How to Break a Rock to Make a Balance

Puzzle Corner
by
Allan J. Gottlieb

This month we are presenting answers to seven problems. In addition to the five December puzzles, we have (the revised) J/A1 and PERM 2. Thus the solution section of this column is long, and I will limit my introductory remarks to noting that Dr. Leo Epstein has been working on an outgrowth of 1974 M/A 2, finding a closed form for

$$S(n) = \sum_{x=1}^{n} x^x.$$

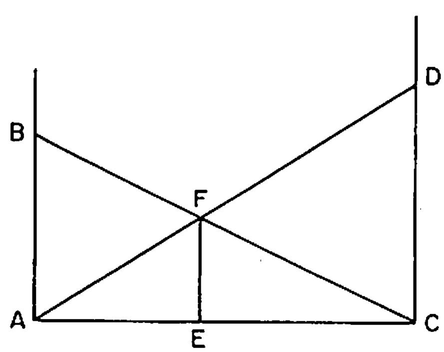
Anyone wishing details should write to him at Argonne National Laboratory, 9700 South Cass Ave., Argonne, Ill., 60439.

Problems

M/A1 We begin this month with a bridge problem from William J. Butler, Jr.: Rodney Yarborough, world's unluckiest bridge player, has been playing bridge for a number of years. During this period he has never received a hand worth even a single point. (Conventional point counting gives four points for each ace, three per king, two per queen, and one per jack. Also, void suits count three, singleton suits two, and doubleton suits one.) Rodney calculates that he has seen more than 1 per cent of the total number of these terrible hands. What is the minimum number of hands that Rodney has seen?

M/A2 Our next problem, related to 1975 M/A 3, is from William Fitch Cheney and Norman M. Wickstrand:

Given AB, CD, and EF perpendicular to AC, find any set of integers x, y, m, w, and h such that $x = \overline{CD}$, $y = \overline{AB}$, $m = \overline{AD}$, $n = \overline{BC}$, $w = \overline{AC}$, and $h = \overline{ET}$.



M/A3 A friendly number-theory problem from Chip Lawrence:

A rural storekeeper in Georgia has a set of balance scales and a rock weighing 40 pounds. A seller from the city is passing through and he luckly has a set of conventional scales. Seizing upon this opportunity, the storekeeper desires to break his rock up so that he can weigh any exact poundage between one and 40 pounds. The city seller, however, plans to charge outrageous rates for the use of his modern scales. What is the minimum number of pieces into which the storekeeper can break his rock and still accomplish his purpose? How much would each weigh? (Rocks may be placed on either or both trays of the balance scales.)

M/A 4 Jack Parsons wants you to find the fourth term for each of the following (related) sequences:

(a) 1, 20, 190
(b) 1, 21, 210
(c) 1, 22, 231

M/A 5 We end with an interesting geometry problem from Richard Brady: Prove that the sum of the distance from any point in or on an equilateral triangle to the three sides of the triangle is constant.

Speed Department

M/A SD1 Continuing in his effort to ease our conversion to metric, R. Robinson Rowe offers the following:

The very up-to-date weathercaster on a local TV station reports both Fahrenheit and Celsius temperatures. Recently, for a nearby city, his two figures for the low temperature that morning were in the ratio of 5 to 1, and, by a coincidence, so were his figures for the high temperature that afternoon. Quick, what was the daily range?

M/A SD 2 An interesting paradox from Sam Gutmann:

Jack Suburb went to the bank to get a mortgage on a new house which cost \$20,000; Jack wanted to pay in 30 yearly installments, at 8 per cent interest. Let x be the yearly payment. Then the total he pays is 30x. The total he owes is \$20,000 plus interest. The interest for the first year is (.08) (30x), for the next year (.08) (29x), for the next (.08) (28x), etc., since in each case those are the amounts he still owes the bank. So, 30x = 20,000 + (.08) (30x) + (.08) (29x) + . . . (.08) (x). Solving, x < 0. Does the bank owe Jack money??

Solutions

PERM 2 Construct as many integers as possible using four 4s; for example, $14 = \sqrt{4} + 4 + 4 + 4$. The greatest integer function is not allowed.

As usual with this type of problem, it is not clear what is legal. As originally stated, the greatest integer function is out. I am also eliminating transcendental functions (T, antilog, etc.) except for $\sqrt{\ }$ ($\sqrt[3]{\ }$, etc., are out). Factorials are OK, but combinations and permutations are not. Finally, .4 = .44444 . . . is also illegal. I realize that these rules are arbitrary, especially the ones concerning $\sqrt{\ }$ and !. I have pooled everyone's results and here is the list up to 100. The solutions for 1 to 30 appeared in July/August, 1976, as the solution to 1976 M/A5. Numbers over 100 will appear in subsequent issues, so there is still time to contribute. How about 73? Impossible?

31. (4 + 4!)/4 + 4!
32. $4^4/(4 \times 4)$
33. $(\sqrt{4}/.4) + 4! + 4$
34. (4! × .4) + .4 + 4!
35. 4! + 44/4
36. 4! + 4 + 4 + 4
37. [(4! + .4)/.4] − 4!
38. (4/.4) + 4! + 4
39. $44 - (\sqrt{4}/.4)$
40. 4! + 4! − 4 − 4
41. $[(\sqrt{4} + 4!)/.4] - 4!$
42. 4! + 4! − (4!/4)
43. 44 − 4/4
44. 44 + 4 − 4
45. 44 + 4/4
46. $44 + 4 - \sqrt{4}$
47. 4! + 4! − 4/4
48. 4! + 4! + 4 − 4
49. 4! + 4! + 4/4
50. 44 + (4!/4)
51. $[(4! - \sqrt{4})/.4] - 4$
52. (4! × 4) − 44
53. $4! + 4! + (\sqrt{4}/.4)$
54. (4!/.4) − (4!/4)
55. $(44/\sqrt{4})/.4$
56. 4! + 4! + 4 + 4
57. $[(4! - \sqrt{4})/.4] + \sqrt{4}$
58. (4! + 4!) + 4/.4
59. (4!/.4) − 4/4
60. 44 + (4 × 4)
61. (4!/.4) + 4/4
62. $(4!/.4) + 4 - \sqrt{4}$
63. $(4^4 - 4)/4$
64. 4! + 44 − 4
65. $(4^4 + 4)/4$
66. (4!/.4) + (4!/4)
67. $[(\sqrt{4} + 4!)/.4] + \sqrt{4}$
68. $4^4/4 + 4$
69. $[\sqrt{4} + 4!)/.4] + 4$
70. (4!/.4) + (4/.4)
71. (4! + 4.4)/.4
72. 44 + 4 + 4!
73.
74. $(4! + 4!) + (4! + \sqrt{4})$
75. $(4! + 4 + \sqrt{4})/.4$
76. (4!/.4) + (4 × 4)
77.
78. $4(4! - 4) - \sqrt{4}$
79. $[(4! - \sqrt{4})/.4] + 4!$
80. (44 − 4!) × 4
81. $(\sqrt{4} + 4/4)^4$
82. $4(4! - 4) + \sqrt{4}$
83. [(4! − .4)/.4] + 4!
84. 4 × (4! − 4) + 4
85. [(4! + .4)/.4] + 4!
86. (4 × 4!) − (4/.4)
87.
88. 44 + 44
89. $[(\sqrt{4} + 4!)/.4) + 4!$
90. (4 × 4!) − (4!/4)
91. $(4 \times 4!) - (\sqrt{4}/.4)$
92. 4! + 4! + 44
93.
94. $(4 \times 4!) - (4/\sqrt{4})$
95. (4 × 4!) − 4/4
96. (4 × 4 × 4!)/4
97. (4 × 4!) + 4/4
98. $(4 \times 4!) + (4/\sqrt{4})$
99.
100. 4 × (4! + 4/4)

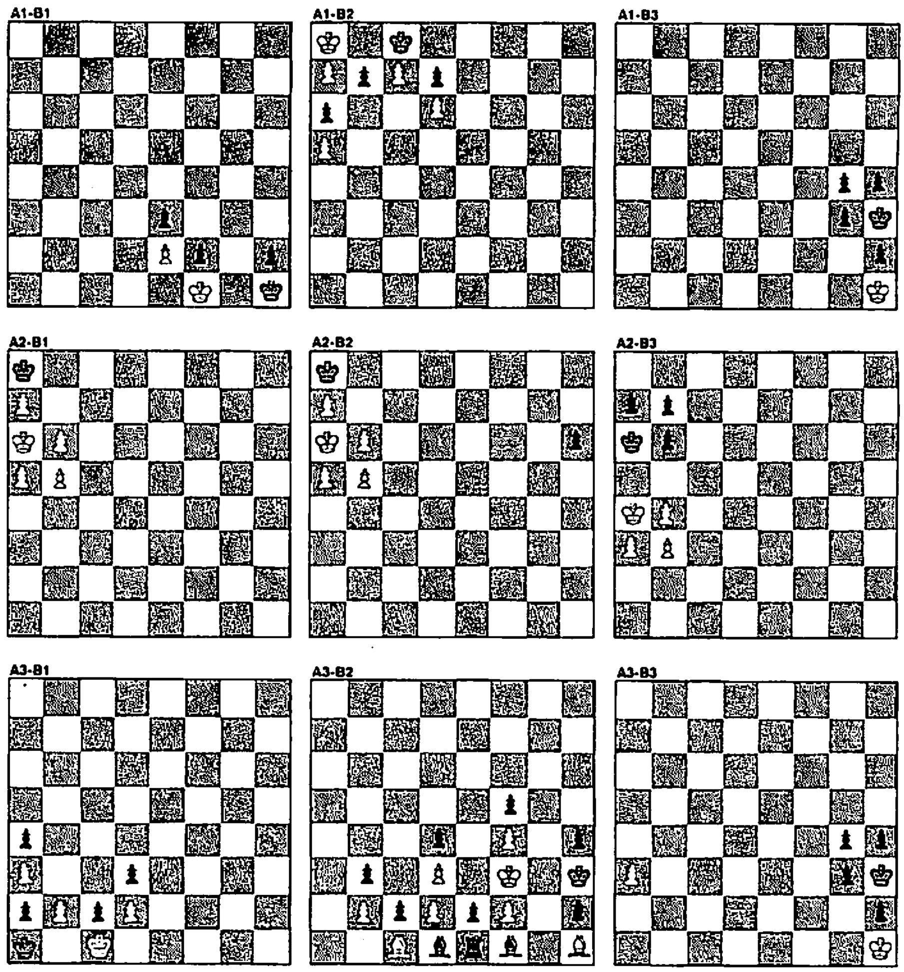
Solutions came from Harry Zaremba,

Sam Jacobs, William J. Butler, Jr., Thomas Jenkins, S. D. Turner (or ∫ dt), Morrie Vasser, George H. Ropes, and Robert Roth.

1976 J/A 1 What is the minimum number of pieces required for a position in which
A. If White is to move,
1. The situation is a stalemate;
2. White must win;
3. Black must win. Or
B. If Black is to move,
1. The situation is a stalemate;
2. White must win;
3. Black must win.

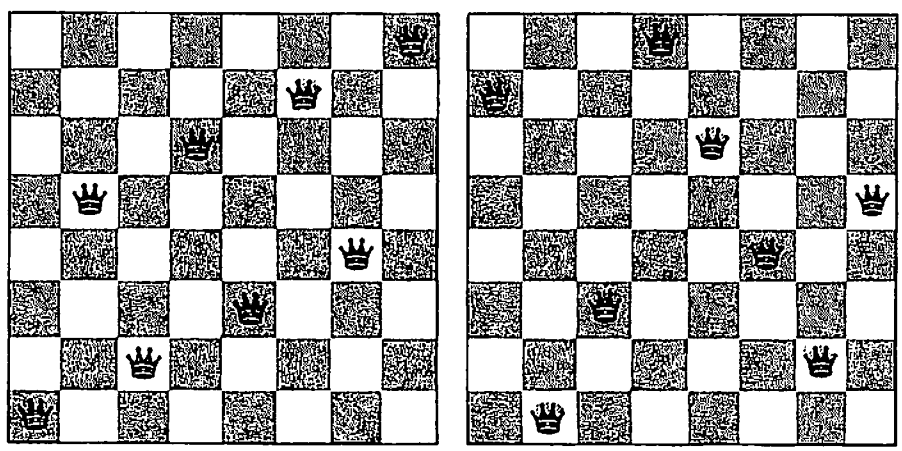
The three requirements when White is to move combine with the requirements when Black is to move to create nine sub-problems. Three comments are in order: pieces include pawns; the sub-problem A = 2, B = 3, for example, requires that when White moves first he wins for any sequence of legal moves, and similarly for Black; and (as clarified in December) you may *not* assume logical play (so, for the sub-problem A = 2, B = 3, for example, you must find a position satisfying: 1) If White moves first, *any* sequence of legal moves leads to a White victory (this is A = 2); and 2) If Black moves first, *any* sequence of legal moves leads to a black victory (this is B = 3). Similar remarks hold for the other eight cases.)

I offer the conglomeration of solutions shown at the right, above, sent by Harry Nelson, Steve Grant, and the proposer, Bill Saidell. (A3 — B2 is from Mr. Nelson — what a thought!) Responses were also received from Eric Jamin, Anthony Coppola, William J. Butler, Jr., and Joseph A. Haubrich.



DEC 1 How should eight queens be placed on the chess board so that the total number of moves is maximum?

Only William J. Butler and R. Robinson Rowe responded to the challenge. I'm quite sure that Frank Rubin, the proposer, will let us know if the solutions are non-optimal; he implied that they are (chess diagrams at right, bottom).



DEC 2 An interesting series is the "paired" series:
(1,1), (2,3), (5,7), (12,17), (29,41), (70,99), . . .
a. What is the next pair?

b. What is the "rule" for constructing the series?
c. Show that the pairs give solutions to the equation:

$$2n_1^2 \pm 1 = n_2^2$$

and that the plus and minus signs alternate. The limit is, of course, $\sqrt{2}$ for the ratio of the pairs.

In his first contribution to Puzzle Corner, Douglas Szper has given us an extremely thorough presentation. Not content with recursion formulas, he derives closed forms using one of the cornerstones of numerical analysis, Newton's Forward Difference Formula. Finally a generalization is given. The advantage of a closed form can be appreciated when one considers that W. C. Johnson's HP 55 needed over ten minutes to calculate the 261st pair using the recursion formulas.

We are given the paired series: (1,1), (2,3), (5,7), (12,17), (29,41), (70,99). The next pair is (169, 239). Let $\{s_i\}$ denote the series {1, 2, 5, 12, 29 . . .} of first components and let $\{t_i\}$ be the series of second components, {1, 3, 7, 17, . . .}. There are many interesting relationships here. The pair (s_n, t_n) can most easily be generated recursively by:

$$s_n = s_{n-1} + t_{n-1}, (s_1, t_1) = (1, 1) \qquad (1)$$

and

$$t_n = s_n + s_{n-1} \qquad (2)$$

Equation (1) can be written as a difference formula:

$$\Delta s_n = t_n, \qquad (3)$$

which states that $\{t_n\}$ is the series of first differences of the series $\{s_n\}$. Examining the difference table of $\{s_n\}$, further relationships shown in this table become ap-

s	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7
1							
	1						
2		2					
	3		2				
5		4		4			
	7		6		4		
12		10		8		8	
	17		14		12		8
29		24		20		16	
	41		34		28		24
70		58		48		40	
	99		82		68		
169		140		116			
	239		198				
408		338					
	577						
985							

parent. The sequence of $\Delta^2 s_n$ is equal to twice the sequence s_n, and similarly for t_n:

$$2s_n = \Delta^2 s_n \qquad (4)$$

$$2t_n = \Delta^2 t_n = \Delta^3 s_n \qquad (5)$$

From (4) we obtain:

$$2s_n = s_{n+2} - 2s_{n+1} + s_n, \text{ or}$$

$$s_{n+2} = 2s_{n+1} + s_n, s_1 = 1, s_2 = 2 \qquad (6)$$

This is a recursive formula for $\{s_n\}$, and we have a parallel formula for $\{t_n\}$:

$$t_{n+2} = 2t_{n+1} + t_n, t_1 = 1, t_2 = 3 \qquad (7)$$

To develop a "rule" for $\{s_n\}$, we examine the forward differences and note that they are the powers of 2, each appearing twice. Using Newton's Forward Difference Formula, we have:

$$\begin{aligned} s_n &= s_1 + [(n-1)/1!]\,\Delta s_1 \\ &\quad + [(n-1)(n-2)/2!]\,\Delta^2 s_1 \\ &\quad + [(n-1)(n-2)(n-3)/3!]\,\Delta^3 s_1 \\ &\quad + \ldots \\ &= ([1 + (n-1)]/1!)2^0 \\ &\quad + ([3 + (n-3)]/3!)2^1(n-1) \\ &\quad (n-2) + \ldots \\ &= \binom{n}{1} 2^0 + \binom{n}{3} 2^1 + \binom{n}{5} 2^2 + \ldots \end{aligned}$$

Thus we can express the even terms of $\{s_n\}$ by a finite sum:

$$s_{2m} = \sum_{i=1}^{m} \binom{2m}{2i-1} 2^{i-1} \qquad (8)$$

and the odd terms by

$$s_{2m-1} = \sum_{i=1}^{m} \binom{2m-1}{2i-1} 2^{i-1} \qquad (9)$$

In general, let $[\![(n+1)/2]\!]$ denote the greatest integer in $(n+1)/2$; then:

$$s_n = \sum_{i=1}^{[\![(n+1)/2]\!]} \binom{n}{2i-1} 2^{i-1} \qquad (10)$$

A summation formula for $\{t_n\}$ could be derived similarly, but a slightly different approach is also interesting. We may use formulas (6) and (7) to extend each sequence back to n = 0. We find that $S_0 = 0$, $t_0 = 1$. Thus we may expand the difference table (illustrated for $\{t_n\}$) as follows:

1					
	0				
1		2			
	2		0		
3		2		4	.
	4		4		
7		6			
	10				
17					

Note that now the 0th advancing differences are powers of 2 and 0, alternating. Thus we have:

$$t_n = \binom{n}{0} 1 + \binom{n}{1} 0 + \binom{n}{2} 2 + \binom{n}{3} 0 + \ldots,$$

where the sum is finite and ends with the term whose coefficient is $\binom{n}{n}$. Thus we may write

$$t_n = \sum_{i=1}^{[\![n/2+1]\!]} \binom{n}{2i-2} 2^{i-1}$$

$$= \sum_{i=1}^{[\![n/2]\!]} \binom{n}{2i} 2^{i} \qquad (11)$$

Using the first recursion relation in equations (1) and 2), if the one pair of the sequence (x,y) satisfies

$$2x^2 - 1 = y^2$$

then the next pair is (x + y, 2x + y) and we have:

$$\begin{aligned} 2(x+y)^2 + 1 &= 2x^2 + 4xy + 2y^2 + 1 \\ &= 2x^2 + 4xy + (2x^2 - 1) \\ &\quad + y^2 + 1 \\ &= 4x^2 + 4xy + y^2 \\ &= (2x + y)^2 \end{aligned}$$

Thus, if any pair satisfies $2s^2 - 1 = t^2$, the next pair satisfies $2s^2 + 1 = t^2$. Likewise, if a pair (x,y) satisfies $2s^2 + 1 = t^2$, then:

$$\begin{aligned} 2(x+y)^2 - 1 &= 2x^2 + 4xy + 2y^2 - 1 \\ &= 2x^2 + 4xy + (2x^2 + 1) \\ &\quad + y^2 - 1 \\ &= 4x^2 + 4xy + y^2 \\ &= (2x + y)^2, \end{aligned}$$

so the next pair satisfies $2s^2 - 1 = t^2$. Therefore, since $2(1^2) - 1 = 1$, the pair (1,1) satisfies the equation

$$2s^2 \pm 1 = t^2 \qquad (12)$$

with the "−" sign, and each pair of the sequence satisfies (12) with alternating signs.

Also solved by Harry Zantopulos, Barry R. Davis, Avi Ornstein, R. Robinson Rowe, Frank Carbin, Doug Hoylman, Emmet J. Duffey, William J. Butler, Jr., Jim Ertner, John F. Chandler, Bruce Fleischer, John E. Prussing, Mary Lindenberg, Row Moore, Harry Zaremba, Stephen F. Wilder, and the proposer, Winslow H. Hartford.

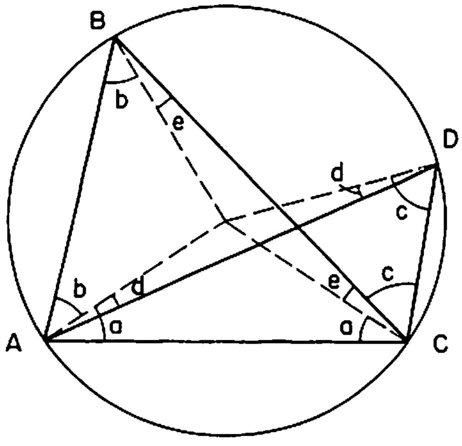
DEC 3 The horopter is the set of points in space which, in traveling from one to another, subtend equal retinal angles. The horopter in a horizontal plane is a circle which passes through the centers of the lenses. In order to show this the necessary theorem was: Given a circle with a chord drawn, the vertical angle of any triangle constructed on that chord will be equal. Phrased that way, of course, it is not true — but if the constraint that the triangles be on the same side of the chord is invoked, then it is true. The problem: prove that given the circle shown, angle ADB = angle ACB.

Many solutions reduced this problem to the theorem that an angle in a circle measures half its inscribed arc. I suspect that the best proof came from an old geometer, but it was all Greek to me. The following solution from Harry Zentopulos solves the original problem directly:

With respect to triangle ABC (*drawing, next page*), angle A plus angle B plus angle angle B plus angle C = 180°. Therefore,

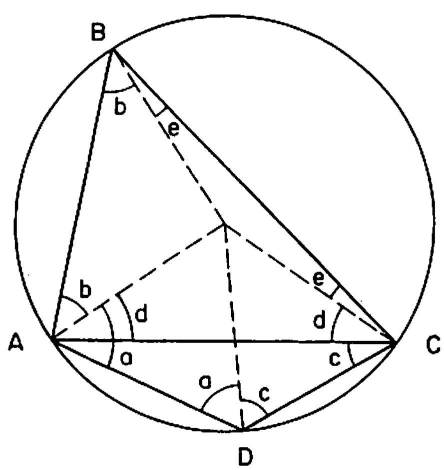
$$\begin{aligned} (a + b) + (b + c) + (a + c) &= 180° \\ 2(a + b + c) &= 180° \\ a + b + c &= 90° \\ b + c &= 90° - a. \quad (1) \end{aligned}$$

With respect to triangle ADC, angle A plus angle D plus angle C = 180°. Therefore,



$(a - d) + (c - d) + (a + c) = 180°$
$2(a + c - d) = 180°$
$a + c - d = 90°$
$c - d = 90° - a. (2)$

Combining (1) and (2),
$b + e = c - d$, and angle B equals angle D.



With respect to triangle ABC above, angle A plus angle B plus angle C = 180°. Therefore,

$(b + d) + (b + e) + (d + e) = 180°$
$2(b + d + e) = 180°$
$b + d + e = 90°$
$b + e = 90° - d.$

With respect to triangle ADC, angle A plus angle D plus angle C = 180°. Therefore,

$(a - d) + (a + c) + (c - d) = 180°$
$2(a + c - d) = 180°$
$a + c - d = 90°$
$a + c = 90° + d.$

Therefore, $(b + e) + (a + c) = 180°$ and angle B plus angle D = 180°.

Also solved by R. Robinson Rowe, Emmet J. Duffy, William J. Butler, Jr., John E. Prussing, Mary Lindenberg, Harry Zaremba, Benjamin Gray, Clem Wang, Joe Lacey, John F. Chandler, Row Moore, Barry Davis, Robert Pogoff, and the proposer, Joe Horton.

DEC 4 For any positive integer n, there are 2^n distinct binary numbers of n binary digits (bits), leading zeros allowed. Here is the list for n = 2, arranged in counting order:

00
01
10
11

In decimal, they would read 0, 1, 2 and 3. In order to solve some problems in digital design, the list should be arranged so only *one* bit changes from line to line, including the wrap-around case (bottom line back up to top line).

Here's one solution for n = 2:

00
01
11
10

Problem: find a method for generating such a sequence for arbitrary n.

Basically there are two ways of wording the solution. Robert Pogoff tells us which bit to change:

With the sequence listed, there are n columns and 2^n rows, for all the changes. Row 1 is the start; row 2 is the first change in column 1. Then: Change column 1 in every second row starting with the second; change column 2 in every fourth row starting with the third; change column 3 in every eighth row starting with the fifth; and change every column I in every 2^Ith row starting with the $(2^{I-1} + 1)$th. The change in the next-higher-numbered row is made whenever none of the lower-numbered rows can be changed according to the above rule. For example, a four-digit binary number is changed thus:

Row	Column: 4	3	2	1	Column no. changed
1	0	0	0	0	start
2	0	0	0	1	1
3	0	0	1	1	2
4	0	0	1	0	1
5	0	1	1	0	3
6	0	1	1	1	1
7	0	1	0	1	2
8	0	1	0	0	1
9	1	1	0	0	4
10	1	1	0	1	1
11	1	1	1	1	2
12	1	1	1	0	1
13	1	0	1	0	3
14	1	0	1	1	1
15	1	0	0	1	2
16	1	0	0	0	1

Note, however, that the columns may be numbered in any order, and the starting number may be any binary between 0 and $2^n - 1$. For example:

Row	Column: 2	1	3	4	Column no. changed
1	1	0	1	0	start
2	1	1	1	0	1
3	0	1	1	0	2
4	0	0	1	0	1
5	0	0	0	0	3
6	0	1	0	0	1
7	1	1	0	0	2
8	1	0	0	0	1
9	1	0	0	1	4
10	1	1	0	1	1
11	0	1	0	1	2
12	0	0	0	1	1
13	0	0	1	1	3
14	0	1	1	1	1
15	1	1	1	1	2
16	1	0	1	1	1

Roy Moore gives us the recursive construction:

Call a sequence with the required property a "gray sequence." 0, 1 is a "gray sequence" for n = 1, and the illustrated solution is a "gray sequence" for n = 2. We now proceed inductively. Suppose $\alpha_1, \alpha_2, \ldots, \alpha_m$ ($m = 2^n$) is a "gray sequence" for the binary numbers of n digits. Then $0\,\alpha_1, 0\,\alpha_2, \ldots, 0\,\alpha_m, 1\,\alpha_M, 1\alpha_{m-1}, \ldots, 1\alpha_1$ is readily seen to be a "gray sequence" for the binary numbers of n + 1 digits.

Also solved by John F. Chandler, R. Robinson Rowe, Emmet J. Duffy, William J. Butler, Jr., Douglas Szper, Bruce Fleischer, Barry Davis and the proposer, Dave Kaufman.

DEC 5 There are 4n tennis players who wish to play 4n − 1 doubles matches, where n equals any positive integer. How can the matches be arranged so that all players play in every match with the limitation that each player plays with each other player once only and against each other player the same number of times? When n equals one the solution is easy and quite obvious. Is there a general solution or formula or system? Is it limited to perhaps n equals 5 or 6?

Only R. Robinson Rowe responded, and with only a limited solution at that: (Perhaps this will be an NS problem sometime in the 1980s.)

This problem reminds me of the progression at bridge parties. There was a number assigned to each table of four players. After playing four deals, scores were totalled and the winning pair at each table advanced to the next table (or from the first table to the last). Each pair then split and became opponents for the next four deals. Many remarked that the progression was extremely fair, because, it seemed, no two players were partners twice. Trying this system for eight players (n = 2), I developed this tabulation:

Table 1		Table 2	
AB	CD	EF	GH
AE	BF	CG	DH
AC	EG	FH	BD
AF	CH	DG	BE
AG	DF	BH	CE
AB	BS	DE	CF
AD	EH	BC	FG

That is, at the start, A and B were partners against CD. C and D advanced and split. A and B stayed and split. It was convenient to have the second column of Table 1 and the first column of Table 2 advance each time. The system is not automatic. For instance, if BE instead of DG had advanced after the fourth rubber, one or the other would have been partner of A again. But there was an obvious alternative. I haven't the patience nor the computer to try this system for n = 3, 4, etc. Parties often had four or more tables, but for four tables it would take 15 rubbers to complete the test of fairness and we never played that many in the evening. So all I can say is that this system works for n = 2.

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The problem is reminiscent of the Kirkman's Schoolgirl Problem. The original particular set-up with 15 girls in a column of threes was not too difficult, but extension to the general case of 3n girls was so widely analyzed that W. W. Rouse Ball devoted an entire chapter to it in his *Mathematical Recreations and Essays.* The general case of this problem might be equally complicated.

Proposers' Solutions to Speed Problems

M/A SD1 Remembering that 32° F. = 0° C. and that increments are in the ratio of 9 to 5, one step gives 41° F. = 5° C. and the second step, 50° F. = 10° C., which is in the ratio of 5 to 1. Stepping down the same way gives in turn 23° F. = − 5° C., 14° F. = − 10° C., 5° F. = − 15° C., − 4° F. = −20° C. — again in the ratio of 5 to 1. So the daily range was 54° on the Fahrenheit scale and 30° on the Celsius scale. Of course, a more elegant solution would use algebra, starting with − 40° F. = − 40° C.

M/A 2D2 The interest due the first year is (.08) ($20,000) ± (.08) (30x), etc.

Allan J. Gottlieb is Coordinator of Computer Activities and Assistant Professor of Mathematics at York College of the City University of New York; he studied mathematics at M.I.T. (S.B. '67) and Brandeis University. Send problems, solutions, and comments to him at York College, 150-14 Jamaica Avenue, Jamaica, N.Y., 11451.

Salisbury

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Before the Arab oil embargo, the United States was committed to an international solution for the problem of Antarctic resources. But since that time, the U.S. agencies with energy responsibilities have begun to lobby for unilateral exploitation of Antarctic wealth. Any such action is believed to be at least a decade away.

It seems likely, in the meantime, that scientists will begin sharing the vast spaces of Antarctica with technologists intent on profiting from its wealth. Because the treaty, as written, ignores resource exploitation, legal authorities say there is little anyone could do if a country such as China moved onto the continent and began to exploit it.

David F. Salisbury is writing for the Christian Science Monitor *from its West Coast office; he is a frequent contributor to the* Review.