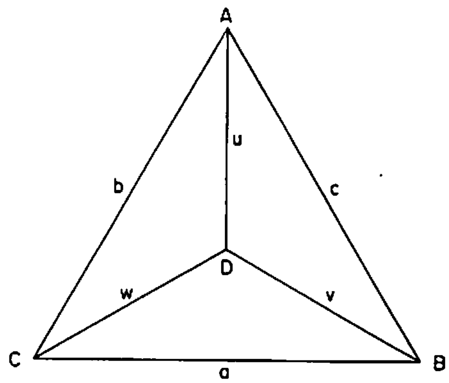
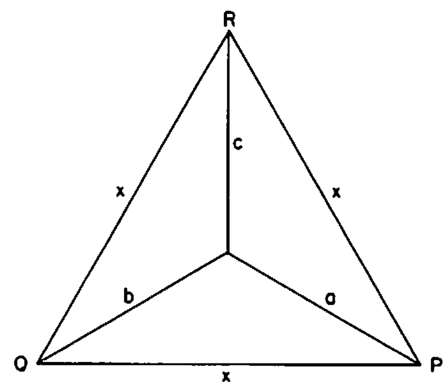
Who Plays (Baseball, Is It?), and Where

Puzzle Corner
by
Allan J. Gottlieb





It's time to answer our yearly problem — and to start on the next one.

Last January we were asked to take the digits 1, 9, 7, and 5; the operators +, −, *(multiply), /(divide), and **(exponentiate); and form the integers from 1 to 100 using each digit once and the fewest possible number of operators. Parentheses were allowed to indicate the order of operation, and in case of a tie a solution using 1 9 7 5 in order was to be favored. The answers appear under "Solutions," below.

Y1976 Now is the time to work on 1 9 7 6. Same rules as above; send your solutions before November 1, 1976 — or, better yet, as soon as you have them.

Problems

Having given you a problem to last for the year, here are some to last for the month:

JAN 1 Our first offering is an eight-card bridge problem from Emmet J. Duffy:

	♠ A 2 ♥ A 2 ♦ A 4 3 2 ♣ –	
♠ K Q ♥ K Q ♦ K Q J 10 ♣ –		♠ J 10 9 ♥ J 10 9 ♦ 9 8 ♣ –
	♠ 4 3 ♥ 4 3 ♦ – ♣ 5 4 3 2	

Clubs are trump. North leads. The problem: North and South to take all eight tricks against any defense.

JAN 2 Mary Lindenberg submits the following problem from the U.S.A. Mathematics Olympiad: Consider two triangles ABC and PQR, shown above. Angle ADB = angle BDC = angle CDA = 120°. Prove that X = u − v − w.

Jan 3 L. W. Sprodin wonders: If you drop a six-inch pencil onto a tiled floor, each tile a 12-inch square, what is the probability that the pencil will cross at least one edge?

JAN 4 The following entertaining problem, entitled "Who Plays Where?," is from Anne Goetting:

1. Andy dislikes the catcher.
2. Ed's sister is engaged to the second baseman.
3. The center fielder is taller than the right fielder.
4. Harry and the third baseman live in the same building.
5. Paul and Allen each won $20 from the pitcher at pinochle.
6. Ed and the outfielders play poker during their free time.
7. The pitcher's wife is the third baseman's sister.
8. All the battery and infield, except Allen, Harry, and Andy, are shorter than Sam.
9. Paul, Andy, and the shortstop lost $50 each at the race track.
10. Paul, Harry, Bill, and the catcher took a trouncing from the second baseman at pool.
11. Sam is involved in a divorce suit.
12. The catcher and the third baseman each have two children.
13. Ed, Paul, Jerry, the center fielder, and the right fielder are bachelors.
14. The others are married.
15. The shortstop, the third baseman, and Bill each cleaned up $100 betting on the fight.
16. One of the outfielders is either Mike or Andy.
17. Jerry is taller than Bill.
18. Mike is shorter than Bill.
19. Each of them is heavier than the third baseman.

Who plays where?

JAN 5 We end on a serious note; the following problem in abstract algebra is from William B. Ackerman: Let S_i be the sequence of polynomials in the variables tr A, tr A^2, tr A^3, . . . defined by the recurrence relations

$S_0 = 1$ and

$$S_N = 1/N(S_{N-1}\,\mathrm{tr}\,A - S_{N-2}\,\mathrm{tr}\,A^2 + S_{N-3}\,\mathrm{tr}\,A^3 - \ldots + (-1)^{N-1} S_0\,\mathrm{tr}\,A^N).$$

(At this point, tr A, tr A^2, etc., should be considered simply abstract variables. For example, tr A^2 is *not* the square of tr A.) The first few polynomials are easily found to be:

$$S_0 = 1$$

$$S_1 = \mathrm{tr}\,A$$

$$S_2 = \frac{1}{2}(\mathrm{tr}\,A)^2 - \frac{1}{2}\mathrm{tr}\,A^2$$

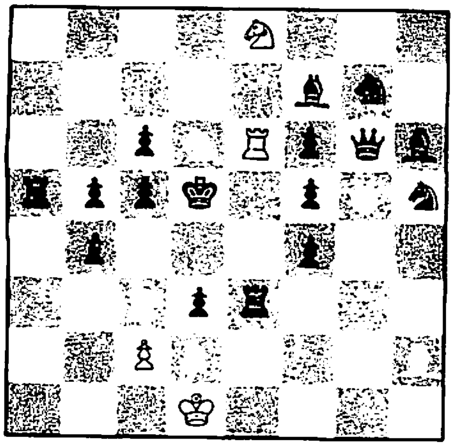
$$S_3 = \frac{1}{6}(\mathrm{tr}\,A)^3 - \frac{1}{2}\mathrm{tr}\,A\,\mathrm{tr}\,A^2 + \frac{1}{3}\mathrm{tr}\,A^3$$

$$S_4 = \frac{1}{24}(\mathrm{tr}\,A)^4 - \frac{1}{4}(\mathrm{tr}\,A)^2\,\mathrm{tr}\,A^2 + \frac{1}{3}\mathrm{tr}\,A\,\mathrm{tr}\,A^3 + \frac{1}{8}(\mathrm{tr}\,A^2)^2 - \frac{1}{4}\mathrm{tr}\,A^4$$

Now let "tr" denote the trace of a matrix, which is just the sum of the elements on the main diagonal. tr A^i means the trace of the matrix A^i, using matrix multiplication to produce A^i and then taking the trace of the result. If A is an N × N matrix, prove that its determinant is the value of S_N.

Solutions

The following are solutions to problems which appeared in the July/August issue.



J/A 1 Starting from the beginning position, what is the minimum number of moves required to reach the following:

Our minimal solution is the following 33 moves from William J. Butler, Jr.:

1.	P — KR4	P — KR3
2.	P — QN4	P — QR4
3.	B — N2	P x P
4.	B — K5	P — QN4
5.	B — R2	R x P
6.	P — N4	P — QB4
7.	P — N5	P x P
8.	P — R5	N — KB3
9.	P — R6	N — R4
10.	P — R7	R — N1
11.	P — R8(Q)	P — N3
12.	Q — Q4	P — K4
13.	P — KB4	KP x Q
14.	N — KB3	P x P
15.	N — R4	P — Q6
16.	N — B5	P x N
17.	B — N2	R — N6
18.	B — B6	P x B
19.	P — K4	R — R4
20.	R — QR3	B — KR3
21.	Q — N4	N — N2
22.	R — B3	N — Q2
23.	K — Q1	N — B3
24.	R — K1	N(B3) — R4
25.	N — R3	Q — N4
26.	P — K5	K — K2
27.	P — K6	K — Q3
28.	P — K7	P — B3
29.	P — K8(N)ck	K — Q4
30.	N — B4	B — K3
31.	N(B4) — Q6	B — B2
32.	R — K6	Q — N3
33.	Q — N5	R — K6

Solutions were also received from Richard I. Hess and the proposer, Paul Reeves.

J/A 2 Find integers $0 < a < b$ such that for all pairs of non-negative integers m and n the linear combinations na + nb fail to include exactly 35 positive integers, one of which is 58.

The following is a slightly modified version of a solution submitted by Frank Rubin:

To start off, find a formula F(a,b) for the number of integers *not* generated in the positive linear combinations

$\{am + bn | 0 \le m, 0 \le n\}$.

To motivate the derivation, consider the case where a = 5 and b = 7. Arrange the integers in five columns, as shown below. Now all integers in the a-th column are generated. In the column containing b, all integers of the form b + ma are generated, and [b/a] integers are not generated, where [x] represents the integer part of x. In the column containing κb, [κb/a] integers are not generated, for $1 \le \kappa < a$.

1	2	3	4	5
6	~~7~~	8	9	~~10~~
11	~~12~~	13	~~14~~	~~15~~
16	~~17~~	18	~~19~~	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~
~~26~~	~~27~~	~~28~~	~~29~~	~~30~~
~~31~~	~~32~~	~~33~~	~~34~~	~~35~~
~~36~~	~~37~~	..		

Now, for general a and b, if a = 1 then F(a,b) = 0. Otherwise, if gcd(a,b) = p, p > 1, then F(a,b) = ∞ since only multiples of p are generated. The remaining case is where $1 < a < b$ and gcd(a,b) = 1. In this case, F(a,b) = [b/a] + [2b/a] + . . . + [(a − 1)b/a]. To reduce this to a closed form, notice that κb/a = [κb/a] + fraction, where the fractional parts for κ = 1, 2, . . . , a − 1 are all distinct. Hence F(a,b) = b/a + 2b/a + . . . + [(a − 1)b]/a − [1/a + . . . + (a − 1)/a] = (a − 1) (b − 1)/2. Now, from the original problem, F(a,b) = 35 so (a − 1)(b − 1) = 70. Possible factorizations of 70 are 1·70, 2·35, 5·14, and 7·10, giving the values a = 2, b = 71; a = 3, b = 36; a = 6, b = 15; and a = 8, b = 11. Since we also have gcd(a,b) = 1, the only possible pairs are a = 2, b = 71; a = 8, b = 11. Since we have the additional fact from the problem that 58 is not generated, a = 2, b = 71 is not possible.

Also solved by Gerald Blum, Winslow H. Hartford, Frank S. Model, R. Robinson Rowe, E. Jamin, William J. Butler, Jr., and Richard I. Hess.

J/A 3 Produce an explicit one-to-one correspondence between the points in the unit interval $0 \le x \le 1$ and in the unit square $0 \le x,y \le 1$.

Many people offered the "obvious" correspondence, which does not quite work. The proposer starts with this correspondence and then fixes it up:

Presented with the task of constructing a single one-to-one correspondence between the points of the closed segment [0, 1] and the points of the closed square region [0, 1] × [0, 1], one may propose the following device for the required association a ⟷ (x, y): write a in decimal form; take the first, third, fifth, etc., digits to be the decimal expression for x, and take the second, fourth, sixth, etc., digits to be the decimal expression for y. Although this scheme is straightforward, it does not produce a one-to-one correspondence because it allows no simple way of consistently avoiding superfluous decimal expressions ending only in repeated 9s. Let us insist from the start that such expressions are not allowed for a, x, or y. Then if a = .50939393 . . ., the proposed device would have x = .5999 . . ., which is not allowed, and, if allowed, would be redundant.

There is a simple way to modify our scheme so that repeated 9s are avoided in a consistent manner. This modification is perhaps best explained by illustration. If a = .239943699925947 . . ., partition the decimal sequence for a into groups of digits as follows:

a = .2/3/994/3/6/992/5/94/7/. . .

Note that every 9 or string of consecutive 9s is grouped with the next digit which is not a 9. Otherwise, the groups are single digits. We now use the resulting sequence of groups to form decimal expressions for x and y in analogy to the original procedure, taking the first, third, fifth, etc., groups for x, and the remaining groups for y. For the case being illustrated we get x = .2/994/6/5/7/. . ., and y = .3/3/992/94/. . . . Technically, our device only yields a one-to-one correspondence between the half open interval [0, 1) and the half open square [0, 1) × [0, 1). However, it is not difficult to use this device to construct a one-to-one correspondence between [0, 1] and [0, 1] × [0, 1], and, in fact, the correspondence can be accomplished in a very general way. Most useful toward this end is the simple one-to-one correspondence now provided between the closed segment [0, 1] and a closed disc of unit radius. We take polar coordinates r, θ about the disc's center and extend the above correspondence a ⟷ (x,y) by setting $\theta = 2\pi x$, $r = 1 - y$, and further matching the end point a = 1 with the center of the disc. It is an easy task to map the closed disc above onto a closed square region, or onto any closed and bounded convex region. This correspondence can be directly accomplished by taking as a center for new polar coordinates r′, θ', a point inside the specified region (for a square, at its center). If $E(\theta')$ denotes the radial coordinate of the point on the region's edge having angular coordinate θ', we extend the above chain of correspondence a ⟷ (x,y) ⟷ (r, θ) to (r′, θ') via the equations $\theta' = \theta$, $r' = E(\theta) \cdot r$.

Responses were received from Thomas Greenway, William J. Butler, Jr., E. Jamin, Richard I. Hess, R. Robinson Rowe, Glenn Ferri, Frank Rubin, and Gerald Blum (in conjunction with G. Cantor).

J/A 4 Find four distinct positive integers such that the sum of any three is a perfect square.

Craig Gander claims the only three solutions less than 100 are

1	22	41	58
9	34	57	78
14	41	66	89

Also solved by John Unger, Gerald Blum, Frank Rubin, R. Robinson Rowe, Richard I. Hess, E. Jamin, William J. Butler, Jr., Frank S. Model, Winslow H. Hartford, Roger Milkman, Walter F. Penny, Emmet J. Duffy, Peter Groot, Harry Zaremba, Arun Trikha, Scott Peterson, Herbert B. Wyman, Neil E. Hopkins, William Benton Fisher, and the proposer, Fritz Olenberger.

J/A 5 The problem was an acrostic, and space is inadequate to reprint it. The solution reads, "We are now in a position to illustrate how kinetic theory can supplement an empirical thermodynamic formula with a physical model," and prints of the original puzzle may be had on request from the Editors of the *Review*, Room E19-430, M.I.T., Cambridge, Mass., 02139.

Solutions were received from Harry Zaremba, Gerald Blum, W. Allen Smith, Nancy Burstein, Michael H. Auerbach, Paul McAllister, Mary Fenocketti, Glenn Rowsam, Roger Milkman, Richard I. Hess, R. Robinson Rowe, and the proposer, Dawn Friedell Jacobs. Several readers commented favorably on the problem, and Ms. Jacobs is to be complimented.

Y1975 From the four digits 1, 9, 7, and 5, construct integers from 1 to 100 using only +, −, *(multiply), /(divide), and **(exponentiate). The best answer for a given number is the one with the lowest "point value," one point being assigned for each occurrence of +, −, *, /, or **. (For a further description, and for an extension of the problem to 1 9 7 6, see the second paragraph of this column.)

No one was able to obtain 23, 41, 55, 71, 86, and 90. Several solutions purport to be exhaustive computer searches, so these numbers are presumably unattainable. The following list is from William R. Kampe III. Others equaled his total of 198 operators.

Number	Score	Solution
1	1	1**579
2	2	(19 − 5)/7
3	1	57/19
4	2	5 − 1**79
5	2	5*1**79
6	2	(51 − 9)/7
7	2	19 − 5 − 7
8	2	91/7 − 5
9	2	9*1**57
10	2	9 + 1**57
11	3	(7 − 5)*1 + 9
12	1	71 − 59
13	2	15 − 9 + 7
14	3	(1**7)*5 + 9
15	3	1**9 + 5 + 9
16	1	91 − 75
17	2	15 − 7 + 9
18	2	91/7 + 5
19	3	(7 − 5)*9 + 1
20	3	5 − 1 + 7 + 9
21	2	17 − 5 + 9
22	3	1 + 5 + 7 + 9
23		
24	1	95 − 71
25	3	5**(9 − 7)*1
26	2	71 − 9*5
27	3	5*7 − 9 + 1
28	1	79 − 51
29	3	(9 − 5)*7 + 1
30	2	(9 − 7)*15
31	2	15 + 7 + 9
32	3	(9 − 7)**5*1
33	3	(9 − 7)**5 + 1
34	1	91 − 57
35	2	51 − 7 − 9
36	3	(5 − 1**7)*9
37	3	(5 − 1)*7 + 9
38	1	57 − 19
39	3	(7 − 1)*5 + 9
40	2	(17 − 9)*5
41		
42	1	59 − 17
43	3	(5 − 1)*9 + 7
44	3	1*5*7 + 9
45	3	5*7 + 1 + 9
46	1	97 − 51
47	2	57 − 1 − 9
48	2	(57 − 9)*1
49	2	51 − 9 + 7
50	3	(1**7 + 9)*5
51	2	59 − 1 − 7
52	2	(59 − 7)*1
53	2	51 − 7 + 9
54	2	5*7 + 19
55		
56	1	75 − 19
57	2	71 − 5 − 9
58	2	1**9 + 57
59	2	(1**7)*59
60	2	(19 − 7)*5
61	3	(1 + 5)*9 + 7
62	2	9*5 + 17
63	3	(1**5)*7*9
64	1	79 − 15
65	2	57 − 1 + 9
66	2	57*1 + 9
67	2	51 + 7 + 9
68	2	(9 − 5)*17
69	3	7*9 + 1 + 5
70	3	(1**5 + 9)*7
71		
72	2	(15 − 7)*9
73	2	79 − 1 − 5
74	2	(79 − 5)*1
75	2	71 − 5 + 9
76	1	17 + 59
77	3	(7 + 1)*9 + 5
78	1	95 − 17
79	2	91 − 5 − 7
80	2	1**5 + 79
81	3	(7 + 9)*5 + 1
82	1	97 − 15
83	2	75 − 1 + 9
84	2	75*1 + 9
85	2	71 + 5 + 9
86		
	2	
88	2	(95 − 7)*1
89	2	91 − 7 + 5
90		
91	2	97 − 1 − 5
92	2	(97 − 5)*1
93	2	91 − 5 + 7
94	1	15 + 79
95	2	(1**7)*95
96	2	15*7 − 9
97	2	(1**5)*97
98	2	(19 − 5)*7
99	3	(9 + 5)*7 + 1
100	3	(1 + 9)**(7 − 5)

Contributions were also received from Richard I. Hess, Edward Friedman, Jim Stuart (my old roommate, Tulsa?), R. Robinson Rowe, Gerald Blum, Harry Zaremba, Craig Presson, William E. Peck, B. W. Letourneau, and Harvey Goldman.

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Norman

Continued from p. 6

for setting broad national energy policy, while E.R.D.A. and the Federal Energy Administration are charged with the task of putting policy into practice. And many other units in the federal government — for instance, the Departments of Transportation and the Interior, the Environmental Protection Agency, the Nuclear Regulatory Commission, the Federal Power Commission, and the Council on Environmental Policy — also have their fingers in the pie.

But the departure of Rogers C. B. Morton as Commerce Secretary and Chairman of the Energy Resources Council has again left a gap at the top of the energy policy apparatus. And with the Administration and Congress increasingly at loggerheads on virtually every aspect of energy policy — not to mention the fact that congressional committee jurisdictions are so hopelessly confused that there's no unit taking a broad legislative view of energy matters — a coherent energy strategy has yet to emerge.

In 1974, when the bill establishing E.R.D.A. was debated in Congress, a few sage observers argued that a much more ambitious reorganization of the federal bureaucracy is needed to cope with energy policy. Representative Mike McCormack (D.-Wash.) observed that simply setting up an energy hardware development agency would leave too many isolated units with a share in energy policy, and that continued confusion could result. He may well be proven correct.

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