Thousands of Rocks and One Pirate

Puzzle Corner
by
Allan J. Gottlieb

Our first yearly problem, Y1975 (*see January, p. 62*) has evoked some interest and some questions, and a few additional comments are in order. First, I do *not* guarantee that there is a full solution; that is, I do not know that there exists an expression according to the rules of the problem for each integer between 1 and 100. Second, I will follow the suggestion of several readers that if two expressions for the same integer yield the same point value and one uses 1975 in the proper order, that one will be selected. Third, decimal points are *not* allowed. Fourth, the asterisk (*) denotes multiplication (standard in computer programming). Parentheses do not denote multiplication; they are only to be used to indicate the order in which operations are to be performed (that is why they are not assigned any point value). Thus 1 + 9 + (7) (5) is illegal and should be written 1 + 9 + 7 * 5 or 1 + 9 + (7 * 5). If parentheses are not used the "normal order of operations" applies: all ** are done first (from *right to left*), then all * and / (from *left to right*), and finally all + and − (from *left to right*).

Since these "few additional comments" have rambled on to such a length, let me simply close this introduction by assuring T. Schaeffer that I am no relation to the pinball magnate who has become rich off his habit; and by reporting that a further solution to O/N1 will appear in June (*see February, p. 62*).

Problems

MAY1 We start with a bridge problem from Michael Kay. After years of proliferation, these problems are suddenly in short supply.

	North	
	♠ A J 5 3 2 ♥ 3 2 ♦ 10 8 ♣ A K 8 5	
♠ Q 10 9 6 ♥ K J 9 7 5 ♦ K 7 3 ♣ 9		♠ 8 7 ♥ 10 4 ♦ 6 4 ♣ Q J 10 7 4 3 2
	♠ K 4 ♥ A Q 8 6 ♦ A Q J 9 5 2 ♣ 6	

South dealt with neither side vulnerable. North-South use Blackwood, and the bidding was:

S	W	N	E
1D	1H	1S	P
3D	P	4D	P
4NT	P	5H	P
6D	P	P	P

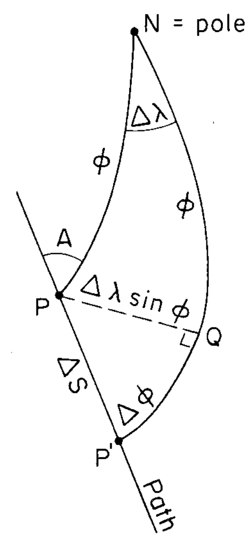
You are to use the bidding (and not the East-West hands) to guide you to South's winning play.

MAY2 Eric Jamin poses the following geometry problem: Given three lengths a, b, and c ($a < b + c$, $b < a + c$, and $c < a + b$), find the side of an equilateral triangle, inside which a point joins the three vertices with distances a, b, and c. (Hint: a geometric solution exists, with only six lines to draw.)

MAY3 Winthrop Leeds wants you to show that for any integer $A > 2$, there exist integers B and C such that $A^2 + B^2 = C^2$.

MAY4 The following problem, entitled "A Rhumb Line Flight," is from R. Robinson Rowe; he describes it as "an exercise in geodesy, on the rhumb line or loxodrome (from the Greek *loxo* meaning 'oblique' and *dromos* meaning 'running' — thus running oblique). As used in navigation, it was very convenient: draw a straight line on a Mercator map from origin to destination and determine a constant azimuth for the entire trip; it isn't much longer than a great-circle course for short voyages or small azimuths and is lots easier to steer." The problem:

— Starting at zero-zero latitude and longitude at 12:00 M on Sunday, Aaron Ott flew his plane at a constant 225 knots loxodromically North 60° West. Where was he at 12:00 M on Monday? Mr. Rowe adds three notes: A ± in the differential equation makes it adaptable to all directions in both hemispheres; here one would use the minus sign because longitude is increasing and colatitude decreasing. A knot is a convenient unit in navigation, being one nautical mile per hour, or one minute of arc per hour; thus 60 knots is one degree per hour and 225 knots is 225/60 = 3.75 degrees of arc per hour. The following differential diagram in spherical trigonometry illustrates the derivation of the general differential equation:



In right triangle PP′Q:

$\tan A = PQ/P'Q = (\Delta\lambda \sin \phi)/\Delta\phi = \text{constant}$

$\sec A = PP'/P'Q = \Delta S/\Delta\phi.$

At limit:

$d\lambda = \tan A \cdot d\phi/\sin \phi$

$dS = \sec A \cdot d\phi.$

MAY5 We close with the following problem from Karl Kadzelski: A band of pirates was chased, and one was caught. A search of the pirate was made, and a description of the location of buried treasure was found; it read: "From the great tree are nine rock formations. Counting from left to right turn around at the ninth rock counting the eighth as ten then again turning around at the first rock counting it as the seventeenth, the second rock the eighteenth, etc. When the number 1,000 is reached, the treasure is buried five paces north of this rock." One of the natives read this description and immediately figured out where the treasure was located without going through all the steps. What formula did he use? And near what rock was the treasure buried?

Speed Department

MAY SD1 Jack Parsons asks:

If eight spades are held by your opponents in bridge (hopefully, spades are not trump), what is the probability of the most probable split?

MAY SD2 John Sowa submitted the following:

You are taking the Graduate Record Examination, a test with multiple-choice questions. The next problem is to evaluate

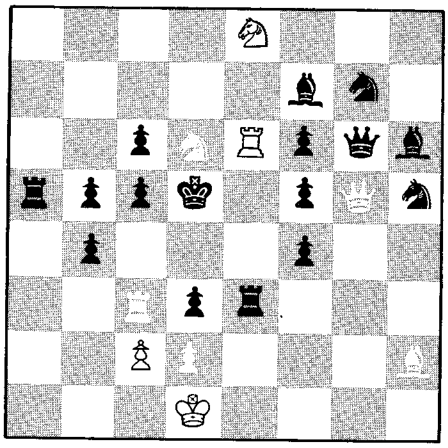
$$\int_0^{\pi} \cos^2 x \, dx$$

You are under severe time pressure. Your mind goes blank, and the only trigonometric identity you remember is $\sin^2 x - \cos^2 x = 1$. But that is all you need! The answer is obvious; without further calculation you select the correct answer. How?

Solutions

The following are solutions to problems published in January:

JAN1 White to play to win:



Although the following (from T. Schaeffer) is not as pretty as some solutions, it is "better" by virtue of using the minimum number of moves, all forced:

1	N — QB7 ck	K — Q5
2	R x P ck	R x R
3	B — N1 ck	R — K6
4	B x R ck	P x B
5	Q x P ck	B x Q
6	P x B mate	

Also solved by H. J. de Garcia, Jr., and son Mark, Richard Hess, Michael Laufer, Winthrop Leeds, Michael Middlebrooke, Ron Moore, Paul Reeves, Frank Rubin, Stephen Strauss, Jerome Taylor, and S. J. Zaroodny.

JAN2 Prove that among triangles of a given perimeter the equilateral has maximal area.

The key is to use the semiperimeter, s = Perim/2. The following is from Avi Ornstein:

Given a triangle with perimeter P, let P = 2s. If the sides are expressed as a, b, and c, then the area, A, can be expressed as

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

If it is an equilateral triangle, $a = b = c$. Assume, however, that $c = P/3$ but $a \neq b$. Then $a + b = 2c$. In finding A, the difference from an equilateral triangle depends on $(s - a)(s - b)$ compared to $(s - c)(s - c)$; this is equivalent to comparing $s^2 - as - bs + ab$ with $s^2 - 2cs + c^2$, and $s^2 - s(a + b) + ab$ with $s^2 - s(2c) + c^2$. Since $a + b = 2c$, the difference in A can be reduced to comparing ab to c^2. Since $a + b = 2c$ (again), take a real number x, such that $a = c + x$ and $b = c - x$. Subsituting these values into ab, one gets $(c + x)(c - x)$, or $c^2 - x^2$. Whatever the value of x, $ab < c^2$. Thus the triangle with sides a, b, and c has an area smaller than that of the equilateral triangle. What about the case of a triangle abd, where $a + b + d = P$ and none equals c? Compare this to a triangle efd, where $a + b = e + f$. Let a and e be the larger number in each pair, and let a and b be closer to one another, which means that $a - b$ is less than $e - f$. Then by using a positive number x, $e = a + x$ and $f = b - x$. In comparing areas, the varying factors are $(s - a)(s - b)$ and $(s - e)(s - f)$ which can be expressed as $s^2 - s(a + b) + ab$ and $s^2 - s(e + f) + ef$, respectively. Since $a + b = e + f$, the differences in areas vary by ab and ef. But $ef = (a + x)(b - x) = ab - ax + bx - xx$. Since $a > b$, $ef < ab$. Thus the area of abd is greater than that of efd. This shows that the closer the values of the sides of a triangle, the greater the area for a given perimeter. Hence an equilateral triangle has the maximum size for P.

Also solved by Gerald Blum, Winslow Hartford, Richard Hess, Ken Kahn, Jack Parsons, John Prussing, Paul Reeves, R. Robinson Rowe, Frank Rubin, Les Servi (the proposer), Dave Taenzer, Smith D. Turner, and Harry Zaremba.

JAN3 What is the maximum time a truly parabolic comet can remain inside the earth's orbit?

I wondered why I found this difficult, but now I know! Knowledge of astronomy is helpful; solutions contained references to Lambert's theorem or equations, neither of which is in Gamow's *Matter, Earth and Sky* (my total "astronomical" knowledge). The following is from Richard Hess:

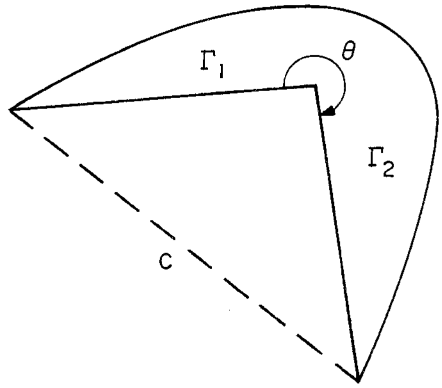
Lambert's theorem gives the time to traverse a parabolic orbit as:

$$t = u^{-1/2}\, 2^{1/2}[s^{3/2} + (s - c)^{3/2}]/3 \quad (\theta \geq 180°)$$

$$t = u^{1/2}\, 2^{1/2}[s^{3/2} - (s - c)^{3/2}]/3 \quad (\theta \leq 180°)$$

where u is the force constant and $s = (r_1 + r_2 + C)/2$. (See diagram at the top of the following column.)

The time expression is maximum when C



is maximum. For a parabolic orbit within the earth's orbit, $r_1 = r_2 = r$ and for maximum time

$$C = 2r \Rightarrow t_{max} = u^{-1/2} 2^{1/2} (2r)^{3/2}/3 = 4r^{3/2}/3u^{1/2}.$$

The period of the earth around the sun is

$$p = 2\pi u^{-1/2}\, r^{3/2} = 1 \text{ year} \Rightarrow r^{3/2} = 1/2\pi \text{ years} \Rightarrow$$

$$t_{max} = 2/3\pi \text{ years} \approx 0.212207 \text{ years} \approx 77.5 \text{ days}.$$

Also solved by Winslow Hartford, John Prussing, and R. Robinson Rowe.

JAN4 Find all x such that $x^x = i$.

Another tough one, but at least it's "only" mathematics. The following is courtesy of Robert Pogoff:

x is complex. Therefore, let $x = a + ib = c \cdot e^{i\theta}$ where $c = \sqrt{a^2 + b^2}$ and $\theta = \arctan(b/a)$. Then follows the development in the box on the next page. Thereafter, Mr. Pogoff proceeds: For $m = 0$, first approximations, a_0 and b_0 are made for a and b respectively. Let $a_0 = 1$, $b_0 = 1$. Closer approximations are:

$$a_1 = a_0 - Y_0/Y'_0$$

$$b_1 = b_0 - Z_0/Z'_0.$$

Substitute these values in equations (12), (13), (14), and (15) to obtain closer approximations:

$$a_{p-1} = a_p - Y_p/Y'_p$$

$$b_{p-1} = b_p - Z_p/Z'_p.$$

Continue until differences $|a_{p-1} - a_p|$ and $|b_{p-1} - b_p|$ are small enough. Repeat the procedure for $m = \pm 1, \pm 2, \pm 3, \ldots$ Note, also, that by a similar procedure, solutions can be obtained for

$$x^x = -i$$

$$x^x = 1$$

$$x^x = -1$$

where the angle in the right hand side of equations (5), (7), (9), and (14) is respectively, $(4m + 3)\pi/2$, $4m\pi/2 = 2m\pi$, and $(4m + 2)\pi/2 = (2m + 1)\pi$. Finding the solutions to x^x = any complex unity vec-

2239	57947	31817	40039	11503	55819	20593	40763
10663	54979	21433	41603	3079	58787	30977	39199
19333	42863	11923	52879	29717	41299	5179	57527
30557	42139	4339	56687	18493	42023	12763	53719
40879	29297	57107	4759	43283	19753	53299	12343
42443	18913	54139	13183	41719	30137	56267	3919
55399	11083	40343	20173	58367	2659	40459	32237
59207	3499	39619	31397	54559	10243	41183	21013

Development of JAN4 Solution by Robert Pogoff (see page 67)

$$x^x = (c \cdot e^{i\theta})^{a+ib} = e^{(\ln c + i\theta)\cdot(a+ib)}$$

$$x^x = e^{(a \ln c - b\theta) + i(a\theta + b \ln c)} = e^{(a \ln c - b\theta)} \cdot e^{i(a\theta + b \ln c)}$$

$$x^x = e^{(a \ln c - b\theta)} \cos(a\theta + b \ln c) + i\, e^{(a \ln c - b\theta)} \sin(a\theta + b \ln c) \quad (1)$$

But $x^x = i$. Therefore

$$e^{(a \ln c - b\theta)} \cos(a\theta + b \ln c) = 0 \quad (2)$$

$$e^{(a \ln c - b\theta)} \sin(a\theta + b \ln c) = 1 \quad (3)$$

From equation (2),

$$\cos(a\theta + b \ln c) = 0$$

$$a\theta + b \ln c = (2n+1)\pi/2 \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \quad (4)$$

$$\sin(a\theta + b \ln c) = \sin[(2n+1)\pi/2] = \pm 1.$$

However, the negative sign does not satisfy equation (3). Therefore n must be even. Let $n = 2m$. Then

$$\sin(a\theta + b \ln c) = \sin[(4m+1)\pi/2] = 1 \quad (5)$$

$$e^{(a \ln c - b\theta)} = 1$$

$$a \ln c - b\theta = 0 \quad (6)$$

and, from equation (4),

$$a\theta + b \ln c = (4m+1)\pi/2.\ m = 0, \pm 1, \pm 2, \pm 3, \ldots \quad (7)$$

Substituting for c and θ, in equations (6) and (7),

$$a \ln(a^2 + b^2)/2 - b \arctan(b/a) = 0 \quad (8)$$

$$a \arctan(b/a) + b \ln(a^2 + b^2)/2 = (4m+1)\pi/2. \quad (9)$$

Solutions to the pair of equations (8) and (9) can be obtained by a version of the Newton-Raphson numerical method. Differentiate (8) with respect to b; $a' = da/db$:

$$a/2\,(2aa' + 2b)/(a^2 + b^2) + a' \ln(a^2 + b^2)/2 = (ba - b^2a')/(a^2 + b^2) + \arctan b/a.$$

Therefore,

$$da/db = a' = \frac{-\arctan b/a}{1 - [\ln(a^2 - b^2)]/2} \quad (10)$$

Differentiate (9) with respect to a; $b' = db/da$:

$$(a^2b' + b^2b')/(a^2 + b^2) + \arctan(b/a) + b' \ln(a^2 + b^2)/2 = 0.$$

Therefore

$$db/da = b' = \frac{-\arctan(b/a)}{1 + \ln(a^2 + b^2)/2} \quad (11)$$

Let

$$Y = a \ln(a^2 + b^2)/2 - b \arctan(b/a); \quad (12)$$

then

$$dY/da = Y' = 1 + \ln(a^2 + b^2)/2 - b' \arctan(b/a).$$

Substituting for b′ from (11)

$$Y' = 1 + \ln(a^2 + b^2)/2 - \frac{[\arctan(b/a)]}{1 + \ln(a^2 + b^2)/2}. \quad (13)$$

Let

$$Z = a \arctan(b/a) + b \ln(a^2 + b^2)/2 - (4m+1)\pi/2 \quad (14)$$

$$dZ/db = Z' = 1 + \ln(a^2 + b^2)/2 + a' \arctan(b/a).$$

Substituting for a′ from (10),

$$Z' = 1 + \ln(a^2 + b^2)/2 + \frac{[\arctan(b/a)]^2}{1 + \ln(a^2 + b^2)/2} \quad (15)$$

tor is similarly solved by substituting the corresponding angle in those equations.

Mr. Pogoff then lists solutions for a, b, and x^x for values of k ranging from −79 to 121; the list is too long for publication here, but readers may obtain it by writing to the Editors of the *Review* at Room E19-430, M.I.T., Cambridge, Mass., 02139.

Also solved by Gerald Blum, Winslow Hartford, Richard Hess, and R. Robinson Rowe.

JAN5 Can you have a magic square with each entry prime? With each entry a distinct prime?

This one is somewhat of a breather after the last two. Roger Milkman submitted over a dozen solutions; one is

73	349	157
277	193	109
229	37	313

The proposer claims his seven-year-old daughter, after being told the definitions, gave the following solution. Personally, I suspect the only seven-year-olds who could generate such a solution come from I.B.M., etc.; the solution appears in the box at the bottom of this page.

Also solved by Gerald Blum, Loren Dickerson, Emmit Duffy, John Feldman, Mrs. Leonard Fenocketti, Richard Hess, Avi Ornstein, Walter Penney, R. Robinson Rowe, and Harry Zaremba.

Better Late Than Never

1973 J/A3 Frank Rubin has explained why integers appeared.

1974 MAY5 Howard Ostar has reponded

1975 O/N1 Winthrop Leeds and William Ackerman have responded.

1975 O/N2, O/N3 William Ackerman has responded.

1975 O/N4 Dean Worcester, Daniel Pratt, and Irving Hopkins have responded.

1975 O/N5 William Ackerman has responded with a very detailed and beautifully organized solution which should be published if space permitted; note that no solution has previously appeared. Copies of Mr. Ackerman's solution can be had from the Editors at Room E19-430, M.I.T., Cambridge, Mass., 02139. (An anonymous reader has also responded.)

DEC1 Responses have come from Emmet Duffy, Joseph Evans, Winslow Hartford, Richard Hess, Fred Price, Ben Roberts, and Stephen Strauss.

DEC2 Responses have come from Emmet Duffy, Winslow Hartford, Richard Hess, Paul Reeves, and Frank Rubin.

DEC3 Responses have come from Michael Goldberg, Winslow Hartford, Richard Hess, Irving Hopkins, Robert Lutton, Robert Pogoff, Paul Reeves, and Frank Rubin.

DEC4 Responses have come from Winslow Hartford, Richard Hess, Craig Presson, and Frank Rubin.

DEC5 Several readers obtained methods requiring more fuel than the printed solution; others used the same method and obtained the same answer. These responses came from Christopher Brooks, Thomas Collins, Gregory Dorner, Emmet Duffy, Richard Hess, Robert Pogoff, Paul Reeves, Frank Rubin, and Stephen

Strauss. Robert Lutton has used a different method and obtained a better result — once more a solution for which there is no space. A copy can be had from the Editors, Room E19-430, M.I.T.

Proposers' Solution to Speed Problems

MAYSD1 Not 4-4 but 5-3, for which the probability is 7/16.

MAY SD2 The curve for $\cos^2 x$ is the same as the curve for $\sin^2 x$ displaced by $\pi/2$. Over the interval $(0,\pi)$, the areas under the two curves are equal. Therefore,

$$\int_0^\pi \cos^2 x\,dx = \tfrac{1}{2}\int_0^\pi (\sin^2 x + \cos^2 x)dx = \tfrac{1}{2}\int_0^\pi dx = \pi/2 .$$

Allan J. Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Assistant Professor of Mathematics at York College of C.U.N.Y. Send problems, solutions, and comments to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y. 11432.

Nisbet

Continued from p. 9

Such a question is in fact difficult to answer without making subjective extrapolations. The levels of these chemicals in remote areas are relatively low; most of the effects likely to take place are sublethal, often subtle functional changes; uncontaminated populations are not available for comparison. Although a number of effects have been reported which are more or less plausible, almost the only well-documented ones are effects on the reproduction of birds. These will be discussed in the next issue.

Ian C. T. Nisbet, who writes regularly for Technology Review, *is a member of the Scientific Staff of the Massachusetts Audubon Society. His Ph.D. (in physics) is from Cambridge University.*

Boulding

Continued from p. 12

of whether income from pure ownership is or is not a reward for some "function." The really significant problem, however, revolves around the syndromes of centralization and decentralization. If we look upon private property as the price we must pay for decentralization of decision-making — and hence for the relative security of an ecological rather than an organismic system — the whole debate looks very different. One could offer many other examples, and a serious attempt to widen agendas in this debate could hardly fail to be productive.

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Book Review

Continued from p. 17

own. But none of these can match the power of the public's collective eye and its visual consensus."

Rolf Jensen in *Cities of Vision,* while agreeing that visual communication is primary, takes the opposite view: "Unfortunately, it is the lot of . . . nearly all planners to be continually confronted with situations in which, in spite of their experience . . . , a lay committee or the public as a whole profess to know better and as frequently insist on pressing their individual and collective points of view against sound advice." And in one of the most interesting criticisms of Lewis Mumford on record, referring to some Mumford Senate testimony: "He claimed that he saw his vital role as one of knocking heads of specialists together, and taking a lofty overall view of their activities which they were incapable of doing themselves. Coming from a layman who does not appear ever to have been involved professionally in city planning, this damaging and presumptious assertion seems to stem from what might have been hoped was the long-dead fallacy of equating specialization with narrowness. The statement was made, as might be expected, without any suggestion of evidence to support the belief that the trained professional expert was a fool who needed to be taught his business by the bystander."

People versus Professionals

Thus, one of the major (largely unspoken) issues in today's urban planning scene is confronted by two very able, experienced, and articulate men. Mr. Clay is a respected urban journalist, editor of *Landscape Architecture Quarterly,* formerly real estate editor of the *Louisville Courier–Journal* and President of the American Society of Planning Officials. Professor Jensen has had 40 years of varied experience in architecture and planning in Britain, Southeast Asia, and Australia; he is Dean of the Faculty of Architecture and Town Planning at the University of Adelaide. *Close-Up* deals with observations and trends, developing new, more accessible phrases for established urban design terms. *Cities of Vision* is a more conceptual and historical presentation, devoted to more professional concerns.

Clay and Jensen are both critical of computerized systems and systems analysis. Clay supports Harvard Professor Alan F. Westin's conclusion that "access to expensive computer systems . . . has turned into 'a factor in consolidating rather than in redistributing government power,' and that access to this new source of power remains so expensive that 'the poor, the black . . . cannot harness computers to their causes.' The gap between the power of experts to manage data and that of the ordinary citizen to have access to it must be narrowed."

While not totally discounting these methodologies, Jensen thinks systems analysis "must inevitably fall far short of the essential creative act required to produce a humane environment. It was neither the plumbers . . . nor the economists who provided the physical entity of the traditional city we admire. . . . The cities of vision . . . thus become the creative masterpieces of enlightened individuals . . ."

Of course, it is clear to planners that no present information system has been adequately coupled with an enlightened individual and or a citizen-interaction process at the city level over a significant period of time. Certainly there is potential here requiring strong governmental leadership and sustenance.

A compromise is perhaps our best course: it is desirable that urban visual values for use in design should transcend, but not ignore, the common denominator of any present transitory population. Human and other resources will be protected if city designers value the *wants* of their contemporaries, the *needs* for their social development, and the *goals* for achieving the best possible society in the future.

"Wants" refers to the routine expressions of individual desire by citizens who have not been exposed to a significant range of options. "Needs" are those things that can be determined through a systematic application of current facts and evaluation systems. "Goals," inherently dealing with values, must be imaginatively developed and constantly and explicitly referenced to the most desirable faces of a changing society.

Clearly, *Close-Up* is devoted to retrieval of and concern for the wants of the current population, and *Cities of Vision* is most substantially concerned with needs and goals. Twentieth-century urban planning has given most weight to our needs, with oscillations toward goals in the first quarter and toward wants in the last. Environmental protection and energy conservation are major forces currently swinging the pendulum again toward goals. The peruser of these books will find himself asking many questions: the relative weights of current needs of individuals, needs of society in general, and goals for future developments; the appropriateness of the valuations given these in past and current decisionmaking processes; and the needs for improvement in the immediate future.

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