How Much Luck in the First Transfusion?

Puzzle Corner
by
Allan J. Gottlieb

I have received several letters concerning my question in the January issue about games of strategy, particularly RISK. David Rosenthal sent a partial computer analysis of RISK. In addition, I have been sent some hints on games in general which I would like to pass along. Liz and Neil Doppelt recommend BROKER and a card game called "Six-Pack Bezique" (Winston Churchill's favorite). Finally, Tom Stockfish suggests writing to Simulation Publications, Inc. (44 E. 23rd St., New York, N. Y., 10010) for a catalogue of their many strategic games.

Problems

J/A 1 The following problem on the subject of "keyword bridge" is from Walter F. Penny:

A scrambled sequence of letters is formed by choosing a keyword (containing no repeated letters) and writing the remainder of the alphabet in order after it. The letters of this sequence are written as capitals on the 26 cards ♠A to ♥2. The letters of another sequence (based on a different keyword) are written as small letters on the 26 cards ♦A to ♣2. A Bridge game, contract 4♥ by S., down one, might be written as shown below.

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W N E S W N E S W N E S W N E S W N E S W N E
d s c g
  G D R O
        p z n m
              H X V Y
              T E C M
                    h w u q
                    x k N l
                        y o e a
                          S U J L
                              F Z P B
                                  f r W i
                                      K b t v
                                      Q A J l
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What are the keywords?

J/A 2 While working on an old problem, $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$, Neil Judell has come up with the following variant: For positive x, let $y_1 = x$, $y_2 = x^x$, and in general $y_n = x^{y_{n-1}}$. Now let $Z_n = \lim_{x \searrow 0} y_n$. In terms of n, what is Z_n?

J/A 3 The following was submitted by Joseph Horton, who notes that the first reported blood transfusion was performed before the nature of blood typing and matching was known; therefore, it was fortunate that luck was on the patient's side. The problem: What was the probability of a successful transfusion (one in which no adverse reaction occurred among major factors)? You need to know:

		Donor			
	Type	**A**	**O**	**B**	**AB**
Recipient	**A**	ok	ok	x	x
	O	x	ok	x	x
	B	x	ok	ok	x
	AB	ok	ok	ok	ok

		Donor	
	rh	**+**	**−**
Recipient	+	ok	ok
	−	x	ok

The distribution of blood types in the population is approximately as follows: A—40 per cent, O—40 per cent, B—15 percent; and AB—5 percent; rh+ —85 percent and rh− —15 per cent. Types and rh's are randomly mixed—that is, 85 per cent of each type is rh+ and 15 per cent is rh−. (My wife Alice, budding immunologist, says that the above is only a rough approximation and that in reality there are many other factors besides type and rh; but for the sake of the problem, ignore her complications.)

J/A 4 Here is a problem from John G. Connine, who describes "a lesser known game of solitaire played here in the snowbelt during long winter nights" (it is also played here in the pollution belt). A standard deck of 52 cards in shuffled and placed face down upon the table. The cards are then turned face up one at a time by flipping over the top card of the face-down stack. As this is done, the player simultaneously calls out the sequence A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A, 2, etc., one call being made for each card flipped over. To win the game, one must go through the deck without matching a card flipped over with the card called. Suits don't matter, so, for example, any 4-spot flipped over on the 4th, 17th, 30th, or 43rd turn results in a loss. "Since winter will surely come again," Mr. Connine would like to know what are the chances of winning the game. How about a second solution for the same game with a 48-card pinnochle deck?

J/A 5 Frank Rubin wants you to place six points inside a unit square so that the nearest two are as far apart as possible.

Speed Department

SPEED 1 The following is from Dick Boyd: Larry and Bill enjoy hot coffee with cream. What strategy should they use in adding the cream to keep the coffee hottest longest?

SPEED 2 Mary Lindenberg asks the following real estate problem: A realtor bought a house for $16,000, sold it for $17,000, bought it back for $18,000, and resold it for $19,000. How much did the dealer make or lose?

Solutions

Following are solutions to problems published in the March/April issue:

M/A 1 On a usual chessboard, define a new piece, the kknight, by letting its move be three up and two over or two up and three over (also allowing down for up, of course). Can you devise a kknight tour—that is, find a series of kknight moves such that each of the 64 squares is landed on exactly once?

This problem requires the touch of a combinatorist for its solution. Since this is one of Frank Rubin's primary interests, it is not too surprising that he did not have too much trouble: No kknight's tour is possible. Notice that at the 16 encircled cells there are exactly two moves available. Either *both* of these moves are taken in the tour, or that cell must be a terminal. Next note that the two moves at 10 and 55 both end at 29 and 36. Either the tour contains the closed circuit 10-29-55-36-10, which is impossible, or one end of the tour is a 10 or 55. The same reasoning shows that the other end of the tour must be at 15 or 50. Since both ends of the tour have now been located, all

(1)	(2)	3	4	5	6	(7)	(8)
(9)	(10)	11	12	13	14	(15)	(16)
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
(49)	(50)	51	52	53	54	(55)	(56)
(57)	(58)	59	60	61	62	(63)	(64)

of the other moves at the circled cells must now be in the tour. But these moves include a closed circuit 1-20-7-29-16-38-64-45-58-36-49-27-1. Since a tour cannot visit the same square twice, no tour is possible.

For more details the reader is urged to see an article of Dr. Rubin's to appear this fall in the *Journal of the Association for Computing Machinery*.

Also solved by R. Robinson Rowe.

M/A 2 Find a closed form for $1^1 + 2^2 + 3^3 + \ldots + n^n$.

Judith Q. Longyear points out that no closed form is known and cites as a reference p. 471 of Vol. 53 of *American Mathematical Monthly*. In this article it is shown that (let $f(n) = 1 + 2^2 + \ldots n^n$) $n^n\left(1 + \frac{1}{4(n-1)}\right) < f(n) < n^n\left(1 + \frac{2}{e(n-1)}\right)$.

Richard T. Bumbry fiddled around with $f(n) \bmod q$ for various q's and has some evidence to suggest that no closed form is possible.

M/A 3 Sometime in the morning it began to snow, and the snow continued at a constant rate all afternoon. A snowplow, which moves a constant volume of snow per unit of time, traveled twice as far between noon and 1 p.m. as it did between 1 p.m. and 2 p.m. When did it begin to snow?

This problem was somewhat more subtle than first appears. Some of the giants of puzzledom (at least of Puzzle Cornerdom) erred by assuming that there was twice as much snow on the ground at 1:30 as at 12:30. This, however, is incorrect. As the following solution (from Robert Pokoff) indicates, we must integrate the inverse proportionality:

Using C's as constants, let:
volume of snow plowed $= V = C_1 t$,
depth of snow $= h = C_2 t$,
Width of plow $= w$, and
distance plowed $= x$.
Then $dV = hwdx = C_2 t\, wdx$.
Then $dV/dt = C_2 w\, t(dx/dt)$.
But, from (1), $dV/dt = C_1$.
Therefore,
$C_2 w\, t(dx/dt) = C_1$,
$dx = C\,(dt/t)$
$x = \int(dt/t)$
If T is the interval in hours from the time the snow started until noon, and x_1 and x_2 are the distances plowed between noon and 1 p.m. and between 1 and 2 p.m., respectively, then

$$x_1 = C\int_T^{T+1}(dt/t) = C\ln[(T+1)/T]$$

$$x_2 = C\int_T^{T+2}(dt/t) = C\ln[(T+2)/(T+1)]$$

But $x_1 = 2x_2$; therefore

$$\ln[(T+1)/T] = 2\ln[(T+2)/(T+1)] = \ln[(T+2)/(T+1)]^2.$$

Therefore $(T+1)^3 = T(T+2)^2$,

$T^2 + T - 1 = 0$, and $T = 0.618$ hours before noon, or 11:22:55.

Responses were also received from R. Robinson Rowe, Avi Ornstein, John E. Prussing, Frank Rubin, Winslow H. Hartford, David Geisler, Ted Mita, Harry Zaremba, Jack Parsons, and the proposor, Doug Hoylman.

M/A 4 This problem was revised in June, and its solution will appear with the solutions to the June problems.

M/A 5 In each of the 16 equal squares shown place a different letter of the alphabet in such order that they will correctly spell eight *different* four-letter words, one word in each of the four horizontal rows (reading from left to right) and at the same time one word in each of the four vertical columns (reading from top to bottom), making a total of eight *different* four-letter words possible. Do not use plurals or proper nouns. All words must be defined in any one dictionary of your choice. How many words can you get?

Only George H. Lopes was able to give a complete solution. All other solutions used the same letter more than once or did not have eight words. In fact, Mr. Lopes asks for extra credit since his solution has nine words (fiat diagonally upward):

CYST
OPAH
RIMU
FLED

All of these words appear in Webster's *New International* (2nd ed.).

Other respondents were Winslow H. Hartford, Frank Rubin, and Jim Schott.

Better Late than Never

PERM 1 As usual several more replies have been received. However due to the time delay between my deadline and your receiving TR, most of the "new" responses received this month contain material already published in previous issues. The one exception is the following, almost humorous, contribution from Stuart D. Casper. His up arrow is the usual teletype method of signifying exponentiation.

300 $= [[\sqrt{9!}]/[\sqrt{3!}]] \cdot 1^7$
400 $= [\sqrt{9!}/3] \cdot [\sqrt{7-1}]$
500 $= [[\sqrt{9!}]/[\sqrt{3!}]] + [\sqrt{[\sqrt{\sqrt{7!}}]!}] - 1$
600 $= [\sqrt{9!}] - [\sqrt{7}] \cdot 1^3$
700 $= 3!! - 17 - \sqrt{9}$
800 $= 9^3 + [\sqrt{7!}] + 1$
900 $= [\sqrt{\sqrt{\sqrt{([\sqrt{\sqrt{9!}}]!)}}}] - (7.1 \cdot 3!)$
1000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot (9 - 3 - 1)$
2000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot (9 + 1^3)$
3000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot ([\sqrt{\sqrt{(9-1)!}}] + [\sqrt{3}]$
4000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot ([\sqrt{\sqrt{9!}}] - 3 - 1)$
5000 $= 7! - ([\sqrt{[\sqrt{3!!}]}] \cdot (9 - 1))$
6000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot [\sqrt{913}]$
7000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot ([\sqrt{3!!}] + 9^1)$
8000 $= [\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot (([\sqrt{[\sqrt{3!!}]}]!)/\sqrt{9}) \cdot 1$
9000 $= [\sqrt{\sqrt{3!!}}] \cdot ([\sqrt{9!}] - [\sqrt{7}] \cdot 1)$
10000 $= ([\sqrt{([\sqrt{\sqrt{7!}}]!)}] \cdot [\sqrt{\sqrt{\sqrt{\sqrt{9!}}}}]) \cdot ([\sqrt{3!!}] - 1)$
10^5 $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (7 + 1 - \sqrt{9})$
10^6 $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (7 - 1^9)$
10^7 $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (7 \cdot 1^9)$
10^8 $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (7 + 1^9)$
10^9 $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (9 \cdot 1^7)$
10^{10} $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (9 + 1^7)$
10^{20} $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (19 + [\sqrt{\sqrt{7}}])$
10^{30} $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow [\sqrt{917}]$
10^{40} $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{\sqrt{7!}}] \cdot ([\sqrt{\sqrt{\sqrt{9!}}}] + 1))$
10^{50} $= [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{7}] \cdot ([\sqrt{\sqrt{\sqrt{9!}}}] + 1))$

(*Continued on p. 63*)

side the United States the situation is not so promising. National sovereignty, in particular, greatly complicates the solution. For instance, India spent $216 million and five years of research in order to join the nuclear club. This was done without the knowledge of the Canadians, who have now realized that their objectives in aiding India and the Indians' reasons for accepting that aid may have been very different. Recent agreements to export nuclear power technology to Egypt and Iran have intensified concern.

"It is essential that we put our own house in order and demonstrate that we take this matter seriously," Dr. Taylor told me in an interview shortly after publication of the book. "At the same time we must push diplomatically for increased safeguards." Public discussion is essential, thinks Dr. Taylor. He hopes that once the governments of the world realize how easy it may be to make a crude bomb out of materials in their possession, they will institute the safeguards necessary to keep the risks of theft down to an "acceptable level".

But what is acceptable? "To give you a flavor of what I, personally mean," explained Dr. Taylor, "suppose the safety and waste disposal problems were solved and nuclear energy was supplying a sizeable fraction of the world's energy needs. And suppose we could keep terrorists from setting off no more than one small-scale Hiroshima every ten years. I would be hard put to say whether that was acceptable or not."

Is this a risk that we will be forced to accept? It would be truly ironic if all the money the U.S. has spent to justify development of the atomic bomb has made such a world the most probable one.

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Boulding

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of alternative futures. A study of the evaluation or goodness function, therefore, may throw some light on what are the more probable futures. Things which are perceived as "too large" are likely to get smaller and things perceived as "too little" are likely to get bigger. If there is strong divergence of evaluations, however, decisions are likely to be frustrated. Decisions of one person will be offset by decisions of another. This perhaps explains in part what might be called the "paradox of decision"—how is it that when everybody does everything for the best that things so often go from bad to worse?

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Chase

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We should embrace the notion of the "negative income tax," which means family income subsidized, if necessary, to meet a decent standard of living for every American; such a plan should replace the majority of our current relief programs. Even President Nixon has advocated this priority.

We need compulsory arbitration for union contracts in all public services essential to the life of the community—fire, police, sanitation, hospitals, and the like. No more economic blackmail by strikers, any more than by Saudi Arabia. To find the able and fairminded arbitrators needed may not be too difficult after the lessons of Watergate. Ask for a list from Messrs. Cox, Richardson, and Ruckelshaus.

We should establish a high-powered Office of Technology Assessment in the federal government to examine new scientific findings and advise on how far they should go into mass production and mass use. Had such an agency existed in 1916 (the year of the Model T), the internal combustion engine might have been curbed instead of becoming, in the words of one despondent critic, "the greatest disaster to overwhelm the human race since the flood." Such an agency has already been set up in a small way in Washington. The generalist hopes for its development in a big way.

Such an agency could be of great help, too, in countering the growing—and largely ignorant—popular attack on many aspects of technology. To use the semantic approach: technology$_1$ is not technology$_2$—an oxygen tent is not an atomic missile. What kind of "technology" are we talking about? What are the referents?

Any generalist who tries to concentrate on issues can in fact think of many other necessary changes—economic, political, and military—to improve the quality of American and global life. The priorities of this particular generalist as the decade to 1984—that ominous date—begins are those listed above.

How would you amend them?

Stuart Chase, who studied at M.I.T. with the Class of 1910 in the course in general engineering, is a prolific writer on subjects relating to economics, communication, and social affairs. Among his major books are The Proper Study of Mankind, The Tyranny of Words, *and* The Most Probable World.

Puzzle

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$$10^{60} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{7!}] - 9 - 1)$$

$$10^{70} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{7!}] \cdot 1^9)$$

$$10^{80} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{7!}] + 9 + 1)$$

$$10^{90} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{\sqrt{\sqrt{\sqrt{([\sqrt{\sqrt{9!}}]!)}}}}] \cdot ([\sqrt{7}] + 1))$$

$$10^{100} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{[\sqrt{\sqrt{7!}}]!}]/(\sqrt{9} - 1))$$

$$10^{200} = [\sqrt{(\sqrt{[\sqrt{\sqrt{3!!}}]!})}] \uparrow ([\sqrt{[\sqrt{\sqrt{7!}}]!}] \cdot 1^9$$

$$10^{300} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{\sqrt{\sqrt{\sqrt{([\sqrt{\sqrt{9!}}]!)}}}}] \cdot [\sqrt{(([\sqrt{[\sqrt{([\sqrt{\sqrt{[\sqrt{[\sqrt{\sqrt{7!}}]!}]}}]!!)}]}])!)}])$$

$$10^{400} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{[\sqrt{\sqrt{7!}}]!}] \cdot [\sqrt{9 - 1}])$$

$$10^{500} = [\sqrt{(([\sqrt{[\sqrt{((\sqrt{9})!!)}]}])!)}] \uparrow [[\sqrt{[\sqrt{\sqrt{7!}}]!}] \cdot ((\sqrt{\sqrt{\sqrt{[\sqrt{3!!}]}}}) + 1)$$

$$10^{600} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{[\sqrt{\sqrt{7!}}]!}] \cdot (\sqrt{9}) \cdot 1)$$

$$10^{700} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{7!}] \cdot (9 + 1))$$

$$10^{800} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{[\sqrt{\sqrt{7!}}]!}] \cdot [\sqrt{\sqrt{\sqrt{9!}}}] \cdot 1)$$

$$10^{900} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ([\sqrt{\sqrt{\sqrt{\sqrt{([\sqrt{\sqrt{9!}}]!)}}}}] \uparrow [\sqrt{7 - 1}])$$

$$10^{1000} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow ((9 + 1) \uparrow [\sqrt{\sqrt{[\sqrt{([\sqrt{\sqrt{7!}}]!)}]}}])$$

$$10^{10,000} = [\sqrt{([\sqrt{\sqrt{3!!}}]!)}] \uparrow (([\sqrt{(([\sqrt{[\sqrt{([\sqrt{\sqrt{[\sqrt{([\sqrt{\sqrt{7!}}]!)}]}}]!!)}]}])!)}]) \uparrow (1 + \sqrt{9}))$$

$$10^{100,000} = (9 + 1) \uparrow (([\sqrt{(([\sqrt{[\sqrt{([\sqrt{\sqrt{[\sqrt{([\sqrt{\sqrt{7!}}]!)}]}}]!!)}]}])!)}]) \uparrow [\sqrt{\sqrt{3!!}}])$$

$$10^{1,000,000} = (9 + 1) \uparrow (([\sqrt{(([\sqrt{[\sqrt{([\sqrt{\sqrt{[\sqrt{([\sqrt{\sqrt{7!}}]!)}]}}]!!)}]}])!)}]) \uparrow 3$$

As Mr. Casper's solution makes clear, getting really large numbers using the greatest integer function is quite possible. I would really appreciate (in the mathematical sense as well) a rigorous proof of **MAY 2** which asserts as a corollary that **PERM 1** (in the loose sense) is possible for all numbers. A "constructive" proof would, of course, be preferred. I might also point out that **PERM 1** (in the strict sense—no greatest integer) is still quite a challenge. In June I published Eric Jamin's list, which has 20 gaps up to 256, and I have no answers beyond that. So how about 152, 155, etc., without greatest integer?

Responses were also received from Dr. Efrem G. Mallach, Ray Ellis, Tom Davis, Bob McConaughy, Ermanno Signorelli.

Proposor's Solutions to Speed Problems

SPEED 1 Add the cream immediately in order to get the mixture temperature down to reduce the loss of heat from radiation. Radiation heat losses go as the fourth power of temperature differential. Hence the mixture will stay hotter longer if the cream is put in at once than if you wait for the coffee to cool and the cream to warm up before mixing.

SPEED 2 Since the total expenses were $34,000 and the income was $36,000, the profit was $2,000.

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