Streaker at the Banquet Table

Puzzle Corner
by
Allan J. Gottlieb

Let me begin with some information. On September 24 and 25 the fourth annual Conference on Mathematics and Statistics will be held at Miami University, Oxford, Ohio. One of the invited speakers is "our" R. Robinson Rowe! Quoting from the announcement, "As in our previous conferences on geometry, statistics, and history of mathematics, the Friday sessions will be directed mainly to college teachers and those on Saturday to secondary school people, with mathematical hobbyists welcome at any time. In addition to the invited and contributed papers, there will be open problem sessions, displays, a book exhibit, and films." Questions may be sent to Professor Donald O. Koehler, Department of Mathematics and Statistics, Miami University, Oxford, Ohio, 45056.

On another topic, Avi Ornstein has noticed a miswording of our yearly problem. We prefer the digits in the order 1 9 7 6 — not in numerical order. And please correct M/A 1 by giving East the Jack and 10 of clubs.

Finally, I have received a solution to NS 2. See the solutions section.

Problems

JUN 1 We begin this month's selection with a bridge problem from the "house organ" of the Charles S. Draper Laboratories, Inc. The dealer is North, and East-West are vulnerable:

North:
♠ 4 2
♥ K 10 8 7 6
♦ A J 5
♣ Q 4 2
South:
♠ A Q 10 3
♥ A Q J 9 5
♦ 4
♣ A 8 3

The bidding:

North:	South:
Pass	1 ♥
3 ♥	6 ♥ (!)
Pass	

The opening lead is ♦ 3; trumps are divided 2-1. Plan the play.

JUN 2 Our second problem is from James Cassidy: Four people — Kevin, Deb, Breck, and Sally — occupied a side table at a recent banquet. During the height of the festivities one of them slipped behind a nearby drapery and a few minutes later emerged to streak down the center aisle and across in front of the head table, out into the corridor, and back to the drapery — shortly to return and reoccupy the vacated chair. When the excitement had died down somewhat the chairman of the banquet committee questioned the four to find out who had reoccupied the vacant chair.

Kevin answered:
1. I sat next to Deb. It wasn't her.
2. Breck or Deb sat to my right.

Deb said:

3. I sat next to Breck.
4. Kevin or Breck was on Sally's right.

Breck replied:

5. I sat across from Sal.

Sally said:

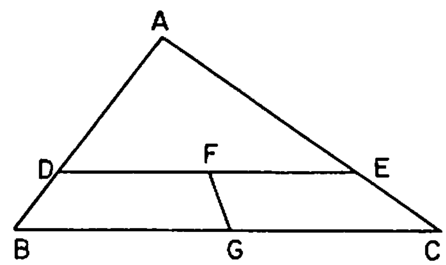
6. Only one of us is lying, and he or she is the guilty one.

Who was the varmint without a garment?

JUN 3 The following problem was suggested by Jeff Kenton: Given a 4 x 4 array with markers in all but one of the squares (as shown); the object is to remove all markers but one by jumping horizontally or vertically (no diagonal jumps allowed), or else to prove that it cannot be done.

X	X	X	X
X		X	X
X	X	X	X
X	X	X	X

JUN 4 A geometry problem from J. Harvey Goldman: Given any triangle ABC, choose points D and E such that BD = EC. Draw FG such that DF = FE and BG = GC. Then prove that FG is parallel to the bisector of angle BAC.



JUN 5 The following problem is from Jack Parsons: A man walking near a lake with a precipitous shoreline sees a girl struggling in the water. He can run twice as fast as she can swim. At what point should he leave the shore to reach her in the shortest possible time? The spatial relationships are: the man is 100 feet from the water, and the distance between the man and girl, parallel to the shore line, is 100 feet. Ground rule: no calculus. (The girl drowns while the man is calculating the best course, but that's irrelevant to the problem.)

Speed Problems

JUN SD1 This one is from R. Robinson Rowe, who says it "should be easy if you know the law 'down under' ": If Ashurst and Bathurst are Aussie neighbors and Ashurst's peacock lays two eggs on Bathurst's land, to whom do the eggs belong; or should each have one?

JUN SD2 Our last offering is from Emmet J. Duffy: Without using logarithm tables, slide rule, "slide rule calculator," etc., determine which is greater, e^{π} or π^{e}.

Solutions

NS 2 A magnetic dipole $m\hat{1}_z$ is situated at the origin of cylindrical coordinates (r,ϕ,z). A charge of q is situated at (2,0,0). In a situation such as this it is well known that the Poynting vector does not vanish so there is an energy flux $\overline{S} = (1/\mu_0)E \times B$ and a momentum $\overline{P} = \epsilon_0\overline{E} \times \overline{B}$ even though the configuration is entirely static (see, for example, Feynman's *Lectures* Volume II, Chapter 27). The problem is to find the total angular momentum of the electromagnetic field about the z axis; that is, find the integral

$$L_z = \int r\bar{I}_\phi \cdot (\epsilon_0 \bar{E} \times \bar{B})\, dV$$

over all space. (Assuming q, m, and ϵ finite and nonzero, then L_z will be finite and not zero).

The following is from I. Volynshchik: The angular momentum is equal to the time integral of the torque about the z-axis when the charge is brought in from infinity. In international units, the vector potential of the dipole is:

$$A_\phi = (\mu_0 m/4\pi) r/(r^2 + z^2)^{3/2}$$

Bring the charge q in along the straight line $(r_0, 0, z_0 + t)$, $-\infty < t \leq 0$, to its final position $(r_0, 0, z_0)$. Then

$$dL_z = qr_0\, \partial A_\phi/\partial z\, (dz/dt)\, dt = qr_0\, dA_\phi; \text{ hence}$$
$$L_z = (\mu_0 mq/4\pi) r_0^2/(r_0^2 + z_0^2)^{3/2}.$$

FEB 1 In a four-move chess game, White's moves were: 1. P — KB3; 2. K — B2; 3. K — N3; and 4. K — R4. On the fourth move, Black delivered mate. What were Black's moves?

The solution is:

P — KB3	P — K3
K — B2	Q — B3
K — N3	Q x P ck
K — R4	B — K2 mate

This solution was discovered by Gerald Blum, Randy Kimble, William J. Butler, Jr., and Benjamin W. Wurzburger.

FEB 2 How many sequences can be formed using the 28 dominoes?

My feeling is that this is a very difficult problem as originally intended. However, it was not clear as worded that *legal* (in the domino sense) sequences were required. Without this proviso, Harry Nelson was easily able to supply the following solution:

Each of the two parts of a domino is either blank or has pips of one to six in number. Seven dominoes have identical parts, from double-blank to double-six, and 21 dominoes have dissimilar parts, from blank to six. If the dominoes are placed end to end, they can be arranged to form 28! different ordered sequences. In every sequence, each of the 21 dominoes having a different number of pips in its parts can be oriented in two different ways. Therefore, the total number of sequences is $(2)^{21} \times 28 \approx 6.39397 \times 10^{35}$.

For legal sequences, the best we have is the proposer's (Eric Jamin) sketch of a solution. Basically, the solution is to find how many complete closed routes exist over a heptagon and all its diagonals, this number being equal to the number of different circular arrangements of the dominoes, excluding doubles (label the vertices of the heptagon 0 to 6; edges and diagonals correspond to the 21 nondouble dominoes having digits equal to those on the two vertices joined by this edge or diagonal). This number is equal to 129,976,320; reverse routes considered different. There are 3^7 ways of inserting the seven double dominoes and 28 ways to break the loop to give a sequence; this gives a grand total of 7,959,229,931,520. Responses also from R. Robinson Rowe, Gerald Blum, and William J. Butler, Jr.

FEB 3 The word "FACETIOUS contains all five vowels (no duplicates), and they occur in alphabetical order. Name another English word (no proper nouns) having the same properties.

Jeffrey A. Miller found three words: ABSTEMIOUS, ABSTENTIOUS, and ARSENIOUS; William J. Butler, Jr., added PARECIOUS, and Harvey M. Elentuck and other readers added FACETIOUS. Other respondents included James Finder, Frank R. Smith, Emmet J. Duffy, Morrie Gasser, Randy Kimble, Harry Zaremba, Gerald Blum, R. Robinson Rowe, Arthur J. (illegible), Harold C. Leighton, and the proposer, Mark D. Yellon.

FEB 4 Any system of locks requires a water supply at its upper level. For the Panama Canal, this supply is Gatun Lake. For which vessel transiting the Canal from the Atlantic to the Pacific does more water flow out of Gatun Lake — an aircraft carrier or a rowboat?

All three possible answers were received. I am *not* an expert on canals, so I will present responses from one proponent of each. Considering the current political maneuverings with the Panama Canal, I should probably let Dr. Kissinger adjudicate the result (despite his connections with my old Cambridge rival). Our first position, for equality, is from E. B. Jarman, the proposer:

The answer — the loss of water from Gatun Lake is identical incident to the passage of both vessels. Both the (ascending) lock entering the lake and the (descending) lock leaving the lake must be considered. Raising the water level of the ascending lock to the lake level, after the vessel enters and the entering gate is closed, requires the same amount of Gatun Lake water for both (a volume equal to the mean horizontal area of the lock multiplied by the height the water level is raised). However, when the vessel enters the lake from this lock, an amount of water equal in weight to that of the vessel must flow back into the lock from the lake to maintain the water level. At this stage of transit, Gatun Lake has lost more water in the case of the aircraft carrier. Proceeding southward toward the Pacific, however, as the vessel moves into the descending lock, it displaces again an amount of water equal to its weight, which flows back into the lake to maintain the water level, exactly compensating for that lost from the lake to the ascending lock for the same vessel (and over and above the equal amounts lost to raise the level of the ascending lock in both cases). Therefore the outflow from the water supply of Gatun Lake is insensitive to the size of the vessel transiting via the canal.

Many readers favored the rowboat; the following from Gerald Blum is typical:

To clarify the problem, assume there is one lock on each side of the lake connecting it to each ocean, and assume the locks are of equal size and the oceans of equal elevation. Let V_L be the volume of water needed to raise a lock from ocean height to lake height, and assume the volume of the lake to be much greater than V_L, so that the lake elevation is unaffected by a lock filling. Now consider a complete cycle of ship passage. Let the volume of water displaced by the ship be V_S. The cycle begins with both locks open to the ocean and closed at the lake end. The ship enters the Atlantic lock, and the ocean door of that lock closes. The lock is filled to lake height, which drains a quantity ($V_L - V_S$) of water from the lake. The lake door of that lock then opens, and the ship sails onto the lake. An additional quantity of water equal to V_S enters the lock from the lake. While the ship is crossing the lake, the Atlantic lock's lake door is closed and the ocean door opened, releasing a quantity V_L of lake water into the Atlantic. Meanwhile, the Pacific lock has closed its ocean door and opened its lake door, sending V_L of water from the lake into the lock. The ship then enters the Pacific lock, and V_S of water returns to the lake. The lake door of the Pacific lock is then closed, the ocean door opened, and the ship sails out into the Pacific as a quantity of lake water equal to ($V_L - V_S$) enters the Pacific. The cycle is complete, and the lake has lost a total quantity of water $= 2V_L - V_S$, this is maximal when V_S is minimal, so more is lost for the rowboat.

R. Robinson Rowe seems to have extra information about the Panama Canal that leads him to take the minority position:

In an ordinary lock, the chamber has a dead volume v at its lower stage and a volume (V + v) at its upper stage. A vessel with displacement D, if locking up, enters the chamber at its lower stage "carrying its displacement with it" — that is, as the vessel advances, water flows around it to its wake. So when the lower gate is closed behind it, the volume of water in the chamber is reduced from v to v − D. Then the culvert valves are opened to raise the water level to its upper stage. Then the volume of water in the chamber is (V + v − D). Thus the water added is (V + v − D) − (v − D) = V. That is, it is independent of the displacement and would be the same for a rowboat as for an aircraft carrier. A similar analysis holds for locking down.

But only one lock in each direction at Panama is an ordinary lock. The other five each way are subdivided with auxiliary gates, so that an aircraft carrier would have to use a full chamber 1,000 feet long, but the row boat would need only a subchamber 600 feet long. The chambers are 110 feet wide, so a passage of an aircraft carrier would use $1000 \times 110 \times 85 \times 2 \times 7.48 = 140{,}000{,}000$ gallons and a rowboat $(1000 + 5 \times 600)/6000 =$ two-thirds as much. As a matter of collateral interest, the unsubdivided lock is the Lower Miraflores Lock. Volumes are for mean tide, 85 feet lower than Gatun Lake. About 90 per cent of transiting vessels are less than 600 feet long and use the smaller chambers. A rowboat would probably be locked with a larger vessel, effectively using no water at all. The facts would differ for two-way locks which use the same water to lock one vessel up and another down.

Responses were also received from Jack Parsons, W. A. Schoenfield, James Finder, Bruce Parker, Arthur J. (illegible), Harry Zaremba, and William J. Butler, Jr.

FEB 5 A man is to pack a carton with equal-sized rectangular blocks. To prevent shifting during shipping, there must be no fault in the packing; that is, no plane may cut through the carton without cutting a block. What is the smallest sized carton, in volume, which he can use if the blocks are 1 x 1 x 2? 1 x 1 x 3? 1 x 2 x 3?

The following self-explanatory pictures are from Robert Pogoff. They are minimal among all responses received. I have not allowed solutions where the carton is not completely filled.

Responses were also received from Gerald Blum, Bruce Parker, Harry Zaremba, Norman M. Wickerstrand, Richard J. Allen, William J. Butler, Jr., and the proposer, Eric B. Jamin.

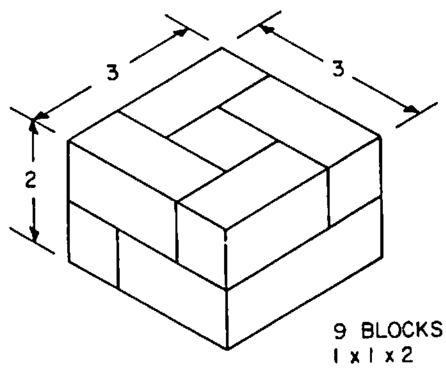
Better Late Than Never

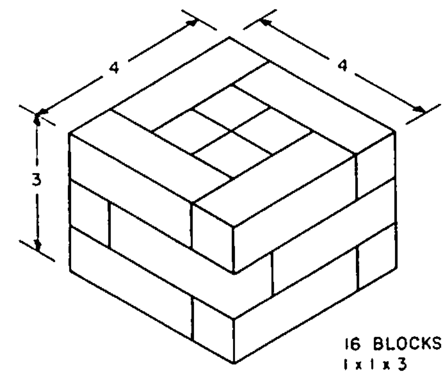
J/A 4 Emmet J. Duffy has submitted a pair of letters in which he shows the existence of N distinct positive integers such that any (N − 1) sum to a square, for any N, equals 0, 2, 4 (mod 6).

O/N 1 The following analysis is from Emmet J. Duffy:

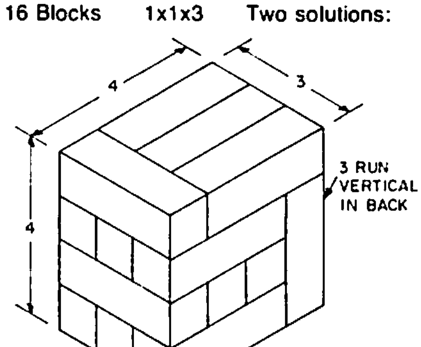
The correct way to play the hand is given in the answer to O/N 1, but some of the odds are not correct. Probability or odds should be based on the cards in the opponents' hands at the time a decision is to be made. Odds based on the opponents' hands at the start of the game will be incorrect if either opponent does not follow suit.

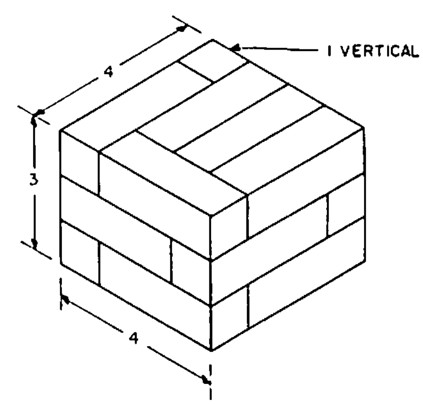
When declarer makes his decision to finesse or play the ♥A, West has six cards and East has seven cards. One way to compute the odds is to determine the



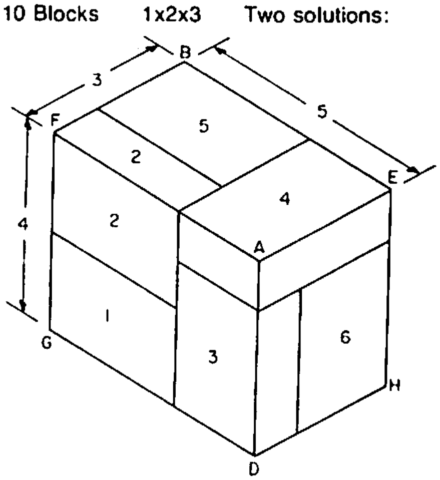


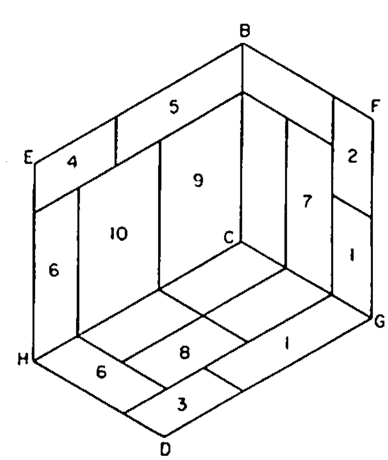
16 Blocks 1x1x3 Two solutions:





10 Blocks 1x2x3 Two solutions:





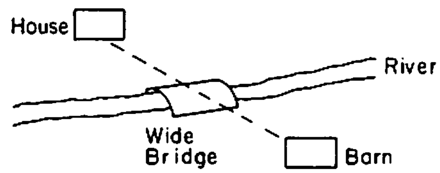
number of different hands West can have, without the heart, compared with the number of hands West can have with the heart. For example, if West started with no spades and dropped three clubs on the three spade leads, the 13 cards are six spades, one heart, and six clubs. West can now hold six clubs for which there is only one combination, or West can hold one heart and five clubs for which there are six combinations. Hence the odds for the finesse are 6:1. If West started with one spade and dropped two clubs on two of the spade leads, the 13 cards are five spades, one heart, and seven clubs. West can hold six clubs for which there are seven combinations, or West can hold one heart and five clubs for which there are 21 combinations. The odds for the finesse are then 3:1. The odds for all possible spade holdings in the West hand can be computed in this manner, but there is a much simpler way to compute the odds. From the six cards in the West hand and the seven cards in the East hand, subtract the number of spades figured to be in each hand. The numbers that remain will give the odds. For example, if West started out with no spades, then East started with nine spades and played three of them on the three spade leads. East now has six spades. Subtracting six from seven leaves East with one card which could be the heart, and West has six cards any one of which could be the heart. Hence the odds that West has the heart are 6:1. The computation is as shown in the table at the top of these two columns. If West and East each play spades on all three spade leads, and there is no knowledge of the spade holdings in either hand, then with six cards in the West hand and seven cards in the East hand, the odds that West has the heart are 6:7 and play of the ♥ A is preferred.

Spades in West hand at start	*Spades in each hand at time of decision*		*Result of subtraction*		*Odds for successful finesse*
	West	*East*	*West*	*East*	
0	0	6	6	1	6 to 1
1	0	5	6	2	3 to 1
2	0	4	6	3	2 to 1
3	0	3	6	4	3 to 2
4	1	2	5	5	1 to 1
5	2	1	4	6	2 to 3
6	3	0	3	7	3 to 7
7	4	0	2	7	2 to 7
8	5	0	1	7	1 to 7
9	6	0	0	7	Impossible

O/N 4 I have received two interesting letters recently on this problem.

Vern Reisenleiter, an ex-navigator, points out that the third solution presented is incorrect. "Distances are distorted in a Lambert Conformal Projection, and track AB is not east-west; it is, approximately, a great circle track."

Peter Welcher attacked the entire problem which ignores the width of the bridge. His sister gives the following solution for a wide bridge:



O/N SD 1 A rebuttal from R. Robinson Rowe:

Claiming that hereditary barrenness is possible, Bob Horvitz in February (*page 70*) cites as the simplest way that two parents be of an Aa genotype, where A = normal and a = barren. He does not explain how either parent acquired the *a* gene. Genes affecting only female descendants may pass through male lines, but a gene of barrenness would have to have a barren female ancestor — and *ipso facto* she would have no posterity! Hence his presumption that Aa genotypes exist is fallacious. Of his "less likely" examples, the first is quite plausible — that there could be a mutation of a gene in one of the parents. But it is plausible only if the mutation occurred to a gene of the mother, and if the still-fertile mother passed the mutant to a daughter, and if the daughter was barren. But then, since the mother was not barren, what the daughter inherited was only the gene of barrenness — *not* hereditary barrenness. It should be noted that barrenness is defined as "absolute inability to conceive" and its etiology is limited, so far as I can find out, to physiology (such as a constricted oviduct), psychology (such as a mental block), or to surgery (such as an hysterectomy). The first might be congenital, but that is not genetic. The mental attitude may stem from parental restraints, but cannot be of genetic origin. And of course surgery is not inherited. Finally, I note that Mr. Horvitz only claims possibilities, with no citation of facts. Such would not suffice for evidence in the courts of Maine, Idaho, Utah, and Iowa. As to the antiquity of the problem, I note that you heard a variant of it when in high school say *circa* 1960. I first heard it as a variant (naming no states) in 1913 when in Harvard — posed by a law student seated at the same table in Memorial Hall. I predict it will never die in spite of nit-picking.

FEB SD 1 Ralph M. Jones noticed an alternate solution wherein circles B and C are internally tangent to circle A. This gives the result $a = 26$, $b = 9$, $c = 14$.

The following have responded to the problems indicated:

O/N 4 Abraham Schwartz and N. Sacid Ozker.

DEC 1, 2, and 3 Eric Jamin.

JAN 1 Richard I. Hess, Bill Blake, and Eric Jamin.

JAN 2 Eric Jamin.

JAN 3 Eric Jamin, Richard I. Hess, Bob Lutton, Gerald Blum, Art Hovey, Vern Reisenleiter, and William J. Butler.

JAN 4 Gerald Blum, Bob Lutton, Ray Sullivan, Stanley Joehlin, Aviva Eichler, William Proctor, and Michael Auerbach.

Proposers' Solutions to Speed Problems

JUN SD1 By the law "down under," peacocks don't lay eggs — only peahens.

JUN SD2 Let $x^{\pi e/x}$ be a function which is e^{π} when x is e and π^{e} when x is π. Differentiating the function with respect to x and setting the derivative equal to zero, it is seen that the maximum value of the function occurs when x is e; hence e^{π} is greater than π^{e}.

Professor Allan J. Gottlieb is Coordinator of Computer Activities at York College of C.U.N.Y.; he studied mathematics at M.I.T. (S.B. 1967) and Brandeis University (A.M. 1968, Ph.D. 1973). Send problems, solutions, and comments to him at York College, Jamaica, New York, 11451.

Letters

Continued from p. 3

hissing sounds often reported. With the gas released, the unstable, writhing mat mass sinks into the Loch, perhaps to be sighted again or photographed on its journey downward by the crew of a scientific submarine.

The lake bottom area is so large that the chance of this phenomenon being created and sighted fairly often is very good.

Eat your heart out, Loch Ness monster!

Robert G. Hahl
Alexandria, Va.

Mr. Hahl is a physical scientist for the U.S. Department of Defense.