Kknights and a Snowplow

Puzzle Corner
by
Allan J. Gottlieb

I've recently encountered two reminders of my increasing age. First, I read an article in which the I.B.M. 7094 was called "one of those historic pre-byte computers." But I remember the evolution of the I.B.M. 704 series which culminated in the 7094 (Model 2 if you want to quibble) and quite a few computers before that. Second, Abby Hoffman recently came to speak at York College, and no one in my upper-level mathematics course had heard of him. When I said that he was a leader in the movement, they asked, "What movement? That night I searched for grey hairs.

Much confusion has arisen concerning two chess problems from 1973 issues—M/A 1 and O/N 1. I hope to clear everything up in the "Better Late Than Never" department of this issue and the solution section of next issue. Though chess problems in combined issues seem to fare badly, I'll try again (see below).

Help! Critical shortage of Speed Problems.

Problems

Mark Lorbiecki found this problem in the *1973 Farmer's Almanac:*

M/A 1 On a usual chessboard, define a new piece, the kknight, by letting its move be three up and two over or two up and three over (also allowing down for up, of course). Can you devise a kknight tour—that is, find a series of kknight moves such that each of the 64 squares is landed on exactly once?

A number theory problem from Akbar Ahmed:

M/A 2 Find a closed form for $1^1 + 2^2 + 3^3 + \ldots + n^n$.

This "snowplow" problem is a favorite of Doug Hoylman:

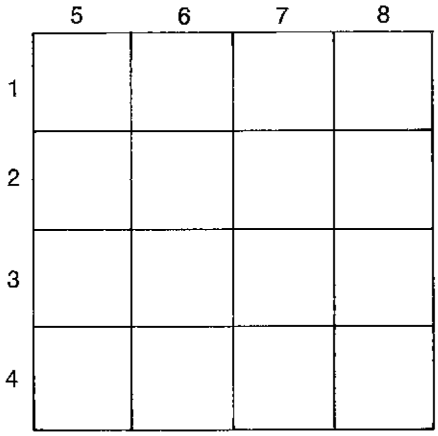
M/A 3 Sometime in the morning it began to snow, and the snow continued at a constant rate all afternoon. A snowplow, which moves a constant volume of snow per unit of time, traveled twice as far between noon and 1 p.m. as it did between 1 p.m. and 2 p.m. When did it begin to snow?

Richard T. Bumbry would like to introduce you to three gods:

M/A 4 The God of Truth and the God of Falsehood are obvious; then there is also the God of Malice, who gives random answers to any question. You are to ask each God one question and determine, from their answers, who is who.

The following is from George L. Uman:

M/A 5 In each of the 16 equal squares



shown place a different letter of the alphabet in such order that they will correctly spell eight *different* four-letter words, one word in each of the four horizontal rows (reading from left to right) and at the same time one word in each of the four vertical columns (reading from top to bottom), making a total of eight *different* four-letter words possible. Do not use plurals or proper nouns. All words must be defined in any one dictionary of your choice. How many words can you get?

Speed Department

Jack Parsons writes:

M/A SD1 A canal is carried across a ravine by means of an aquaduct. The total water load is 4,000 tons. A boat weighting 100 tons is towed slowly across. What is the maximum total load on the aquaduct?

The following is from Charles A. Piper:

M/A SD2 Given a glass of wine and a glass of water, remove a teaspoonful of wine and stir it into the water. Now remove a teaspoonful of this mixture and stir it into the wine. Is there more wine in the water or more water in the wine?

Solutions

The following are solutions to problems published in the December, 1973, issue:

DEC 1 Construct a hand on which North-South can make seven of any suit but cannot make seven no-trump. You are allowed to make up the East and West hands but you must allow for best play by the defenders.

Norman Sleep proposes the following hands:

North: ♠ A K Q J; ♥ A K Q J; ♦ A K Q J 9; ♣ —

West: ♠ 8 7 6; ♥ 8 7 6; ♦ 10 7 6 5 4; ♣ 6 5

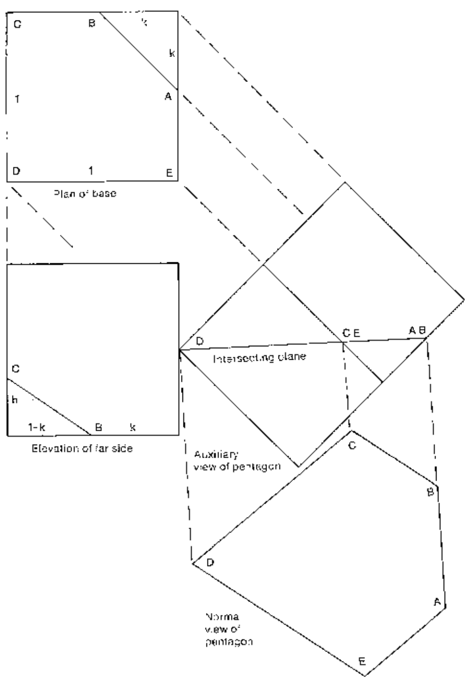
East: ♠ 5 4 3 2; ♥ 5 4 3 2; ♦ 3 2; ♣ 4 3 2

South: ♠ 10 9; ♥ 10 9; ♦ 8; ♣ A K Q J 10 9 8 7

The only difficulty is making seven diamonds against a spade or heart lead. In this case N-S win the first six tricks with three hearts and three spades from North but ruff the sixth in South. Now cash two clubs, discarding North's last spade and heart. Leading a club sets up a winning trump coup.

Also solved by Neil Cohen, Jacques Ludman, Harry Nelson, and the proposer, Charles E. Blair.

DEC 2 A hexahedron has three regular



Note that BC is parallel to DE and AE to CD. Such parallelism must be characteristic of any pentagonal section, and hence none can be a regular pentagon.

sections: an equilateral triangle, a square, and a regular hexagon. Prove that it cannot have a fourth—namely, a regular pentagon.

R. Robinson Rowe found such an easy solution what I am tempted to think this should have been a speed problem. However, few were able to find such an easy solution (and I was not among them); and Mr. Rowe then expands his discussion to include a solution which was not originally required: The simplest proof that it cannot be regular is (1) the plane must intersect exactly five faces of the cube; (2) of these five faces, two pairs are parallel; (3) if a plane intersects two parallel planes, the traces are parallel lines; (4) hence the pentagonal section will have two pairs of parallel sides; but (5) a regular pentagon does not have two parallel sides. Q.E.D.

As a sidelight, Mr. Rowe continues to derive the symmetrical pentagon with three equal sides:

Let the five intersected faces of the cube be the bottom and four sides—that is, an open cubical box. Let the intersecting plane pass through an upper corner D and cut the base in line AB, the hypotenuse of an equilateral triangle with legs k. Let this plane cut the lateral edges at C and E. To make AB = BC = AE, we derive the quartic in k:

$$k^4 - 2k^3 - 6k^2 + 14k - 5 = 0$$

which has four real roots: −2.615, 1.701, 2.457, and 0.45731. Since k must be less than 1, only the last is pertinent, whence AB = BC = AE = 0.64673307. The other two sides of the pentagon are CD = DE = 1.1917.

Also solved by Frank Rubin and Harry Nelson.

DEC 3 A body of N legislators is divided up into various (possibly overlapping) committees. An executive committee is formed consisting of at most one representative of each committee (the same individual will represent every committee of which he is a member). The executive committee is always chosen to be as large as possible without violating the rule that only one member from any other committee may be on the executive committee. How large is the executive committee?

The intent of this problem was to ask for the size of the executive committee for any set of regular committees. Only the proposer interpreted this correctly, and his solution will follow. Gerald Blum and R. Robinson Rowe thought that you were to find the maximum possible executive committee that could occur. They point out that if each of the N legislators is a one-man committee, the executive committee will include all the legislators. Mr. Rowe has included a piece of "Rowe history" which I cannot resist printing:

"Here is an analogous historical fact. My paternal grandfather, William Rowe, was a staunch Republican living in Grand Rapids, Mich., when he, like many other 'carpetbaggers,' sought a fortune by moving into the defeated South. He settled in Judsonville, Ark. About 1880, at the urging of friends, he became a 'hopeless' Republican candidate for the legislature, but by a political miracle he was elected. He was the only Republican in the legislature. Its rules prescribed that each committee include a representative of the minority party, so my grandfather was a member of every committee and the busiest member of the legislature!"

Let the committees be $C_1, C_2, \cdots, C_n$. Let the members of committee C_i be C_{i1}, $C_{i2}, \cdots, C_{im_i}$. Also, let the symbol x_{ij} represent the Boolean condition that member c_{ij} is on the executive committee. Then the condition that the executive committee contains a member of committee C_i is represented by the Boolean expression $X_i = x_{i1} \vee x_{i2} \vee \cdots \vee x_{im_i}$. The condition that the executive committee contains a member of every committee is expressed as the conjunction $X = X \wedge X_2 \wedge \cdots \wedge X_n$.

By putting this expression in disjunctive normal form (ie, multiplying out) all valid executive committees are represented as terms of the expression. (Since no negated variables appear, the disjunctive normal form is unique after absorbtion has been applied, that is $a \vee a \wedge b = a$) The term with the most factors (conjuncts) represents the largest valid committee. It is not, however, necessary to generate all terms to obtain the largest term. At stage k of the expansion, that is $E_k = X \wedge X_2 \wedge \cdots \wedge X_k$, $k < n$, let M_k be the largest number of factors in any term. Then any term with F factors, such that $F + (n - k) \leq M_k$ can be deleted, since it cannot lead to an executive committee with more than M_k members. This substantially reduces the number of terms in the final expression, X.

DEC 4 Each of n men has a preference ordering of n women; each of the n women has a preference ordering of the n men. A marriage is an assignment of the n men to the n women. A marriage is called unstable if some man prefers another woman to his present mate and the preferred woman also prefers this man to her present mate. Can you always find stable marriages?

The following is from Melvin Jameson: Yes. As everyone knows, a stable marriage results from following a proper courtship procedure—for example, the following. (Although the roles of men and women are interchangeable as far as the existence proof is concerned, I have assigned the active role in the courtship to women for the sake of added interest.) Arrange the n women in an arbitrary order and send them out courting one by one. Each begins by approaching the man of her first choice. If he does not already have a partner, or if he prefers her to his current partner, they form an alliance and the old partner, if any, hits the road. If, on the other hand, the man prefers his current partner to her, she must go on to her second choice. She thus proceeds in order of decreasing preference until she finds a man who will have her. If she is the rth woman, this will be at worst the rth man she approaches, as only $(r - 1)$ couples previously existed. Each woman is thus initially paired so that every man with whom she would like to commit instabilities has a partner he prefers to her. If a woman is bumped from a pairing, she looks around for a new mate. The men she prefers to her old partner all rejected her in the first place because each already had someone he liked better; if any has changed in the meantime it can only be for someone with an even higher ranking—our bumped bachelorette cannot look there for companionship. She must continue in order of decreasing preference from her ex-partner. As above, she will end up with at worst her rth choice and have no opportunity for infidelity. Thus a finite number of attempted courtships after the rth woman sallied forth onto the lists of romance, we have r couples such that no woman can find a man she prefers to her partner without a partner he prefers to her. When all n women have had their chances to go a-courting, we will have found, in a finite number of steps, n such couples—which constitutes a stable marriage.

Also solved by Walter Hill, R. Robinson Rowe, Frank Rubin, and the proposer, Hal Varian.

DEC 5 Find a three-by-three array of integers all of whose rows, columns, and diagonals add up to the same number without this number being a multiple of three.

A	B	C
	D	

A	B	C
	D	
A+B −D	A+C −D	B+C −D

The following is from John T. Rule: In any 3×3 magic square, consider the top row and middle square as shown. A, B, C, and D may be the same or different. The sum, S, of every row, column, and diagonal is therefore $S = A + B + C$. Thus we can evaluate the bottom squares as in the right-hand diagram. Since this last row must also sum to $A + B + C$, we obtain $(A + B - D) + (A + C - D) + (B + C - D) = A + B + C$, or $S = A + B + C = 3D$. Hence S is a multiple of 3.

Also solved by Donald Aalkin, Gerald Blum, Frank Evans, Avi Ornstein, R. Robinson Rowe, Frank Rubin, Norman Sleep, and Paul deVegvar.

Better Late Than Never

M/A 1 The history of this is as follows: The problem was first published in March/April, then revised in June. Then in December, 1973, the solution was given along with the other June problems. Unfortunately, the problem was restated in its incorrect March/April form. Even more unfortunately, the analysis given was wrong. Let us have Harry Nelson, the proposer, set us straight once and for all: The analysis given in December is nearly right. At step 8 we find, "Black's KRP was also taken elsewhere." This need not be the case; in fact, Black's *promoted* KRP is the piece at R3. The sequence is as follows: Black's KRP captured White's QB (at Black's square KN4), then proceeded to become a promoted piece on square N8. From the position we know that the promotion was not to a Bishop, since color is wrong; and not to Queen or Rook, since then White's King would be in check. Thus the missing piece is a Knight.

To completely pin things down, I have composed a 27-move game which yields

1	P–Q4	P–KB4	18	K–R2	N–B3
2	B–N5	P–QR3	19	PxN	P–N8(N)
3	P–K3	PxB	20	PxR	B–Q2
4	Q–B3	P–B5	21	PxN	0–0–0
5	Q–B6	PxQ	22	PxQ	K–N1
6	B–R6	P–B6	23	R–N4	N–R3
7	PxP	P–N5	24	R–R1	B–B3
8	N–B3	R–R4	25	R(1)–R4	PxR
9	N–N5	N–B3	26	NxB	R–B1
10	N–KR3	N–K5	27	PxR(Q)	. . .
11	N–N5	Q–Q4			
12	N–R7	PxN	The game probably		
13	P–QR4	PxB	continued		
14	P–R5	Q–N2	27	. . .	KxQ
15	R–R4	P–N6	28	R–N7	K–Q1
16	0–0	P–N7	29	R–N8 mate	
17	P–R4	R–Q4			

the desired position shown at the top of this column.

Also solved by Theodore Edison, Douglas Hoylman, James Kuti, Ted Mita, Arthur Polansky, Frank Rubin, and Norman Sleep.

The following have responded to the problems indicated:

MAY 5 Winthrop Leeds, Frank Rubin

JUN 1 Frank Rubin

JUN 5 Robert Colwell, Frank Rubin

J/A 2 Walter Hill

O/N 2 Gerald Blum, S. Daher, Michael Thelen

O/N 3 Ronald Appel, Gerald Blum, Almerio Amorim Castro, Winston Chan, Peter Grood, Howard Lockwood, Avi Ornstein, Bruce Parke, Larry Rosenblum, Andrew and David Smith

Robert Simon has raised the limit to 148. His examples from 116 through 148 follow:

116 $(3! - 1)! - 7 + \sqrt{9}$
117 $(7 + 1 - 3)! - \sqrt{9}$
118 $(3! - 1)! - 9 + 7$
119 $(3 - 1)^7 - 9$
120 $[(3 \cdot 7) - 9]/0.1$
121 $97 + (3 + 1)!$
122 $(3! - 1)! + 9 - 7$
123 $(7 + 1 - 3)! + \sqrt{9}$
124 $(9 \cdot 13) + 7$
125 $(3 - 1)^7 - \sqrt{9}$
126 $7 \cdot 9 \cdot (3 - 1)$
127 $9/0.1 + 37$
128 $97 + 31$
129 $(7 + 1 - 3)! + 9$
130 $(19 \cdot 7) - 3$
131 $(3 - 1)^7 + \sqrt{9}$
132 $139 - 7$
133 $19 \cdot 7!/(3!)!$
134 $137 - \sqrt{9}$
135 $0.1 \cdot (3!)! + (9 \cdot 7)$
136 $19 \cdot 7 + 3$
137 $(3 - 1)^7 + 9$
138 $(17 + 3)! \sqrt{9}!$
139 $(19 \cdot 7) + 3!$
140 $137 + \sqrt{9}$
141 $[(7 \sqrt{9}!)/0.3] + 1$
142 $71 (\sqrt{9}!/3)$
143 $137 + (\sqrt{9})!$
144 $(3! - 1)! + (7 - \sqrt{9})!$
145 $(7 - 3)! \cdot (\sqrt{9}!) + 1$
146 $137 + 9$
147 $7[(3/0.1) - 9)]$
148 $37(\sqrt{9} + 1)$

ON 4 Gerald Blum, Almerio Amorim Castro, Peter Groot, Joseph Keilin, Avi Ornstein, Bruce Parke, David and Andrew Smith

O/N 5 Gerald Blum, Peter Groot, John Rotramel, David and Andrew Smith

JAN SD1 Donald E. Savage points out that the last line of his solution was not

(Continued on p. 76)

Nisbet

Continued from p. 5

The Stress of Climatic Change

Much of this stress arises from the intense competition between plants for limited resources such as light, water, and nutrients. But, in addition, many long-lived plants such as forest trees may be stressed by climatic factors. We now know that climatic variables such as temperature and rainfall can change markedly in periods of a few centuries or even decades. Yet some trees live for hundreds of years: a tree that starts to grow in an ideal climate and survives the rigorous competition between saplings may live for the rest of its life in a climate to which it is not well adapted. Changes in mean temperature of 1° to 2°F. have been recorded within this century. Yet a difference in mean temperature of only 2°F. is found, for example, between areas in New York state where the "natural" forest trees are gray birches and other northern species, and between parts of North Carolina which sustain shortleaf pines and other southern species.

Although the composition of a forest can be very closely related to patterns of variation in climatic factors such as temperature, exposure, and moisture, it may actually be poorly adjusted to the present-day climate because of its long response time. Stresses resulting from this lack of adjustment may be especially severe now, as a result of the rapid climatic changes in the twentieth century.

Disease and Readjustment

There is much evidence that fungus diseases are important agents leading to readjustment of forests to climate. Some of the most dramatic recorded changes in forest composition, such as the elimination of the American chestnut by blight 50 years ago, and the continuing losses of American elms to Dutch elm disease, have indeed been caused by fungi. Similarly in agriculture, fungus diseases occasionally cause severe damage to selected crop varieties: the Irish potato blight of 1865 is the classic example, and recent outbreaks of wheat rust and corn blight continue the process of selection.

These extremely devastating outbreaks may not be closely related to climate, but other recorded cases are. Many fungus diseases of trees are closely limited by climatic factors such as temperature and moisture; accordingly local outbreaks usually take place in association with unusual weather such as frosts, fogs, heat waves, or droughts. As the climate changes, "unusual" types of weather become more usual and the outbreaks become more frequent. At the same time the trees may be more stressed by the climate and more susceptible to the disease. Fungi are mobile and widespread, so the trees are usually continuously exposed: as with human disease, the trees usually succumb only when unusual conditions erode their natural resistance. It is likely that air pollution is sometimes the factor that tips the scales.

Even if we could prove that it is, what significance should we ascribe to this? Our experience of the extreme adjustments would indicate that they represent substantial losses on a human value system. All who knew the American chestnut regret its loss; the American elm is part of the New England heritage and much money has been spent in attempts to save it.

On the other hand, one could argue that if forest composition is now poorly adjusted to local climate, a factor that hastens the process of adjustment is "beneficial." Some forests in Pennsylvania, for example, are now heavily dominated by oaks: outbreaks of defoliating insects, followed by root fungus diseases, are killing many oaks and allowing other trees to grow in their place. It has been seriously argued that these forests have "too much oak" and that the insect-fungus combination is improving them by diversifying them. Likewise it could be argued that Iowa has "too much corn" and that its problems of insect pests and blights are a consequence of unwise monoculture. The individual farmer may gain more profit from corn than from mixed crops, but there is an argument that if all external costs were counted, a more diversified system might provide greater net benefits to society as a whole.

The conflict between values is apparent. In managed systems, such as agriculture and commercial forestry, we place primary value on marketable productivity; in unmanaged systems we place value on subjective characters which include diversity of species. We would like more diversity even in the managed systems if this could be reconciled with short-term economic goals. There is little direct evidence that low-level air pollution reduces the diversity that is valued in natural systems, but we have some sound reasons to believe that it may act in conjunction with other stress factors to do so.

Ian C. T. Nisbet, Associate Director of the Scientific Staff of the Massachusetts Audubon Society, holds a Ph.D. in physics from Cambridge University (1958.) He currently is working on population biology and chemical pollution.

Puzzle

Continued from p. 73

printed. Please add, "The distance traveled is v times this, or $L/[1 - \cos(2\pi/N)]$."

Speed Department Solutions

The following are the proposers' solutions to this month's Speed Department problems:

M/A SD1 4,000 tons. The only way the extra load could be carried to the aquaduct would be by an increase in the depth of the water due to the displacement of the boat. This would happen in a lock, but in a canal the displaced water flows away; the depth and the load remain the same.

M/A SD2 Neither. As the volumes are the same before and after, any water in the wine bottle must be exactly compensated for by wine in the water bottle.

Allan J. Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973) he is now Assistant Professor of Mathematics at York College of C.U.N.Y. Send problems, solutions, and comments to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y., 11432.