How to Pack a Box of Boxes

Puzzle Corner
by
Allan J. Gottlieb

I often receive inquiries about why a response appears in the "Better Late Than Never" category when the writer believes it was submitted "on time." The answer is that my column is sent to the Editors *two months* before you read it. For example, today is December 1, and my deadline for the February issue is December 10; thus, nothing received after about December 8 could be included in this issue.

Continuing the policy started in December, I now present a problem which appeared previously but was not completely solved. But let me correct an error in the January issue: NS 1 first appeared in the April, 1966, issue of *Tech Engineering News*, the M.I.T. undergraduate engineering magazine; it reappeared in *Technology Review* in November, 1966. "Puzzle Corner" began in *Tech Engineering News* during my junior year at M.I.T. (1965-66) and was picked up by *Technology Review* the year after. Unfortunately, I have no record of the first year's columns. If any reader has back issues of *Tech Engineering News*, I would greatly appreciate copies of my first months' attempts at journalism.

Problems

NS 2 This month's "not-previously solved" offering appeared in February, 1967, in *Tech Engineering News* and *Technology Review* (the column was then syndicated); it was submitted by Nicholas J. Pippenger, who believed it was an original problem never before published:

A magnetic dipole $m\bar{1}_z$ is situated at the origin of cylindrical coordinates (r,ϕ,z). A charge of q is situated at (2,0,0). In a situation such as this it is well known that the Poynting vector does not vanish so there is an energy flux $\bar{S} = (1/\mu_0)E \times B$ and a momentum $\bar{P} = \epsilon_0 \bar{E} \times \bar{B}$ even though the configuration is entirely static (see, for example, Feynman's *Lectures* Volume II, Chapter 27). The problem is to find the total angular momentum of the electromagnetic field about the z axis; that is, find the integral

$$L_z = \int r\bar{1}_\phi \cdot (\epsilon_0 \bar{E} \times \bar{B})\, dV$$

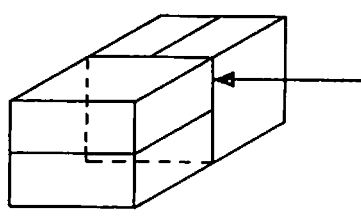
over all space. (Assuming q, m, and ϵ finite and nonzero, then L_z will be finite and not zero).

FEB 1 We begin our regular selection with a chess problem from Bob Kimble and an old lawyer, Alexander Alekhine: In a four-move chess game, White's moves were 1. P–KB3 2. K–B2, 3. K–N3, and 4. K–R4. On his fourth move, Black delivered mate. What were Black's moves?

FEB 2 Eric Jamin wants to know how many sequences can be formed using the 28 dominoes.

FEB 3 The following problem is from Mark D. Yellon: The word *FACETIOUS* contains all five vowels (no duplicates) and they occur in alphabetical order. Name another English word (no proper nouns) having the same properties.

FEB 4 Captain Eric B. Jamin notes that any system of locks requires a water supply at its upper level. For the Panama Canal this supply is Gatun Lake. The question is: for which vessel transiting via the canal from the Atlantic to the Pacific does more water flow out of Gatun Lake — an aircraft carrier or a rowboat?



FEB 5 We end with a practical problem from Frank Rubin: A man is to pack a carton with equal-sized rectangular blocks. To prevent shifting during shipping, there must be no fault in the packing. That is, no plane may cut through the carton without cutting a block. What is the smallest sized carton, in volume, which he can use if the blocks are 1 x 1 x 2? 1 x 1 x 3? 1 x 2 x 3? No credit can be given for the trivial solution in which the size of the carton equals the size of the block.

Speed Department

FEB SD 1 A quickie geometric problem from Mary Lindenberg: Three mutually tangent circles have centers A, B, and C and radii a, b, and c, respectively. The lengths of segments AB, BC, and CA are 17, 23, and 12, respectively. Find the lengths of the radii.

FEB SD 2 We close with a "problem" from Edward Friedman, who suggests that the advent of metrication in the United States makes useful a good understanding of unit conversions. Consider this one from an old 1953 M.I.T. physics quiz: "How many microphones are there in a megaphone?" or its companion: "How many millipedes are there in a centipede?"

Solutions

The following solutions are to problems published in the October/November, 1975, issue.

O/N 1 Given the following hands held by North and South, show how a bid of seven diamonds could be made; neither East nor West bid, and West's lead was ♠2.

♠ K
♥ A Q 9 8 7 6 2
♦ 9 8 4
♣ J 10

♠ A Q J
♥ J 10 5 3
♦ A K Q 5
♣ K 6

As I noted in December, William J. Butler, Jr., is our chief oddsmaker for bridge; he will not stoop to using the usual binomial approximations. Once again he seems to have given the most meticulous answer:

First, South must play for an even three-three break in diamonds. A four-two split is fatal even if the ♦J ♦10 is a doubleton. (The ♦A ♦K would catch the ♦J ♦10 and the ♦9 would win the next trick, but South would be unable to reach his hand to pull the last trump.) Since South's spades can be used to discard dummy's clubs, the problem boils down to how to play the heart suit. South's first six tricks are the high diamonds and high spades. (While leading the last diamond might induce one of the defenders to discard the outstanding small heart I will assume that they would discard clubs and not solve the declarer's problem.) At the seventh trick, South leads ♥J. (West could still make a mistake by "covering an

Possible West hands with . . .	No hearts	1 heart	2 hearts	Total
No spades	—	40	180	220
1 spade	180	3240	6480	9900
2 spades	6480	51840	60480	118800
3 spades	60480	282240	211680	554400
4 spades	211680	635040	317520	1164240
5 spades	317520	635040	211680	1164240
6 spades	211680	282240	60480	554400
7 spades	60480	51840	6480	118800
8 spades	6480	3240	180	9900
9 spades	180	40	—	220
				3695120

honor with an honor.") If West is void of hearts, the jig is up. If West plays the singleton ♥K, the problem is solved. If West plays the ♥4, South consults the table above:

Total possible West hands $= \binom{26}{13} = 26!/[13!(26-13)!] = 10{,}400{,}600$
(Select 13 from 26 remaining cards)
Total West hands with three diamonds $= \binom{6}{3}\binom{20}{10} = 3{,}695{,}120$
The "hearts" columns of the table are computed as follows:
$\binom{6}{3}\binom{9}{C}\binom{2}{H}\binom{9}{S}$, assuming three diamonds out of six, with
C = number of clubs
H = number of hearts, and
S = number of spades.
The "total" column of the table is computed as follows:
(6/3)(9/S)(20/CH), with three diamonds out of six and
S = number of spades
CH = number of clubs and hearts.
If West had only a singleton spade, there are 1,620 possible combinations that would allow him a singleton ♥4 and 6,480 combinations for a doubleton ♥K. Hence the odds are four to one in South's favor if he lets the ♥J ride. If West had two spades, the odds are 60,480 to 25,920 in favor of letting the ♥J ride. If West had three spades, the odds favor the finesse by 211,680 to 141,120. (This distribution may not be determined with precision, but the "spot" cards may offer a clue.) If West had four spades the choice is even. If West has more than four spades, then the play of the ♥A is indicated. If the only information on spade distribution gives West at least three spades but no more than six (all spot cards ignored), then the play of the ♥A is indicated. (Odds over finesse are 917,280 to 801,360.)

Also solved by Winslow H. Hartford, R. Robinson Rowe, Richard L. Hess, Elmer C. Ingraham, and the proposer, Kenneth Barbour.

O/N 2 Is it possible to construct a 3 x 3 x 3 magic cube in which the elements are not all identical?

Gerald Blum proves the following:
Lemma: The central element of a 3 x 3 magic square of magic sum S = S/3.
Proof of Lemma:
Let the magic square be

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array}$$

so that e is the central element. Write the following five of the eight "magic" equations:
a + b + c = S, g + h + i = S, b + e + h = S, a + e + i = S, c + e + g = S. From these we can quickly derive e + i = b + c and e + c = h + i, from which we obtain e − b = c − i and e− h = i − c. Adding these last two to b + e + h = S gives 3e = S or e = S/3. Q.E.D.

From this point on I will use the "pictorial" argument of Eric Jamin: Let s = S/3. Blum's Lemma shows us that the cube has these three layers piled on top of each other:

$$\begin{array}{ccc} & & \\ & & \\ & s & \\ & & \\ & & \end{array} \qquad \begin{array}{ccc} & s & \\ s & s & s \\ & s & \end{array} \qquad \begin{array}{ccc} & & \\ & s & \\ & & \end{array}$$

But to be a magic square,

$$\begin{array}{ccc} & s & \\ s & s & s \\ & s & \end{array} \quad \text{must be} \quad \begin{array}{ccc} s & s & s \\ s & s & s \\ s & s & s \end{array}.$$

So we have:

$$\begin{array}{ccc} & s & \\ s & s & s \\ & s & \end{array} \qquad \begin{array}{ccc} s & s & s \\ s & s & s \\ s & s & s \end{array} \qquad \begin{array}{ccc} & s & \\ s & s & s \\ & s & \end{array}.$$

The extra s's on the top and bottom cube come from looking at two vertical cross sections passing through the central element. But now the top and bottom must also be all s's.

Also solved by William J. Butler, Jr., Richard I. Hess, R. Robinson Rowe, Frank Rubin, Judith Q. Longyear, Avi Ornstein, P. V. Heftler, and the proposer, Roger Lustic.

O/N 3 A two-part problem:
A. Complex numbers of the form $c \equiv a[e_0] + b[ie_1]$ have *one* kind of "complex conjugation," $c^* \equiv a[e_0] - b[ie_1]$. Here $[e_0][e_0] \equiv [e_0]$, $[e_0][ie_1] \equiv [ie_1][e_0] \equiv [ie_1]$, and $[ie_1][ie_1] \equiv -[e_0]$. This conjugation has the property $(cc')^* = c'^*c^* = c^*c'^*$. Note that ≡ is used to represent "equals by definition." Complex quaternions (or "hypercomplex" numbers)

$$q \equiv a[e_0] + b[ie_0] + c[e_1] + d[ie_1] + e[e_2] + f[ie_2] + g[e_3] + h[ie_3] \equiv \sum_{n=0}^{3} a[e_n] + b[ie_n]$$

have *two* kinds of "complex conjugation" with the property $(qq')^* = q'^*q^*$. The hypercomplex number basis elements have a simple multiplication table: $[e_0] \longleftrightarrow 1$ (i.e., $[e_0]$ acts like 1), $[ie_1] \longleftrightarrow i$, $[e_1][e_2] = [ie_3] = -[e_2][e_1]$ and cyclic 1, 2, 3, $[e_1][e_1] = [e_2][e_2] = [e_3][e_3] = [e_0]$, $[ie_n] \longleftrightarrow i[e_n]$ for multiplication. Find the conjugations q^* and $q^\#$ and prove them unique.
B. In physics, there is a 16-element extension of this number system which also seems to describe reality in a fundamental way. We can write it $E = a_n[e_n] + b_n[ie_n] + c[f_n] + d[if_n]$, where again sum on n = 0, 1, 2, 3. Let E ≡ e + f for short. Then $E^+ \equiv e^* + f^\#$ and $E^\wedge \equiv e^\# + f^*$ and $E^\vee \equiv e^\# - f^*$, and all have the property $(EE')^{conj} = E'^{conj}E^{conj}$. The question is open as to whether any other conjugations can be defined with this property. The multiplication table is as follows: e ⟷ f and e ⟷ f; that is, f is like e with respect to $()^*$ and $()^\#$; e and f are like (+) and (−) in multiplication, ef → f, ee → e, ff → e, fe → f; if f is the *left* multiplier only, then fe is like ee given before ($[f_1][e_2] = [if_3]$); if f is the *right* multiplier then $()^{\#*}$ is taken on the *left* before the regular multiplication is done ($[e_1][f_2] \rightarrow [e_1]^{\#*}[f_2] = -[e_1][f_2] \rightarrow -[if_3]$ and $[if_1][f_2] \rightarrow [if_1]^{\#*}[f_2] = +[if_1][f_2] \rightarrow -[e_3]$).

The proposer, James Edmonds, offers:

$$q^*\Big(\sum_{n=0}^{3} a^n(e_n) + b^n(ie_n)\Big) = \sum_{n=0}^{3} a^n(e_n) - b^n(ie_n)$$

$$q^\#\Big(\sum_{n=0}^{3} a^n(e_n) + b^n(ie_n)\Big) = a_0(e_0) + b_0(ie_0) - \sum_{n=1}^{3} a^n(e_n) + b^n(ie_n).$$

Richard I. Hess submitted the following:
Find the conjugations satisfying $(qq')^* = q'^*q^*$, where q and q′ are quaternions.
1. Each $q = ae_0 + be_1 + ce_2 + de_3$, where a, b, c, and d are complex numbers.
2. The multiplication table is as follows:

	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	e_0	ie_3	$-ie_2$
e_2	e_2	$-ie_3$	e_0	ie_1
e_3	e_3	ie_2	$-ie_1$	e_0

3. Complex conjugation must generally be defined as:

$$q^* = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} \\ C_{10} & C_{11} & C_{12} & C_{13} \\ C_{20} & C_{21} & C_{22} & C_{23} \\ C_{30} & C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = cq$$

4. Let the columns of C be C_0, C_1, C_2, and C_3.
5. $(e_0e_0)^* = e_0^*e_0^* \Rightarrow (e_0 - e_0^*)e_0^* = 0 \Rightarrow$ either $e_0^* = 0$ or $e_0^* = e_0$.
6. Suppose $e_0^* = 0$, then $(e_0e_i)^* = e_i^*e_0^* \Rightarrow e_i^* = 0$. Thus one conjugation is

$$C = \begin{bmatrix} 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix} \quad ; q^* = 0 \text{ for all } q.$$

7. Suppose $e_0^* = e_0$, then $(e_ie_i)^* = e_0 = e_i^*e_i^* = C_iC_i \Rightarrow$
$C_{0i}^2 + C_{1i}^2 + C_{2i}^2 + C_{3i}^2 = 1$
$2C_{0i}C_{1i} = 2C_{0i}C_{2i} = 2C_{0i}C_{3i} = 0$
Either $C_{1i} = C_{2i} = C_{3i} = 0$; $C_{0i} = \pm 1$ $(i = 1,2,3)$
Or $C_{0i} = 0$; $C_{1i}^2 + C_{2i}^2 + C_{3i}^2 = 1$ $(i = 1,2,3)$

7a. Suppose $C_{1i} = C_{2i} = C_{3i} = 0$; $C_{0i} = \pm 1$ $(i = 1,2,3)$
then $(e_ie_j)^* = -ie_k^* = e_j^*e_i^*$ (i,j,k cyclic) $\Rightarrow -iC_k = C_jC_i = \pm e_0 \Rightarrow$ contradiction.

7b. Suppose $C_{0i} = 0$, $C_{1i}^2 + C_{2i}^2 + C_{3i}^2 = 1$ $(i = 1,2,3)$
then $(e_ie_j)^* = -ie_k^* = e_j^*e_i^*$ (i,j,k cyclic)
$\Rightarrow C_{1i}C_{1j} + C_{2i}C_{2j} + C_{3k}C_{3j} = 0$ $(i \neq j)$
$C_i \cdot C_j = C_k$ (i,j,k cyclic)
These conditions force the 3 x 3 part of C to be orthogonal with determinant.
$C = +1$ and $e_0^* = e_0$.

8. Two types of conjugation are $C = 0$ and $C =$ orthogonal. Example:

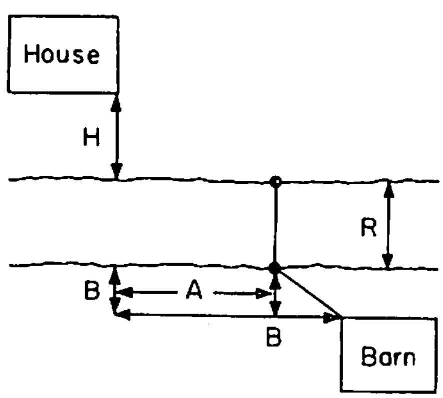
$$e_0^* = e_0$$
$$e_1^* = \sqrt{1/2}\,e_1 + \sqrt{1/2}\,e_3$$
$$e_2^* = e_2$$
$$e_3^* = -\sqrt{1/2}\,e_1 + \sqrt{1/2}\,e_3$$

Judith Q. Longyear also responded.

O/N 4 A farm is cut by a river flowing from east to west, the house on one side, the barn on the other. The farmer wants to build a north-south bridge from his house to his barn. Where should he locate the bridge to minimize his walk from house to barn?

The following pair of solutions were submitted by James W. Shearer:

First, consider the straightforward analytical approach, where distances are defined as shown.



Let y be the farmer's walking distance; then

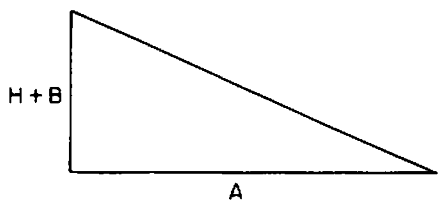
$$y = (H^2 + X^2)^{\frac12} + R + [B^2 - (A - x)^2]^{\frac12}.$$

We then minimize y(X) by setting dy/dx = 0:

$$0 = X/(H^2 + X^2)^{\frac12} + (X - A)/[B^2 + (A - X)^2]^{\frac12}$$
$$1/[(H/X)^2 + 1] = 1/[(B/A - X)^2 + 1],$$

whence $X = HA/(H + B)$ and $A - X = BA/(H + B)$.

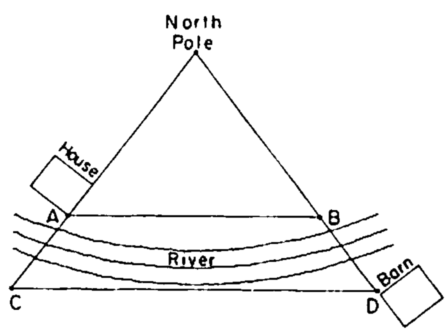
A more intuitive way of arriving at the solution is to realize that the distance R across the north-south bridge is invariant. So the minimum distance is given by the diagonal of the triangle with sides A and (H + B). The value of X needed to locate the bridge is then found from similar triangles:



$X/H = A/(H + B)$.

For another method, consider this response from Mark F. Mitchell:

It is assumed that the problem lies entirely within the Northern Hemisphere. To make this answer obvious by inspection, the sketch has been redrawn in Lambert Conformal Conic Projection so that the meridians converge at the North Pole. Dimensions are, of course, distorted.



Line AB is the shortest east-west path, so that a bridge erected at BD would afford the shortest available route from house to barn.

Also solved by R. Robinson Rowe, Glenn Ferri, R. V. Heftler, Raymond Gaillard, W. A. Schoenfeld, Antonio Carlos Barbosa De Oliveria, Jack Parsons, Richard Hanau, John E. Prussing, Mickey Haney, David J. Pogoff, Charles R. Owen, Jim Ritcey, Richard Ryan, George D. Lawrence, Neil E. Hopkins, Ted Mita, Judith Q. Longyear, Winslow H. Hartford, Gerald Blum, Eric Jamin, William J. Butler, Jr., Frank Rubin, Richard I. Hess, and the proposer, Ray Brinker.

O/N 5 Prove that any set of n integers (not necessarily distinct) contains a nonempty subset whose sum is divisible by n.

I liked this problem, and so did several readers. The solution selected, from Judith Q. Longyear, seems particularly clear.

Let $A = \{a_1, a_2, \ldots, a_n\}$ be a set of n integers. The n sums a_1, $a_1 + a_2$, $a_1 + a_2 + a_3, \ldots, a_1 + \ldots, + a_n$ have various remainders on division by n. Let us say $a_1 + \ldots, + a_i$ leaves remainder X_i. Then there are n sums and n possible remainders $0, 1, \ldots, n - 1$. If some $X_i = 0$ then $a_1 +, \ldots, + a_i$ is a multiple of n. If not, then there must be $i<j$ for which $X_i = X_j$ (although the sums are possibly different.) But then $a_{i-1} + a_{i-2} + \ldots + a_j = a_1 \ldots + a_j - (a_1 + \ldots + a_i)$ leaves the remainder $X_j - X_i = 0$. The set $\{a_{i-1}, \ldots, a_j\}$ is not empty since $i < j$.

Also solved by Eric Jamin, William J. Butler, Jr., Frank Rubin, Richard I. Hess, Ted Mita, P. V. Heftler, R. Robinson Rowe, Peter Kleeman, Joe Martin, and Richard C. Bradley.

O/N SD 1 Maine, Idaho, Utah, and Iowa do not recognize hereditary barrenness as legal justification in a suit by a husband for divorce. Why?

As you know I do not usually print solutions to speed problems. However, I must do so this time as Bob Horvitz gives an important correction. My wife, Alice, (an M.D.-Ph.D. student) says that Mr. Horvitz is right. So I guess I was wrong. In fact, I've been wrong for over a dozen years. I first heard a variant of this problem when in high school. I take some comfort in my company as I have the greatest respect for the proposer, R. Robinson Rowe. Mr. Horvitz writes:

Unless I've missed the point of O/N SD 1, I think you and R^3 may have forgotten some elementary genetics. Hereditary barrenness is indeed quite possible. The simplest way for it to occur would be for the two parents to be of an Aa genotype, where A = normal and a = barren, with a recessive to A; hence an Aa individual is capable of reproducing. If two such Aa individuals have children, half will be aa and thus be of hereditary barrenness.

There are other, less likely ways for an individual with heriditary barrenness to arise. Consider two examples:

1. This person could carry a spontaneous (dominant) mutation, hence making the genotype of her/his parents irrelevant.
2. One parent could carry a second mutant gene which "suppresses" the effect of a gene for hereditary barrenness she/he also carries; i.e., where B = barrenness and b = normal, if B is dominant to b and if S is capable of suppressing the effect of B and is dominant to s; a parent of genotype sSBb would not be barren but could produce a barren child (ssBb) as a result of mating with a normal (e.g., ssbb) individual.

I suspect that one could come up with numerous examples of hereditary barrenness from the medical literature, but I hope that for a mathematics column a theoretical result is good enough.

Better Late than Never
J/A 3 Frank Carbin has responded.
J/A 4 R. L. Bishop has responded, and I failed to mention the proposer.
J/A 5 J. Estermann has responded.

Proposer Solutions to Speed Problems
SD 1

$$\begin{aligned} a + b &= 17 \\ b + c &= 23 \\ c + a &= 12 \\ \hline 2(a + b + c) &= 52 \\ a + b + c &= 26 \end{aligned}$$

Subtract each of the first three equations from the last to obtain a = 3, b = 14, and c = 9.

Allan J. Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Assistant Professor of Mathematics at York College of C.U.N.Y. Send problems, solutions, and comments to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y., 11432.

Letters

Continued from p. 4

not sleep. He sat up all night to write a poem in praise of the young researchers. To scientists all over China, this spontaneous act provided more incentive than any monetary reward possibly could.
T. C. Tsu
Pittsburgh, Pa.

Gas Is Better
Whereas we agree with the overall methodology employed by Ogden Hammond and Martin B. Zimmerman in "The Economics of Coal-Based Synthetic Gas" (*July/August, pp. 42-51*), we do not concur with their conclusion. There are two basic oversights which, it turns out, were responsible for making the case for electric energy over gas energy for space heating.

First, with a few exceptions for "town gas" and hydrogen production at relatively small output rates, the gas-from-coal technology which forms the basis of the author's analysis is pre-commercial when measured against current U.S. gas demands. Current coal-to-gas technology therefore lacks the cost-saving developments and technological improvements that would surely ensue in the first few years of any new commercial process. This fact must be faced by any utility pondering the decision to invest, or not to invest, in any given technology. Consolidated Natural Gas Co. has invested large sums in coal reserves, and is actively evaluating sites for gas-from-coal facilities. But we are delaying commitment to commercial plants representative of the technology cited in the article in the belief that there will be far more attractive options available when we are ready to proceed.

Second, when the authors compare an electric heat pump with a coefficient of performance (COP) of 2.4 and a gas burner with an efficiency of 0.5 they are comparing a thing of the future with something becoming rapidly out-of-date. Specifically, the best of presently available electric heat pumps, operated in our service area, achieve a COP (or, we would prefer, an SPF, or season's performance factor) of 1.7. One of the electric utilities operating in our service area (Cleveland Electric Illuminating Co.) will not vouch for an SPF of more than 1.5 when asked by a customer what can be expected. A COP of 2.4 over a season (in our service area) would require an electric heat pump with extra-large heat transfer surfaces plus some means of modulating the compressor. These features may become practical some day, but are not now available.

The gas burner with a COP of 0.5 represents the worst applications of past-technology gas heating equipment.

Until the electric utility industry finds a practical means of applying all of the waste heat rejected by its generating facilities to space heating, there is no way that electric energy can match the bottom-line efficiency of the on-site use of gas.
L. G. Massey
Cleveland, Ohio

Mr. Massey is Associate Director, Research of Consolidated Natural Gas Service Co.

Boulding

Continued from p. 5

in it, we can visualize the problem. This is an invitation to contemplate appalling human catastrophe, made worse by climatic instability. Yet neither Bangladesh nor anyone else is doing anything about it. The second Club of Rome report, *Mankind at the Turning Point*, by Mihajlo Mesarovic and Eduard Pestel, suggests that catastrophe is nearly inevitable in a large part of the tropics without rapid reduction of human fertility and large transfers of capital from the rich countries to the poor. It is not wholly certain, of course, that these measures are necessary to secure the tropics from disaster, but the probability is high. The probability that they will not be carried out seems even higher. Perhaps the most that can be hoped is that we will learn how to learn from catastrophe when it occurs.

The breeder reactor is fraught with even more awesome uncertainties. Plutonium is probably the most poisonous substance known to mankind. It has a half-life of about 24,000 years, so that after 100,000 years an eighth of it remains in the environment. It will be produced in increasing quantities by the presently contemplated breeder reactors. Plutonium involves a low probability of very large catastrophes, increasing as times goes on unless a substitute source of energy is found. If the probability of a major catastrophe due to plutonium is only 0.1 per cent a year, the probability in 100,000 years is so high as to be virtually certain. With plutonium, we risk profound change in the planet's evolution. I confess I am glad the decision to build the breeder does not rest with me.

Kenneth E. Boulding is Professor of Economics and Director of the Institute of Behavioral Science at the University of Colorado.

Nisbet

Continued from p. 11

viduals fall victim before the age of 40, a fact which testifies to the wide range of human susceptibility. We must remember that in estimating the risks posed by very low levels of carcinogens in the environment, we are ipso facto concerned with predicting the response of the most highly susceptible individuals. There is no evidence in research on cigarette smoking of either a threshold dose or a threshold time.

The data on lung cancer in nonsmokers suggest a probability comparable to smoking almost one cigarette per day. Thus, nonsmokers are already at substantial risk. It is not clear to what extent their lung cancer is caused by other carcinogens and to what extent it is caused by involuntary exposure to cigarette smoke. But it is clear that nonsmokers are well-advised to avoid cigarette smoke: the more susceptible among them are expected to be affected by any incremental increase in dose.

Ian C. T. Nisbet, who writes regularly for Technology Review, *is Associate Director of the Scientific Staff of the Massachusetts Audubon Society. His Ph.D. (in physics) is from Cambridge University.*

Book Reviews

Continued from p. 29

energy: solar energy, wind power, fuel cells, and, where possible, geothermal and ocean-thermal energy.

We may find that one of these sources of renewable energy will not be practicable, or that our clients wish to maintain "normal" comfort conditions that will require supplemental fossil fuel-based systems. Then we should explore ways to re-