A

The "interchange" instruction yields:

M D C A B

The "recall" instruction yields:

M C B A M

The "store" instruction yields:

M D C B A

These operations are very similar to those of the HP-35 calculator, except that arithmetic operations do not reduplicate D in the top register and there is no "roll" instruction. The problem can just as well be worked on paper without a calculator; it is:

Starting from an initial condition of a "1" in the first register:

0 0 0 0 1,

it is possible to generate any rational number using the above nine operations. The problem is to generate a given number in the minimum number of operations; my favorite is the number 355/113, a close approximation to pi.

JUN 5 Our last problem, from Emmet J. Duffy, could be considered number-theoretic, but I prefer to think of it as auto-mechanic-theoretic:

Six width gauges are permanently mounted on a ring. They can measure any width from 0.001" to 0.031" in 0.001" steps by using individual gauges or by sliding two, three, four, five, or six consecutive gauges together. Find five ways to do this, giving the widths of the gauges and their order on the ring.

Speed Department

JUN SD1 A suburban speed problem to help people mow, posed by none other than R. Robinson Rowe:

Mowing my rectangular lawn, I cut 13 inches on each pass. After two circuits I had cut 16 per cent and after two more circuits 31 per cent of my lawn. How many circuits before I was done?

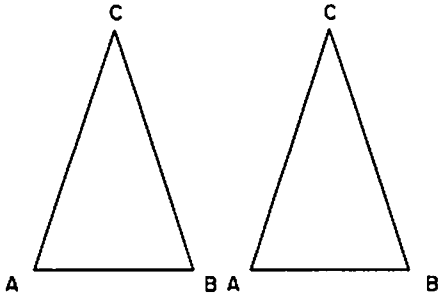
JUN SD 2 A very interesting economic challenge from Hal Varian: Consider a sealed-bid auction. Each person has a "true value" for the good being auctioned off. But depending on each person's beliefs about what other people's valuations might be, the actual bids of each person will not necessarily be equal to his true value. This auction system does not result in true revelation of preference. The problem is to derive a simple variant of this scheme so that it never pays to lie about your true bid. You may assume no collusion.

Solutions

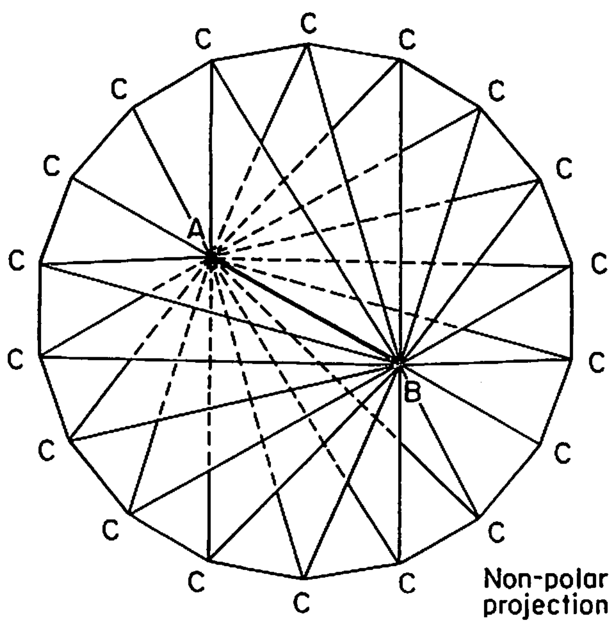
This issue will clear up three of the "Never Solved" problems.

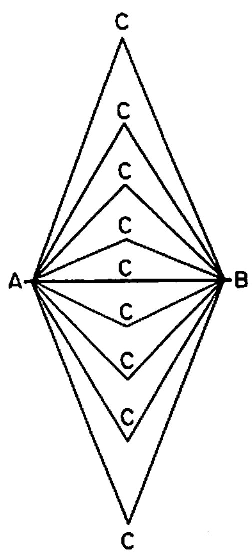
Judith Q. Longyear comments that many of her topological friends have been going crazy over NS4. So here is the answer: the singly infinite torus is homeomorphic to the infinite jailcell, whereas the doubly infinite torus is not. For details, see the problem section after Chapter 1 of Spivak's *Differential Geometry*, Vol. 1.

NS 5 If a pair of triangles is not co-polar, the joins of corresponding vertices form a triangle and so do the intersections of corresponding sides. The original pair of triangles has been transformed into a second pair which can be transformed into a third and so on. How does the sequence of pairs of triangles behave?



This one has had *me* baffled since it first appeared. I really do not follow the terminology at all, so I present the following, from Benjamin Gray, without claiming to understand it:
The joins of corresponding vertices form a triangle, and so do the intersections of corresponding sides. The joins are to connect by line or lines, to be contiguous or in contact, and to unite or come in contact.
1. Vertice of angle A in contact with angle A; vertice of angle B in contact with angle B; vertice of angle C connected by line to angle C.
2. Intersections of corresponding sides: CB intersects CB, CA intersects CA.
3. Triangles formed by joins and intersections are CCA and CCB.
4. Angles C following the points of the compass are not copolar.
5. The result is shown in projection and end view:





NS6 This is the one about the cruise to the Lesser Antilles, with a diagram to be completed. The crew of five were the skipper, first mate Joseph, navigator Peter, deck hand Moses, and cook Able. They all voted for Eisenhower. The total miles shown on the taffrail log was twice the number for the first nine days plus exactly 200 miles. However, we had the log carefully checked and found that for each mile registered we had sailed 6,120 feet, so the distance sailed was slightly greater than that shown on the log. As to the crew, it so happened that if Peter had been 14 years older the skipper would have been twice the average age of his crew. Also if the skipper had been 13 years older his age would have equaled the sum of the ages of the three youngest members of the crew. The dimensions of the boat, sail area, and ages of crew can now be easily ascertained by completing the diagram and using the following clues.

Across
1 Yards sailed in nine days
5 Age of first mate
6 Twice the age of Joseph
7 Miles logged in nine days minus 1 down
11 Square of 4 down minus 2 down
13 Total miles logged
15 Age of Moses
16 Length overall
17 1 down reversed

Down
1 Cube of beam in yards or square of draft in feet
2 Miles logged in nine days
3 Area of mizzen times beam
4 Two times 5 across
8 Length overall times draft
9 Area of mainsail or twice area of mizzen plus sum of digits of 11 across
10 Area of mainsail plus length overall
12 Low water length plus length overall plus draft plus beam
14 Age of Able

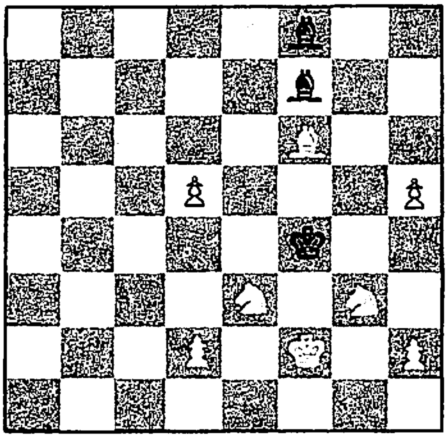
Another clue: the problem was concocted some years ago.
This story has a happier ending: many solutions have arrived, as predicted in February. William B. Blake supplied the following derivation:

The solution can be found after solving step one down. The cube of the beam must be the square of the draft, and this must be a two-digit number; 64 is the only number under 100 and greater than 10 which is both cube and square. Step one across can then be found by checking for three-digit numbers which when multiplied by 2,040 (the number of yards in 6,120 feet) gives a six-digit number which begins with 6, and whose second digit is the first digit of the three-digit number. The only numbers which satisfy these conditions are 312 and 636,480. Step six across is then found by multiplying step five by 2. Step seven across is then found by subtracting steps two down and one down. Steps thirteen across, eleven across, and eight down can be found by multiplication and subtraction of previous steps. Then, all of the rest of the steps, except the age of Pete can be found easily by substitution of previously-known facts. The age of Pete can then be found by remembering that the voting age in 1956 (they all voted for Eisenhower) was 21; both 18 and 28 fit the age equations, but 28 is the only age high enough to vote. So Pete must have been 28. Thus, the age of the skipper is (26 + 27 + 28 − 13) or 68.68 also satisfies the first age equation: [(26 + 27 + 28 + 41) + 14]/2, which equals 68, so the solution checks out. Summarizing, then, here are the required numbers:
Ages: Skipper — 68 years; Joseph — 41 years; Peter — 26 years; Moses — 27 years; and Able — 28 years.
Dimensions: length — 58 feet; draft — 8 feet; beam — 12 feet; area of mainsail — 827 square feet; area of mizzensail — 407 square feet.
Distances: total miles logged — 824; miles logged in nine days — 312; yards sailed in nine days — 636,480.

1 6	2 3	6	3 4	4 8	0
5 4	1	■	6 8	2	■
■	7 2	8 4	8	■	9 8
10 8	■	11 6	4	12 1	2
13 8	14 2	4	■	15 2	7
16 5	8	■	17 4	6	■

Also solved by William A. Bundy, B. Dennis Sustare, Robert Hess, Jim Kempner, Eugene G. Kovach, William J. Butler, Jr., Mr. and Mrs. D. Szper, Edwin Nordstrom, R. Robinson Rowe, Michael Jung, Robert Lutton, Ronald Ort, Oljan Repic, and John F. Chandler.

FEB 1 White to mate in two:



This problem is far less eccentric than previous chess offerings. All solutions were identical: White's first move is P — Q3. If Black replies B x RP, White mates by N x B. Any other Black move is followed by N — K2 (mate).

Solutions received from Howard Ostar, John Schuster, Anthony Coppola, Linsay Faunt, James L. Larsen, Robert Hisiger, Avi Ornstein, Dan Albert, Jim Kempner, William J. Butler, Jr., Ronald Ort, Donald Barnhouse, David M. Johnson, Bruce Stangle, John F. Chandler, Benjamin Rouben, Elliott Roberts, Peter Groot, and Oleg J. Devorn.

FEB 2 The 11 football players of Croam play five-on-a-side matches against each other, the 11th player being referee. One match is played with each possible choice of teams and referee. Can you schedule the 1,386 matches so that each individual team plays its six matches on six different days of the week (Sundays being off limits to football on Croam)?

There were no takers. R. Robinson Rowe thinks it unlikely that such a schedule exists.

FEB 3 Solve:

$$x_1 + x_2 + x_3 \qquad = a^2$$
$$x_1 + x_2 \qquad + x_4 = b^2$$
$$x_1 \qquad + x_3 + x_4 = c^2$$
$$\qquad x_2 + x_3 + x_4 = d^2$$

where each symbol is an *integer* and the four x's are unique.

The following was submitted by Robert Lutton:
By applying Cramer's Rule or by just suitably combining the equations, the system of equations can be transformed to:

$$3x_1 = a^2 + b^2 + c^2 - 2d^2$$
$$3x_2 = a^2 + b^2 - 2c^2 + d^2$$
$$3x_3 = a^2 - 2b^2 + c^2 + d^2$$
$$3x_4 = -2a^2 + b^2 + c^2 + d^2$$

The only way that each of these equations can apply for integer values of all symbols is for a^2, b^2, c^2, and d^2 each to be divisible by 3. However, trying it for a = 3, b = 6, c = 9, and d = 12 gives a negative value for x_1, which was disallowed. With a few trials it can be found that with a = 24, b = 27, c = 30, and d = 33 you get $x_1 = 9$, $x_2 = 198$, $x_3 = 369$, and $x_4 = 522$. This is just the set of smallest values; there are an infinite number of solutions. For instance, a = 300, b = 303, c = 306, and d = 309 gives $x_1 = 28161$, $x_2 = 30006$, $x_3 = 31833$, and $x_4 = 33642$.

Also solved by Peter Groot, John F. Chandler, Mr. and Mrs. D. Szper, Naomi Markovitz, William J. Butler, Jr., Avi Ornstein, Harry Zaremba, Winslow H. Hartford, Emmet J. Duffey, W. Allen Smith, Frank Carbin, John E. Prussing, Harvey M. Elentuck, Timothy Maloney, R. Robinson Rowe, and the proposer.

FEB 4 For all N > 0, find N positive integers (not necessarily distinct) whose sum and product are equal.

This one was so easy even I solved it: For N = 1, use 1 = 1. For N > 1, use

$$\underbrace{1 + 1 + \ldots + 1}_{N-2} + 2 + N =$$
$$\underbrace{1 \cdot 1 \cdot \ldots \cdot 1}_{N-2} \cdot 2 \cdot N.$$

The proposer, Mickey Haney, found an additional solution for odd N > 1:

$$\underbrace{1 + \ldots + 1}_{N-2} + 3 + (N+1)/2 =$$
$$\underbrace{1 \cdot 1 \cdot \ldots \cdot 1}_{N-2} \cdot 3 \cdot (N+1)/2.$$

Also solved by R. Robinson Rowe, Ronald Ort, Benjamin Rouben, M. Kaufman, Naomi Markovitz, George Flynn, Harvey M. Elentuck, John E. Prussing, W. Allen Smith, Winslow H. Hartford, Harry Zaremba, Avi Ornstein, William J. Butler, Jr., Mr. and Mrs. D. Szper, John F. Chandler, and Peter Groot.

FEB 5 Construct a 4 × 4 × 4 magic cube (equal sums along horizontals, verticals, in-outs, and diagonals) consisting of 64 distinct numbers.

Only the proposer, Robert Mills, could solve this problem; and his answer is that such a magic cube is impossible:
Use an additive constant so that the sum for each row, column, etc., is zero.
Using Cartesian coordinates to label elements, we get

$$0 = 0 - 0 - 0 + 0 + 0 =$$

(111+212+313+414)
−(111+222+333+444)

−(112+212+312+412)
+(112+222+332+442)

−(113+213+313+413)
+(113+223+333+443)

+(114+213+312+411)
−(114+223+332+441)

+(411+412+413+414)
−(441+442+443+444)

= 2 (411+414−441−444)

So (411+414−441−444) = 0.

Similarly,

(411−414+441−444) = 0,

so that 411 = 444, contrary to the requirement.

Proposer's Solutions to Speed Problems

JUN SD1 In modules of 26 inches, the uncut area can be expressed as

$$A = x(x + c),$$

where c is the constant excess of length over width. This is a quadratic, for which the second difference will be constant. Since the first two first differences were proportional to 16 and 15, the next six will be proportional to 14, 13, 12, 11, 10, and 9, for a total of 100 per cent. At two circuits each, that meant 12 more circuits. (It wasn't asked, but if you must know, the lawn is 54′2″ by 17′4″.)

JUN SD2 The good is sold to the highest bidder, but he has only to pay the second-highest bid. In this auction, it never pays to lie about your true bid! First, you would never want to bid less than your true bid, since increasing your bid increases your chances of getting the good without affecting what you have to pay. Consider what can happen if you bid *more* than your true value. Then the second-highest bid could be more than your true bid — in which case you have to pay more than the item is worth. On the other hand, the second highest bid could be less than your true bid — but then you would have gotten the good by bidding your true bid.

Allan J. Gottlieb, who is Coordinator of Computer Activities and Assistant Professor of Mathematics at York College of the City University of New York, studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973). Send problems, solutions, and comments to him at York College, 150-14 Jamaica Avenue, Jamaica, N.Y., 11451.

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Letters

Continued from p. 2

and public transportation most effective, are the two modes comparable in passenger miles. Space heating in Sweden is far different than in Germany because all rooms in Sweden are heated and living space per capita is nearly as great as in the U.S. Differences in use per unit of climate arise from significantly greater levels of insulation and weatherization in Sweden compared to the U.S., and not from the number of apartments in the stock or the number of rooms that are heated. We note in our study that economic factors — the price of energy — play a big role in stimulating Swedes to use energy more efficiently than Americans do for similar activities. The economic stimulus towards energy efficiency probably outweighs "fundamental differences in attitudes" you mention, though it is accurate to say that energy use in Sweden appears to approach its own optimal level of use, measured economically, to a greater extent than in the U.S. We also note that you described energy use in these countries in terms of megawatts per $1,000 of GNP, when the correct units were megawatt-hours per $1,000.

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Dr. Schipper is an energy specialist for the Energy and Resources Program of the University of California at Berkeley. Dr. Lichtenberg is Chairman of the program's Energy and Resources Group. — Ed.